



PONTIFICIA UNIVERSIDAD CATÓLICA DE CHILE
ESCUELA DE INGENIERÍA

OPTIMAL CONTROL OF THE MODULAR MULTILEVEL MATRIX CONVERTER WITH DISTRIBUTED ENERGY STORAGE FOR GRID SUPPLY AND MACHINE DRIVES

JOSÉ IGNACIO GAJARDO ROJAS

Thesis submitted to the Office of Research and Graduate Studies
in partial fulfillment of the requirements for the degree of
Master of Science in Engineering

Advisor:
JAVIER PEREDA

Santiago de Chile, June 2024

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Gratefully to my family and friends

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TABLE OF CONTENTS

ACKNOWLEDGEMENTS	v
LIST OF FIGURES	ix
LIST OF TABLES	xi
ABSTRACT	xii
RESUMEN	xiv
1. INTRODUCTION	1
1.1. Modular Multilevel Cascaded Converters	3
1.2. Control of the M3C	5
1.2.1. Traditional Control Schemes	5
1.2.2. Advanced Control Schemes	8
1.3. Distributed Energy Storage Integration on MMCs	10
1.4. Hypothesis	12
1.5. Objectives	12
1.6. Contributions	13
1.7. Thesis Structure	13
2. THEORETICAL FRAMEWORK	14
2.1. Modeling of the M3C	14
2.2. The Decoupling Map	16
2.3. The Electrical Machine Equations	19
2.4. The state-space model	21
2.5. Power Flow Analysis	22
2.6. Instantaneous Power	26
2.7. Summary	31
3. PROPOSED OPTIMAL CONTROLLER	32

3.1. Inner Control Loop (CCS-MPC)	33
3.2. Outer Control Loop	35
3.2.1. Input Reference Generator	35
3.2.2. Speed Controller	36
3.2.3. Power Controller	36
3.2.4. Circulating Current Optimization	37
3.2.5. Common-Mode Voltage Optimization	40
3.2.6. Optimization Algorithm	41
3.3. Practical Issues	43
3.3.1. Flux Observer	43
3.3.2. Grid Observer	45
3.3.3. Power Reference	47
3.3.4. Steady-State Control Input Reference	49
3.4. Summary	51
4. SIMULATION AND EXPERIMENTAL RESULTS	52
4.1. Experimental Setup	52
4.2. Simulation Results	52
4.2.1. Current Controller	55
4.2.2. Machine Control	57
4.2.3. Optimal Power Control	57
4.3. Experimental Results	61
4.3.1. Current Controller	62
4.3.2. Machine Control	64
4.3.3. Optimal Power Control	64
4.3.4. Low Speed Operation	67
4.4. Summary	70
5. CONCLUSIONS AND FUTURE WORK	71
REFERENCES	74

APPENDIX	82
A. Solution to the predictive control problem	83
B. Glossary	85
B.1. Abbreviations	85
B.2. Set and Indexes	86
B.3. Variables	87
B.4. Vectors	88
B.5. Matrices	88

LIST OF FIGURES

1.1	High-power converters classification.	2
4figure.1.2		
1.3	Proposed topology (a) Modular Multilevel Matrix converter circuit diagram (b) Branch circuit with N number of submodules (SM) (c) Full-Bridge SM topology. (e) Respresentation of the distributed energy storage	11
2.1	Simplify representation of the M3C-DES (a) M3C circuit diagram (b) Simplify representation of one arm.	14
2.2	Representation of the decoupled map of the M3C using the Double- $\alpha\beta$ transformation.	19
2.3	Power flow through the M3C: (a) The depiction of power exchange between sub-converters at the output port (purple) and the power flow between clusters (green). (b) The portrayal of power interaction between sub-converters at the input port (purple) and the power transfer between clusters (green).	26
2.4	Representation of $\alpha\beta$ vectors. (a) shows the traditional Clarke transform, which allows obtaining a component in phase with a and its ninety-degree rotation. In (b) is shown the effect produced by the rotation of the vector abc	27
3.1	Overall Control Strategy	32
3.2	Reference design controller for the machine side: Mechanical Speed controller.	36
3.3	Two Steps Sequential Optimization Method	43
4.1	Experimental Setup.	53
4.2	Simulation: Current controller performance (a) Input current (b) Output current (c) Circulating current (d) Modulated voltage (e) Voltage spectrum	54

4.3	Simulation: Machine control performance (a) Machine current (b) Mechanical speed (c) Estimated Rotor flux (d) Electrical torque (e) Power in the branches	56
4.4	Simulation: Unbalanced power operation (a) Total power at the converter ports (b) Input current (c) Output current (d) Circulating current (e) Common-mode voltage (f) Power in the branches	59
4.5	Simulation: Unbalanced power operation (a) Circulating current (b) Common-mode voltage (c) Power in the branches	60
4.6	Experimental: Current controller performance (a) Input current (b) Output current (c) Circulating current (d) Common-mode voltage (e) Power in the branches	63
4.7	Experimental: Machine control performance (a) Machine current (d) Rotor flux (c) Mechanical speed	65
4.8	Experimental: Unbalanced power operation (a) Input current (b) Output current (c) Circulating current (d) Common-mode voltage (e) Power in the branches .	66
4.9	Experimental: Overall control strategy, considering low speed speed operation (Ch 1) Input current i_u (Ch 2) Output current i_a (Ch 3) Branch current i_{ua} (Ch 4) Branch current i_{wa} (Ch 5) Cluster voltage v_{ua} (Ch 6) Common-mode voltage v_{n0}	68

LIST OF TABLES

2.1	Feasible operating modes for the M3C-DES, where input power is positive (+) and output power is negative(-)	23
4.1	Converter Parameters.	53
4.2	Control Parameters	61

ABSTRACT

The integration of hybrid storage systems plays a fundamental role in harnessing renewable energy, electrifying processes, and strengthening the electrical grid. Power converters play a crucial role in enabling the integration of new technologies. In this context, cascade modular multilevel converters are the subject of recent studies due to their benefits in incorporating storage systems into the grid.

Highlights the modular multilevel matrix converter (M3C), which is gaining popularity due to its ability to offer the advantages of a multilevel converter and direct connection between two alternating current (AC) systems. Additionally, this converter facilitates the integration of a distributed energy storage system (DES) in its sub-modules, simplifying the management of energy sources. Despite its notable benefits, this specific application of the converter has not yet been proposed in the academic field.

This thesis proposes the integration of a DES through a control algorithm based on the power imbalance inter-cluster in the M3C. In the first phase, an analysis of equations outlined in recent research is conducted, followed by a mathematical development to deduce current equations and instantaneous power expressions. Finally, an optimal control method, based on an optimized power controller for the reference generator followed by a model predictive controller as the current controller, is formulated using the proposed system of equations.

The proposed control system was initially assessed through simulations and subsequently validated in an experimental environment. This validation encompasses power injection into the electrical grid and the operation of a variable-speed motor, all in conjunction with an inter-cluster power imbalance.

Keywords: Modular Multilevel Converter (MMC). Modular Multilevel Matrix Converter (M3C), Distributed Energy Storage System (DES), Model Predictive Control (MPC), Non-Linear Optimization.

RESUMEN

La integración de sistemas de almacenamiento híbridos desempeña un papel fundamental en el aprovechamiento de energías renovables, la electrificación de procesos y el fortalecimiento de la red eléctrica. Los convertidores de potencia juegan un papel crucial al posibilitar la integración de nuevas tecnologías. En este contexto, los convertidores modulares multinivel en cascada están siendo objeto de estudios recientes debido a sus beneficios en la incorporación de sistemas de almacenamiento a la red.

Destaca el convertidor modular multinivel matricial (M3C), que está ganando popularidad gracias a su capacidad para ofrecer las ventajas de un convertidor multinivel y la conexión directa entre dos redes de corriente alterna (AC). Además, este convertidor facilita la integración de un sistema de almacenamiento distribuido (DES) en sus sub-módulos, simplificando la gestión de las fuentes de energía. A pesar de sus notables beneficios, este tipo de aplicación específica del convertidor no ha alcanzado una amplia popularidad en el ámbito académico.

Esta tesis se enfoca en examinar la integración de un DES mediante un algoritmo de control fundamentado en el desequilibrio de potencia *inter-cluster* en el M3C. En una primera fase, se lleva a cabo un análisis de las ecuaciones delineadas en investigaciones recientes, seguido de un desarrollo matemático para deducir las ecuaciones de corriente y las expresiones de potencia instantánea. Por último, se formula un método de control óptimo basado en cada sistema de ecuaciones propuesto.

El sistema de control propuesto se evalúa inicialmente mediante simulaciones y posteriormente se valida en un entorno experimental. Esta validación incluye la inyección de potencia a la red eléctrica y la operación de un motor a velocidad variable, todo ello en conjunto con un desequilibrio de potencia *inter-cluster*.

Palabras Claves: Conversor Modular Multinivel (MMC), Conversor Modular Multinivel Matricial (M3C), Sistema de Almacenamiento Distribuido (DES), Control Predictivo por Modelo (MPC), Optimización No-Lineal.

1. INTRODUCTION

Renewable energy sources (RESs), including wind turbines, photovoltaic solar systems, solar-thermal power, biomass power plants and hydropower turbines, are poised to play an integral role in future power generation systems. However, the utilization of RESs, despite their substantial potential, can pose challenges due to their variable and intermittent nature. Energy storage systems offer a solution by mitigating the stochastic nature and abrupt fluctuations of RESs for brief periods. This helps prevent loss of load events and negates the necessity of initiating additional generating plants (Mohd, Ortjohann, Schmelter, Hamsic, & Morton, 2008).

In the realm of transport electrification, battery energy storage systems have nearly covered the needs of light-duty vehicles. Nevertheless, heavy-duty vehicles are still in the early stages of development, facing challenges related to energy demands too high for batteries. As a solution, integrating fuel cells into a distributed energy storage framework via various power converters has become imperative (Haxhiu, Abdelhakim, Kanerva, & Bogen, 2022). However, the use of multiple power converters can increase the losses and decrease the quality of current in the system.

Power electronics plays a pivotal role in modern society's electrification, primarily facilitated by power converters that efficiently convert electrical energy. Continuous exploration of new topologies and applications in this field has notably advanced the development and integration of modern industrial products, continually enhancing efficiency. While many established topologies persist in industrial use, such as older or classical designs, the emergence of modular multilevel cascaded converters presents an alternative with enormous potential for the future of power electronics (van Wyk & Lee, 2013).

Multilevel converters can operate at higher voltage levels and improve the quality of output voltage, thereby reducing the total harmonic distortion (THD) in the electrical current compared to conventional two-level converters. This feature diminishes the need for

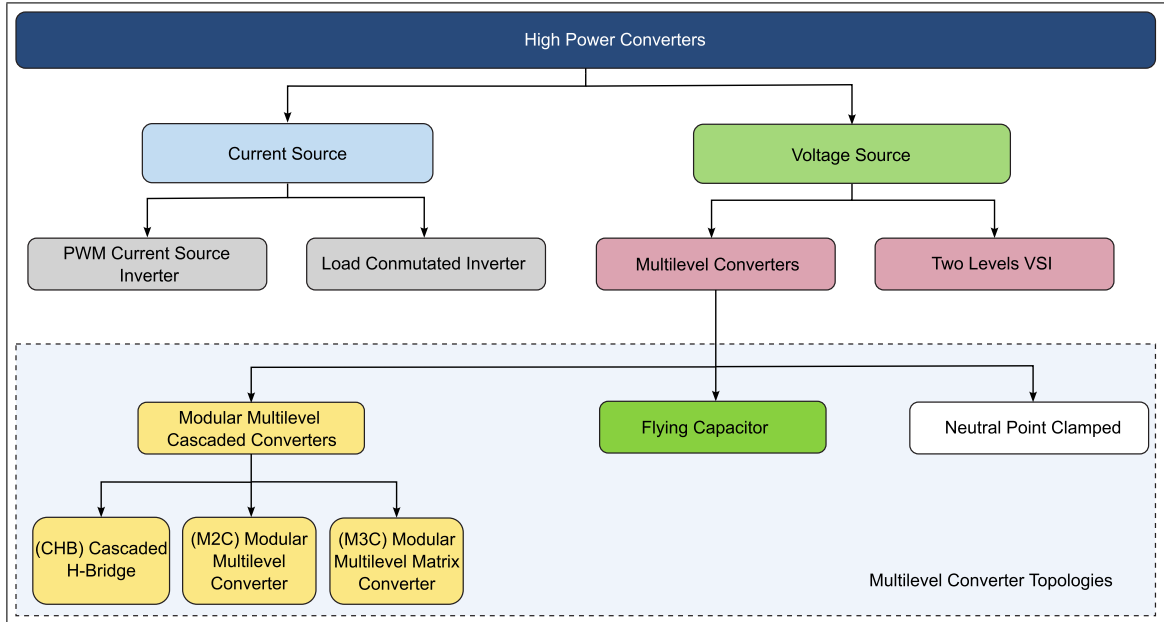


Figure 1.1. High-power converters classification.

larger filtering inductors, making multilevel converters extremely attractive to various industries. Currently, researchers worldwide are investing significant efforts into enhancing the performance of multilevel converters (Franquelo et al., 2008).

The common topologies for multilevel converters include the neutral point clamped converter (NPC), flying capacitor converter, and modular multilevel cascaded converters (MMCs), as shown in Figure 1.1. Currently, multilevel inverters are extensively utilized in high-power applications within medium voltage levels (Rodriguez, Lai, & Peng, 2002). These converter designs have led to the creation of several patents, including one featuring a MMC integrated with energy storage (Towsend, Zelaya de la Parra, Hasle, & Tengner, European Patent 3251205, Jul. 2015). To address the integration of energy storage within power systems, this work proposes the utilization of modular multilevel cascaded converters combined with distributed energy storage sources. This approach aims to consolidate the advantages of a hybrid storage system while harnessing the inherent benefits offered by these specific converters.

1.1. Modular Multilevel Cascaded Converters

The modular multilevel cascaded converter stands out for its numerous advantages when compared to traditional power converters. One of its key strengths lies in its inherent modularity, which grants it a remarkable fault tolerance capability owing to the presence of a high level of redundancy within its structure. This redundancy allows the converter to continue functioning even in the event of certain module failures or faults, ensuring enhanced reliability and uptime in power systems. Moreover, the utilization of these voltage levels may lead to higher efficiency compared to other topologies (Franquelo et al., 2008). This increased efficiency not only contributes to reduced energy losses but also renders MMCs highly suitable for applications demanding high-power operations at medium voltage levels.

There are three main topologies of MMCs: the Cascaded H-Bridge, the Modular Multilevel Converter, and the Modular Multilevel Matrix Converter, as illustrated in Figure 1.1. Each of these topologies comprises a specific number of branches, which consist of an inductor and a cluster with multiple sub-modules (SM) connected in series. The quantity of branches is contingent upon the type of energy conversion required, whether it involves DC/AC or AC/AC, as depicted in Figure 1.2. Commonly utilized SMs include the half-bridge (HB) and the full bridge (FB). HB cells are capable of producing solely zero and positive voltages, resulting in an inherent DC component in the cluster voltage; therefore, this type of SM is exclusively used in the M2C. In contrast, the FB can function across various converters, enabling its utilization in both AC or DC connections (Perez, Bernet, Rodriguez, Kouro, & Lizana, 2015).

The modular multilevel cascaded converters offer distinct advantages such as scalability, exceptional waveform quality, and reduced filtering component sizes. These advantages have propelled the development of MMCs into commercial products in recent years. The Cascaded H-Bridge (CHB) finds typical applications in battery energy storage systems (BESS) (Kawakami et al., 2014a), photovoltaic grid integration (PV) (Yu,

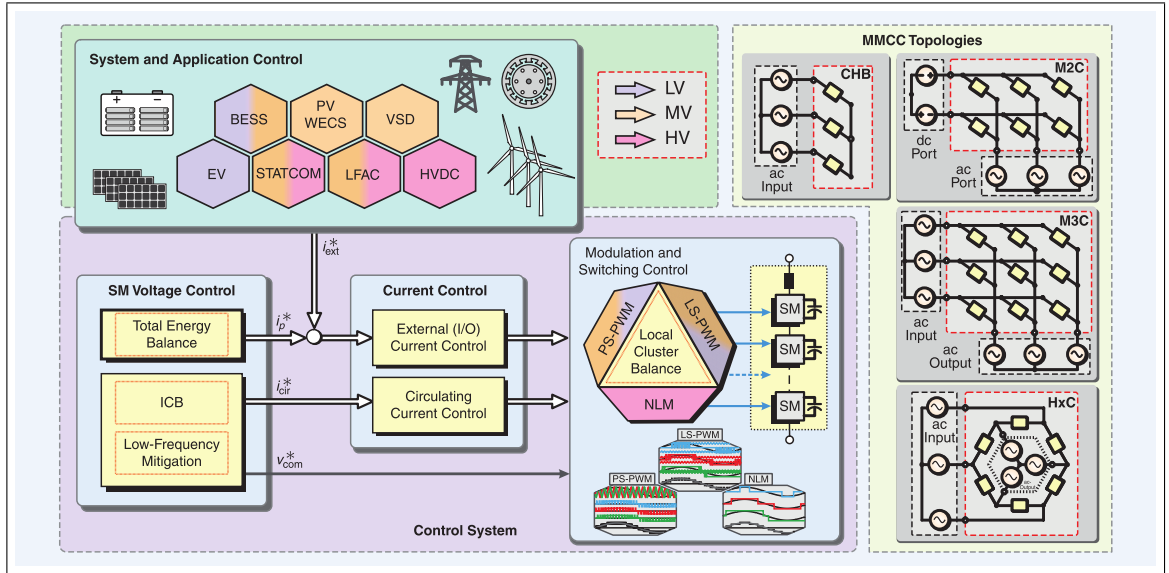


Figure 1.2. Control system for MMCCs: At the top left-hand side, there are some examples of applications. Meanwhile, on the right-hand side are the circuit topologies for traditional MMCs, and at the bottom, the traditional control objective for MMCCs is presented (Burgos-Mellado et al., 2023).

Konstantinou, Townsend, Aguilera, & Agelidis, 2017a), and Static Synchronous Compensators (STATCOM) (Farivar, Townsend, Hredzak, Pou, & Agelidis, 2017). Similarly, the Modular Multilevel Converter (M2C) has garnered significant attention for High Voltage Direct Current (HVDC) applications (Perez et al., 2015). However, AC/AC conversion is an area still under extensive study and is primarily dominated by back-to-back power converters. Additionally, direct AC/AC conversion is largely governed by cycloconverters (Kumar & Zare, 2019), resulting in a high injection of current harmonics into the grid. In this context, this study aims to explore the application of the Modular Multilevel Matrix Converter (M3C) for direct AC/AC power conversion, specifically for motor drives and grid feeding, forming or supporting.

Shown in Figure 1.2, the modular multilevel matrix converter (M3C) or Triple Star Cells Bridge (TSCB) comprises nine branches, forming three sub-converters interconnected in a star configuration. Initially introduced in 2001 without the inductor (Erickson

& Al-Naseem, 2001), it wasn't until 2009 that it was presented with the branch inductor, enabling the nine branches to function as a controlled current source (Oates, 2009). M3C's capability of direct AC-to-AC conversion makes them well-suited for various applications, including power system interconnections (50-60 Hz), wind energy conversion systems (WECS), flexible AC transmission systems (FACTS), low-frequency AC transmission systems (LFAC), and motor drives (Diaz et al., 2020).

Despite showcasing numerous benefits, the most significant drawback of MMCs is their high cost. The utilization of a substantial quantity of semiconductors, passive components, and a considerable array of control, measurement, and protection components makes their implementation on an industrial scale challenging. To render them more feasible, ongoing research focuses on developing control systems capable of efficiently and securely operating these converters. However, in the case of the M3C, it presents intricately coupled internal states and power oscillations at specific frequencies.

1.2. Control of the M3C

Control schemes for power converters and drives have constantly been evolving according to the development of new power devices. In the case of the M3Cs, various methods employing classical control theory have been proposed. Additionally, there has been evidence of the emergence of advanced control strategies in recent years.

1.2.1. Traditional Control Schemes

The M3C is commonly utilized with floating capacitors in its sub-modules. Consequently, the primary control objectives encompass both input and output currents, along with ensuring capacitor voltage balancing across all operational points. The most prevalent control systems are structured with proportional-integral (PI) controllers, overseeing a decoupled representation of branch currents. This approach involves employing the Clarke

and Sigma-Delta ($\Sigma\Delta$) transform and was initially introduced in 2012 (Kammerer, Kolb, & Braun, 2012).

The subsequent advancements adopted a similar approach, leading to experimental results (Kawamura, Hagiwara, & Akagi, 2014). Within the same study, the power components were established in the $\alpha\beta$ framework, thereby defining the circulating currents. The control of these additional degrees of freedom was achieved through the utilization of a proportional controller. Furthermore, a definition was formulated for the zero sequence or common-mode voltage. This was aimed at producing adequate power to balance the capacitor voltage under low currents on the machine side.

Traditionally, among the control task to be performed, the SM capacitor voltage regulation is fundamental to achieve the a high-performance current control. Usually, the SM capacitor-voltages is performed by implementing (R. Cuzmar, Montenegro, Mora, Pereda, & Aguilera, 2023):

- (i) Total Energy Balancing (TEB) Control: Regulate the energy stored in all capacitors to the desired level by transferring power to or from the ac ports.
- (ii) Inter-cluster Balancing (ICB) Control. Distributes the power of the M3C among all its clusters, determining the imposition of internal command signals for the CMV and the circulating currents, without affecting the ac ports. However, the ICB should generate minimal circulating currents to increase the efficiency and power density of the converter.
- (iii) Low-Frequency Oscillations (LFO) Mitigation. Mitigates large oscillations in SM capacitor voltages by injecting CMV and circulating currents with high harmonic. It is required only at the critical points of operation, when the M3C operates at a very low frequency, but most acutely when these frequencies approach or equal
- (iv) Local-Cluster Balancing (LCB) Control. Locally regulate the capacitor voltage of each SM in each cluster. It is usually implemented at the modulation stage.

The control schemes proposed in the literature typically address the TEB and ICB objectives. While there are some works that aim to tackle the LFO objective, they are relatively few in number.

Since the publications of Kawamura et al. (2014), several new methodologies have been introduced. Particularly noteworthy are certain studies focused on wind energy conversion (Diaz et al., 2017), enabling the integration of external control loops into classical control systems. Additionally, achieving a fully operational frequency range has been made possible through vector control techniques (Diaz et al., 2019). The latter involves the utilization of the $\Sigma\Delta$ framework, which was initially introduced in 2012. This framework aims to attain a geometric orientation of the four circulating currents (Kammerer, Gommeringer, Kolb, & Braun, 2014).

The conventional control approach for the outer loop or voltage balancing involves using PI controllers within a centralized controller. However, in certain cases, the internal loop or current controller is separately implemented for each branch. This is accomplished by employing nine proportional-resonant (PR) controllers (Wang et al., 2019). The inception of the first PR controller was in 2003, and it can be interpreted as a PI controller when the natural frequency of its transfer function aligns with the frequency of the controllable variable (Zmood & Holmes, 2003).

Taking into account the benefits and modularity associated with the traditional approach, it is essential to note that using multiple single-input-single-output (SISO) controllers for a complex system, surpassing the number of variables over degrees of freedom, diminishes control robustness compared to a multiple-inputs-multiple-outputs (MIMO) controller. Additionally, linear controllers, tailored for specific operating conditions, result in reduced closed-loop performance when the converter operates significantly away from the desired point (Lim, Levi, Jones, Rahim, & Hew, 2014). Therefore, the advancement of sophisticated control strategies might be necessary.

1.2.2. Advanced Control Schemes

As mentioned earlier, the primary control objectives for the M3C include current tracking and capacitor voltage balancing. Advanced control strategies have been proposed in literature to address these objectives separately, with the predominant method being model predictive control (MPC). MPC strategies have been a focal point of research due to their simplicity in implementation within a multivariable system, exhibiting rapid dynamics, and allowing incorporation of nonlinearities and constraints into the control law. However, MPC comes with a heavier computational burden compared to other control strategies (Vazquez, Rodriguez, Rivera, Franquelo, & Norambuena, 2017). Given the substantial number of variables in the M3C, specific techniques have been enhanced to facilitate the implementation of MPC in this converter.

Nowadays, Finite Control Set Model Predictive Control (FCS-MPC) is among the most widely used control methodologies in the academy. However, its implementation requires evaluating all potential input states of the converter. In the case of the M3C, the total number of feasible voltages amounts to 3^{9N} , making it unfeasible for practical system implementation. To reduce the computational burden, Sequential Finite Control Set methods have been proposed, as highlighted in (Mora, Espinoza, Díaz, & Cárdenas, 2015). This method updates only one sub-module per branch and separately controls the current of each branch, thereby reducing the total number of combinations to $3 \times 9 = 27$ per sample time. Subsequent advancements in this area have utilized a long-horizon predictive control approach, leading to experimental results (R. H. Cuzmar, Mora, Pereda, Poblete, & Aguilera, 2023).

To mitigate the computational expenses associated with FCS-MPC, recent research suggests retaining the modulation stage while regulating the circulating current using Continuous Control Set Model Predictive Control (CCS-MPC). Consequently, the four control loops dedicated to circulating current tracking are substituted with a single controller. This controller conducts a singular optimization process, incorporating box constraints to adjust the manipulated variable. This adjustment aims to attain the tracking of the four

circulating currents (Urrutia, Cardenas, Clare, & Watson, 2021). Similarly, (R. H. Cuzmar, Mora, Pereda, Aguilera, et al., 2023) proposes a computationally efficient MPC for modular multilevel matrix converter, presenting simulation and experimental results.

While this study focuses on an energy storage approach, it's crucial to examine various energy balancing methods employing advanced control strategies. Among these, the widely used approach involves CCS-MPC structured around the energy management of floating capacitors (Urrutia, Cárdenas, Clare, Díaz, & Watson, 2022). Experimental results highlight the effectiveness of this proposition. Subsequently, a similar method is explored in another work (R. Cuzmar et al., 2023), incorporating the common-mode voltage (CMV) in the equation, addressing the ICB and LFO control objectives. Furthermore, this approach integrates box constraints to establish a circulating current reference within safe operation limits for the converter.

Reducing the computational burden of the control scheme is one of the primary goals for researchers. In this context, a distributed control strategy emerges as a potential solution. A comprehensive review and categorization of this control strategy are presented in (Burgos-Mellado et al., 2023). Building upon this classification, a decentralized control strategy is proposed in (Yao, Liu, & Lu, 2019). Here, a central controller handles inter-cluster voltage balancing, while local controllers manage intra-cluster balancing and current control.

The aforementioned control strategies predominantly center around the management of floating capacitors, with limited studies focusing on M3C with energy storage. Integrating distributed energy storage necessitates modifications either to the power converter or the control strategy. The subsequent section will delve into the integration of energy storage, providing a brief review of modular multilevel cascaded converters incorporating hybrid energy storage.

1.3. Distributed Energy Storage Integration on MMCs

Only a few studies have been published utilizing the Modular Multilevel Matrix Converter for energy storage integration (Das et al., 2020). In this case, a modified converter incorporates a dual active bridges (DAB) connected to the H-bridges, facilitating the incorporation of a battery energy storage system (BESS). This configuration adopts the conventional energy balancing scheme to regulate the capacitor voltage at the DAB output. However, unlike the popularity of the CHB and M2C, the M3C has not gained significant traction in the integration of distributed energy storage. Nevertheless, similar control techniques and converter modifications could be applied with adaptations to accommodate M3C for a distributed energy storage integration. In this work, no modifications were made to the traditional M3C. Therefore, the review will focus on control strategies.

The CHB converter has been extensively investigated for its integration into photovoltaic (PV) systems. However, being the simplest multilevel cascaded converter, its limited degrees of freedom only permit the injection of CMV in the star connection and circulating currents in the delta connection (Yu, Konstantinou, Townsend, Aguilera, & Agelidis, 2017b; Yu, Konstantinou, Hredzak, & Agelidis, 2016). This approach has undergone thorough examination concerning control systems and technical constraints for various connections of the converter. An advanced control strategy was introduced to enable fast dynamics during unbalanced operations (Aguilera et al., 2017; R. H. Cuzmar, Pereda, & Aguilera, 2021). In this context, alternative methods utilizing an instantaneous reference generator employing the Clarke transformation have been proposed, simplifying the computation of the zero-sequence voltage (Aguilera, Acuna, Rojas, Konstantinou, & Pou, 2019).

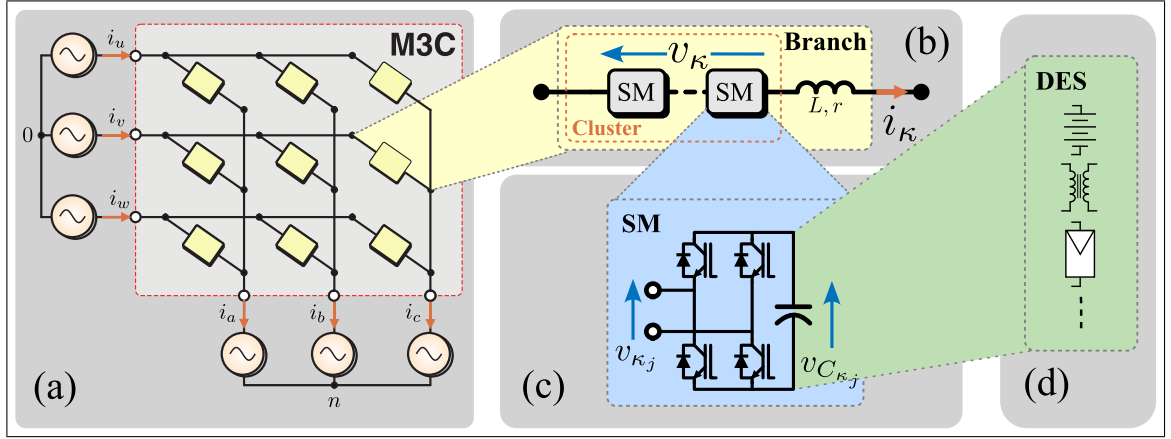


Figure 1.3. Proposed topology (a) Modular Multilevel Matrix converter circuit diagram (b) Branch circuit with N number of submodules (SM) (c) Full-Bridge SM topology. (e) Representation of the distributed energy storage

The study of the CHB was not solely limited to PV systems. In its initial stages, various works explored the integration of battery energy storage. Certain traditional control schemes were employed to generate a CMV reference for state of charge (SOC) balancing in a 500 kW real prototype (Kawakami et al., 2014b). Presently, advanced control strategies, such as model predictive control for SOC balancing, have been proposed. These advancements have resulted in experimental results within a single-phase CHB converter, enabling the incorporation of constraints into the optimization problem (Liang et al., 2023).

Conversely, the M2C has been under investigation for energy storage integration, aiming to function as a multiport power converter. An extensive examination of the Modular Multilevel Converter (MMC) with distributed energy storage was conducted in (Salvadores, 2023). Aligning with the CHB's principles, achieving unbalanced power operation between branches is the objective. However, owing to the inclusion of a third port, the predominant focus in research proposals has been on average power control utilizing the filtered components of the branch power (Münch, Görges, Izák, & Liu, 2010).

Due to the highly coupled states and power oscillations in the M3C, employing digital filters complicates the implementation of a similar approach. In this context, this work aims to investigate unbalanced power operation without relying on digital filters within the Modular Multilevel Matrix Converter with distributed energy storage (M3C-DES), whose topology is illustrated in Fig. 1.3. Furthermore, to demonstrate effectiveness across a wide range of frequencies, an electrical machine is powered at the output port.

1.4. Hypothesis

The following hypothesis is stated for this research project:

- H An optimal control strategy, based on the theory of instantaneous power and complemented by an internal predictive current control, effectively governs a modular multilevel matrix converter with distributed energy storage (M3C-DES). This control scheme manages both network supply and machine drive operations, generating optimal references for circulating currents and common mode voltage, and showing robust performance, particularly during unbalanced power operation scenarios.

1.5. Objectives

This MSc. research project aims to investigate the operation of the M3C-DES. The research described in this thesis pursues to accomplish the following objectives:

- O_1 To develop the model of the M3C-DES, considering the electrical machine model on the output current.
- O_2 To propose an internal control loop, based on model predictive control methods, capable of control the input, output and circulating currents.
- O_3 To design an external control loop, based on the instantaneous power theory, capable of generate an optimal combination of common-mode voltage and circulating current for intra-cluster power imbalance

1.6. Contributions

The contributions of this work can be summarised as follows:

- C_1 An internal control loop based on Continuous Control Set Model Predictive Controller (CCS-MPC), including the electrical machine equations in the output current dynamic equations.
- C_2 An external controller based on a optimization algorithm capable to achieve imbalanced power under different operating point injecting circulating currents and common-mode voltage.
- C_3 Experimental validation of the proposed control scheme is provided, including the imbalance power in the transient and the steady state of the system, under variable speed operation of the electrical machine.

1.7. Thesis Structure

Chapter 1 introduces the concept of an M3C-DES, along with state-of-the-art control schemes for both the M3C and energy storage integration.

Chapter 2 outlines the comprehensive theoretical framework and derives key equations necessary for understanding the proposed controller. Additionally, a power flow analysis is conducted to comprehend the operational aspects of the M3C-DES.

Chapter 3 proposes the control scheme employed to attain the targeted operational points. Furthermore, it delves into specific implementation issues related to the scheme.

Chapter 4 presents the experimental and simulated results that serve as validation for the implementation of the control scheme.

Chapter 5 presents the conclusions of the thesis.

2. THEORETICAL FRAMEWORK

In this Chapter, the M3C-DES dynamic model is introduced. Building upon prior research (Kammerer et al., 2012; Kawamura et al., 2014), the Double- $\alpha\beta 0$ framework is employed to represent the decoupled dynamics of the converter. Furthermore, an examination of the internal power flow within the M3C-DES is provided, leveraging circulating currents and common-mode voltage to attain unbalanced power operation. Lastly, an induction machine model is incorporated to enable direct operation of the electrical machine with the internal controller.

The primary assumptions for this section are that the voltage of each submodule is fixed, meaning that external power conversion in the SMs DC-link will not be considered or analyzed. Finally, a balanced grid is assumed, so an unbalanced grid connection will not be taken into account.

2.1. Modeling of the M3C

The Modular Multilevel Matrix Converter with Distributed Energy Storage (M3C-DES) is composed of nine arms with N number of sub-modules per cluster (Fig. 1.3). A simplified version of the M3C-DES is presented in Fig. 2.1, where the voltage source of each arm represent the cluster voltage (v_κ) and i_κ is the branch current.

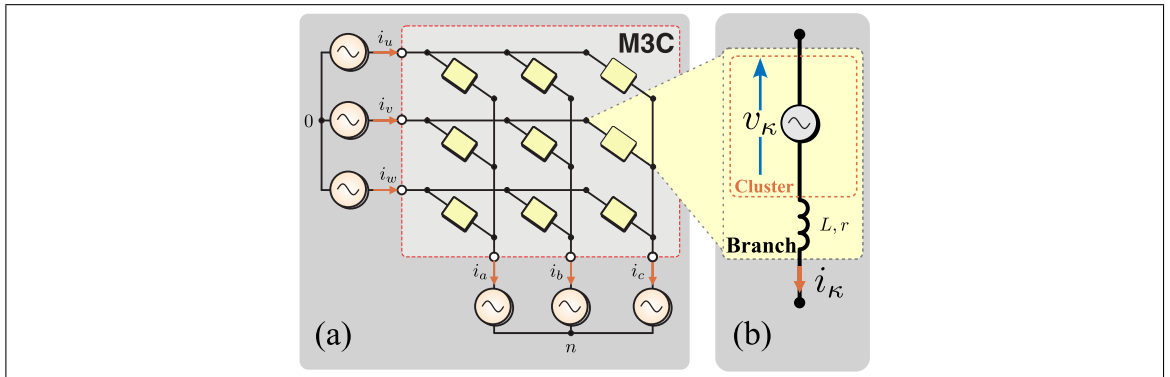


Figure 2.1. Simplify representation of the M3C-DES (a) M3C circuit diagram (b) Simplify representation of one arm.

To introduce the voltage-current model of the M3C, let's consider the set of nine branches $\kappa \in \mathcal{K} = \{1 \dots 9\}$, a set of three phases (or nodes) at the input-side of the converter, $\phi \in \Phi = \{a, b, c\}$, and another set of three nodes at the output-side, $\chi \in \mathcal{X} = \{u, v, w\}$. These statements lead us to the following definitions.

$$\mathbf{v}_\Phi = \begin{bmatrix} v_u & v_v & v_w \end{bmatrix}^T \quad (2.1a)$$

$$\mathbf{v}_\mathcal{X} = \begin{bmatrix} v_a & v_b & v_c \end{bmatrix}^T \quad (2.1b)$$

$$\mathbf{v}_\mathcal{K} = \begin{bmatrix} v_1 & \dots & v_9 \end{bmatrix}^T \quad (2.1c)$$

The vectors $\mathbf{v}_\Phi$ and $\mathbf{v}_\mathcal{X}$ are the input and output voltage of the converter. Meanwhile, $\mathbf{v}_\mathcal{K}$ is the vector of the nine cluster voltages, synthesized by:

$$v_\kappa = \sum_{j=1}^N v_{C_{\kappa_j}} u_\kappa \quad (2.2)$$

where, $v_{C_{\kappa_j}}$ is the DC-link voltage of the sub-module j in the cluster κ whose value is fixed in v_{dc} and $u_\kappa \in [-1 \ 1]$ is the manipulate variable for the average model.

Considering that the M3C is connected between two three-phase ports (Fig. 2.1). Applying Kirchhoff's Voltage Law (KVL), the continuous-time dynamic model of each cluster current i_κ , can be expressed via:

$$\mathbf{P}^T \begin{bmatrix} \mathbf{v}_\Phi \\ \mathbf{v}_\mathcal{X} \end{bmatrix} = L \frac{d\mathbf{i}_\mathcal{K}}{dt} + r\mathbf{i}_\mathcal{K} + \mathbf{v}_\mathcal{K} + \mathbb{1}_{9 \times 1} v_{n0} \quad (2.3)$$

Here L and r are the inductance and resistance of the arms in the converter, respectively, which are assumed equal for all the branches; v_{n0} is common-mode voltage. Moreover, $\mathbf{P}$ is the incidence matrix who give us the relationship of the cluster currents and the input ($\mathbf{i}_\Phi$) and output ($\mathbf{i}_\mathcal{X}$) current ports.

$$\begin{bmatrix} \mathbf{i}_\Phi \\ -\mathbf{i}_X \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 \end{bmatrix} \mathbf{i}_\mathcal{K} \quad (2.4)$$

$\underbrace{\hspace{15em}}_{:=P}$

where,

$$\mathbf{i}_\Phi = \begin{bmatrix} i_u & i_v & i_w \end{bmatrix}^T \quad (2.5a)$$

$$\mathbf{i}_X = \begin{bmatrix} i_a & i_b & i_c \end{bmatrix}^T \quad (2.5b)$$

$$\mathbf{i}_\mathcal{K} = \begin{bmatrix} i_1 & \dots & i_9 \end{bmatrix}^T \quad (2.5c)$$

The cluster currents i_κ comprises three elements: the input side (ϕ), the output side (χ), and the circulating current (z).

$$i_\kappa = \frac{1}{3}(i_{\kappa\phi} + i_{\kappa\chi}) + i_{\kappa z} \quad (2.6)$$

2.2. The Decoupling Map

To simplify the representation in (2.3), a decoupled map for the M3C (Diaz et al., 2017) can be obtained. For this purpose, the Double- $\alpha\beta 0$ transformation is applied.

The Clarke transform has several benefits in three-phase or single-phase system applications. However, when there is more than one frequency it is necessary to extend the definition of the $\alpha\beta 0$ framework, given origin to the Double- $\alpha\beta 0$ which was firstly used by (Kawamura et al., 2014) and the definition in this work is given by:

$$\mathbf{D}'_{\alpha\beta 0} = \begin{bmatrix} \frac{\sqrt{2}}{3} & -\frac{\sqrt{2}}{6} & -\frac{\sqrt{2}}{6} & \frac{\sqrt{2}}{3} & -\frac{\sqrt{2}}{6} & -\frac{\sqrt{2}}{6} & \frac{\sqrt{2}}{3} & -\frac{\sqrt{2}}{6} & -\frac{\sqrt{2}}{6} \\ 0 & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & 0 & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & 0 & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} \\ \frac{\sqrt{2}}{3} & \frac{\sqrt{2}}{3} & \frac{\sqrt{2}}{3} & -\frac{\sqrt{2}}{6} & -\frac{\sqrt{2}}{6} & -\frac{\sqrt{2}}{6} & -\frac{\sqrt{2}}{6} & -\frac{\sqrt{2}}{6} & -\frac{\sqrt{2}}{6} \\ 0 & 0 & 0 & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} \\ \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & \frac{1}{6} & \frac{1}{6} & -\frac{1}{3} & \frac{1}{6} & \frac{1}{6} \\ 0 & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & 0 & -\frac{1}{2\sqrt{3}} & \frac{1}{2\sqrt{3}} & 0 & -\frac{1}{2\sqrt{3}} & \frac{1}{2\sqrt{3}} \\ 0 & 0 & 0 & \frac{1}{\sqrt{3}} & -\frac{1}{2\sqrt{3}} & -\frac{1}{2\sqrt{3}} & -\frac{1}{2\sqrt{3}} & \frac{1}{2\sqrt{3}} & \frac{1}{2\sqrt{3}} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & -\frac{1}{2} & 0 & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix} \in \mathbb{R}^{9 \times 9} \quad (2.7)$$

This linear transformation maps a nine-phase system with two coupled frequencies (Figure 2.1). For example, transforming $\mathbf{i}_{\mathcal{K}} = [i_1 \dots i_9]^T$ into a decoupled Double- $\alpha\beta 0$ reference frame, the following relationship is obtained.

$$\mathbf{i}_{\alpha\beta 0}^{\alpha\beta} = \begin{bmatrix} \mathbf{i}_{\alpha\beta}^0 \\ \mathbf{i}_0^{\alpha\beta} \\ \mathbf{i}_{\alpha\beta}^{\alpha\beta} \\ i_0^0 \end{bmatrix} = \mathbf{D}'_{\alpha\beta 0} \mathbf{i}_{\mathcal{K}} \quad (2.8a)$$

$$\mathbf{i}_0^{\alpha\beta} = \begin{bmatrix} i_0^{\alpha} & i_0^{\beta} \end{bmatrix}^T \quad (2.8b)$$

$$\mathbf{i}_{\alpha\beta}^0 = \begin{bmatrix} i_{\alpha}^0 & i_{\beta}^0 \end{bmatrix}^T \quad (2.8c)$$

$$\mathbf{i}_{\alpha\beta}^{\alpha\beta} = \begin{bmatrix} i_{\alpha}^{\alpha} & i_{\beta}^{\alpha} & i_{\alpha}^{\beta} & i_{\beta}^{\beta} \end{bmatrix}^T \quad (2.8d)$$

where, $\mathbf{i}_{\alpha\beta}^0$ and $\mathbf{i}_0^{\alpha\beta}$ represent the components from the input and output ports, respectively. The vector $\mathbf{i}_{\alpha\beta}^{\alpha\beta}$ maintains the double frequency composition, referred to as circulating currents, and i_0^0 is the zero-sequence current, which equation can be omitted by removing the last row of $\mathbf{D}'_{\alpha\beta 0}$ and the resulting matrix is denoted as $\mathbf{D}_{\alpha\beta 0} \in \mathbb{R}^{8 \times 9}$.

Applying (2.7) to (2.3) results in the following M3C decoupled model:

$$L \frac{d}{dt} \mathbf{i}_{\alpha\beta 0}^{\alpha\beta} = \sqrt{3} \mathbf{P}_{\alpha\beta 0} \begin{bmatrix} \mathbf{v}_{\phi}^{\alpha\beta 0} \\ \mathbf{v}_{\chi}^{\alpha\beta 0} \end{bmatrix} - r \mathbf{i}_{\alpha\beta 0}^{\alpha\beta} - \mathbf{v}_{\alpha\beta 0}^{\alpha\beta} - 3 \begin{bmatrix} \mathbb{0}_{8 \times 1} \\ v_{n0} \end{bmatrix} \quad (2.9)$$

where,

$$\mathbf{P}_{\alpha\beta 0} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & -1 \end{bmatrix} \in \mathbb{R}^{9 \times 6} \quad (2.10)$$

The input side current is depicted by $\mathbf{i}_{\alpha\beta}^0$, similarly to the output side $\mathbf{i}_0^{\alpha\beta}$. Conversely, the circulating current is represented by $\mathbf{i}_{\alpha\beta}^{\alpha\beta}$. Additionally, the vector $\mathbf{v}_{\phi}^{\alpha\beta}$ and $\mathbf{v}_{\chi}^{\alpha\beta}$ are the input and output grid voltages in the $\alpha\beta 0$ framework. Moreover, the control voltage vector in the Double- $\alpha\beta 0$ framework is given by:

$$\mathbf{v}_{\alpha\beta 0}^{\alpha\beta 0} = \begin{bmatrix} v_0^{\alpha} & v_0^{\beta} & v_{\alpha}^0 & v_{\beta}^0 & v_{\alpha}^{\alpha} & v_{\beta}^{\alpha} & v_{\alpha}^{\beta} & v_{\beta}^{\beta} & v_0^0 \end{bmatrix}^T \quad (2.11)$$

Finally, under the assumption of a balanced grid, where $v_{\phi}^0 = v_{\chi}^0 = 0$, and considering that the neutral points are not interconnected, leading to $i_0^0 = 0$, it follows that:

$$v_0^0 = -3v_{n0} \quad (2.12)$$

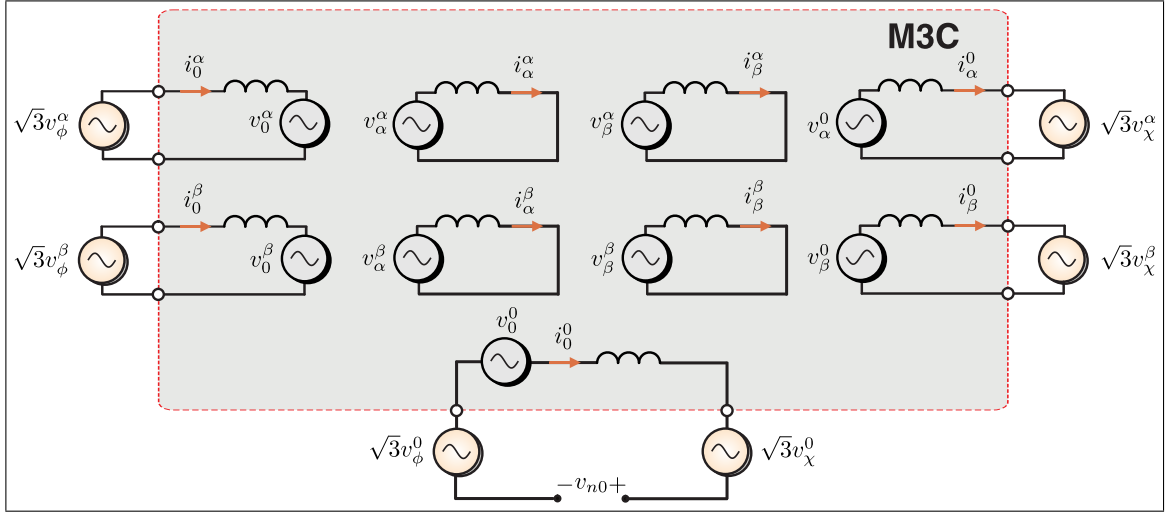


Figure 2.2. Representation of the decoupled map of the M3C using the Double- $\alpha\beta 0$ transformation.

As it was mentioned before (2.9) are the decoupled representation of the nine equation represented by (2.3), and the graphic representation can be found on Fig. 2.2. Applying Kirchhoff Current Law (KCL) (Kawamura et al., 2014), the following relationships are obtained:

$$\mathbf{i}_{\phi}^{\alpha\beta} = \sqrt{3}\mathbf{i}_0^{\alpha\beta} \quad \mathbf{i}_{\chi}^{\alpha\beta} = \sqrt{3}\mathbf{i}_{\alpha\beta}^0 \quad (2.13)$$

where, $\mathbf{i}_{\phi}^{\alpha\beta}$ and $\mathbf{i}_{\chi}^{\alpha\beta}$ are the input and output current in the $\alpha\beta$ framework.

2.3. The Electrical Machine Equations

For specific applications, certain modifications to (2.9) may be necessary. In this study, we propose the utilization of DES-M3C for motor drives and grid feeding.

In order to develop an appropriate control strategy, it's necessary to include the equations of the electrical machine at the output side of the converter. The stator current $\mathbf{i}_s$

and the rotor flux ψ_r in the $\alpha\beta$ are selected as state variables, then the equivalent equations of the stator and rotor dynamics of a squirrel-cage induction machine are obtained (Rodriguez & Cortes, 2012):

$$\mathbf{i}_s + \tau_\sigma \frac{d}{dt} \mathbf{i}_s = -j\omega_k \tau_\sigma \mathbf{i}_s + \frac{k_r}{R_\sigma} \left(\frac{1}{\tau_r} - j\omega \right) \psi_r + \frac{\mathbf{v}_s}{R_\sigma} \quad (2.14)$$

$$\psi_r + \tau_r \frac{d}{dt} \psi_r = -j(\omega_k - \omega) \tau_r \psi_r + L_m \mathbf{i}_s \quad (2.15)$$

where,

$$\begin{aligned} \omega &= \rho \omega_m \\ \tau_s &= \frac{L_s}{R_s} \\ \tau_r &= \frac{L_r}{R_r} \\ \sigma &= 1 - \frac{L_m^2}{L_s L_r} \\ k_r &= \frac{L_m}{L_r} \\ k_s &= \frac{L_m}{L_s} \\ R_\sigma &= R_s + R_r k_r^2 \\ \tau_\sigma &= \frac{\sigma L_s}{R_\sigma} \end{aligned} \quad (2.16)$$

To effectively operate the induction machine, the power converter must have the capability to regulate the stator current $\mathbf{i}_s$. Given that the electrical machine is connected to the output port, it is important to note that $\mathbf{i}_s = \sqrt{3} \mathbf{i}_{\alpha\beta}^0$. Additionally, from this point onwards, the rotor flux ψ_r will be referred to as $\psi_r^{\alpha\beta}$. Finally, using (2.13) and assuming the stator as reference frame, the equations (2.14), (2.15) can be rewritten as:

$$\sqrt{3}\sigma L_s \frac{d}{dt} \mathbf{i}_{\alpha\beta}^0 = -\sqrt{3}R_\sigma \mathbf{i}_{\alpha\beta}^0 + k_r \left(\frac{1}{\tau_r} \mathbb{I}_{2 \times 2} - \mathbb{J}\omega \right) \boldsymbol{\psi}_r^{\alpha\beta} + \mathbf{v}_\chi^{\alpha\beta} \quad (2.17)$$

$$\frac{d}{dt} \boldsymbol{\psi}_r^{\alpha\beta} = \sqrt{3} \frac{L_m}{\tau_r} \mathbf{i}_{\alpha\beta}^0 - \left(\frac{1}{\tau_r} \mathbb{I}_{2 \times 2} - \mathbb{J}\omega \right) \boldsymbol{\psi}_r^{\alpha\beta} \quad (2.18)$$

where, $\mathbb{J}$ is the imaginary unity on his matrix form or the rotation matrix and $\mathbf{v}_\chi^{\alpha\beta} = \mathbf{v}_s$. Then, combining dynamic equation of the output side defined in (2.9) and (2.17) an expression of the output current is obtained:

$$\frac{d}{dt} \mathbf{i}_{\alpha\beta}^0 = -\frac{1}{L_{out}} \left(\mathbf{v}_{\alpha\beta}^0 + r_{out} \mathbf{i}_{\alpha\beta}^0 + \sqrt{3}k_r \left(\mathbb{J}\omega - \frac{1}{\tau_r} \mathbb{I}_{2 \times 2} \right) \boldsymbol{\psi}_r^{\alpha\beta} \right) \quad (2.19)$$

where, $L_{out} = L + 3\sigma L_s$ and $r_{out} = r + 3R_\sigma$. Similarly, assuming the use of a inductor (L_g) in the grid connection, the dynamic of the inductive load can be directly considered in the model:

$$L_{in} \frac{d}{dt} \mathbf{i}_0^{\alpha\beta} = \sqrt{3} \mathbf{v}_\phi^{\alpha\beta} - \mathbf{v}_0^{\alpha\beta} - r_{in} \mathbf{i}_0^{\alpha\beta} \quad (2.20)$$

where, $L_{in} = L + 3L_g$.

2.4. The state-space model

Finally a representation in state space can be realized. For that purpose, the system states and the control input, can be expressed via:

$$\mathbf{x}(t) = \begin{bmatrix} i_0^\alpha & i_0^\beta & i_\alpha^0 & i_\beta^0 & i_\alpha^\alpha & i_\beta^\alpha & i_\alpha^\beta & i_\beta^\beta \end{bmatrix}^T \in \mathbb{R}^8 \quad (2.21)$$

$$\mathbf{u}(t) = \begin{bmatrix} u_1 & u_2 & u_3 & u_4 & u_5 & u_6 & u_7 & u_8 & u_9 \end{bmatrix}^T \in \mathbb{R}^9 \quad (2.22)$$

Then, the continuous-time model is obtained:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{E}\mathbf{v}(t) \quad (2.23)$$

where,

$$\mathbf{v}(t) = \begin{bmatrix} \mathbf{v}_\phi^{\alpha\beta} & \boldsymbol{\psi}_r^{\alpha\beta} \end{bmatrix}^T \in \mathbb{R}^4 \quad (2.24)$$

$$\mathbf{A} = \begin{bmatrix} -\frac{r}{L_{in}}\mathbb{I}_{2 \times 2} & \mathbb{0}_{2 \times 2} & \mathbb{0}_{2 \times 4} \\ \mathbb{0}_{2 \times 2} & -\frac{r_{out}}{L_{out}}\mathbb{I}_{2 \times 2} & \mathbb{0}_{2 \times 4} \\ \mathbb{0}_{4 \times 2} & \mathbb{0}_{4 \times 2} & -\frac{r}{L}\mathbb{I}_{4 \times 4} \end{bmatrix} \in \mathbb{R}^{8 \times 8} \quad (2.25a)$$

$$\mathbf{B} = N \cdot v_{dc} \cdot \begin{bmatrix} -\frac{1}{L_{in}}\mathbb{I}_{2 \times 2} & \mathbb{0}_{2 \times 2} & \mathbb{0}_{2 \times 4} \\ \mathbb{0}_{2 \times 2} & -\frac{1}{L_{out}}\mathbb{I}_{2 \times 2} & \mathbb{0}_{2 \times 4} \\ \mathbb{0}_{4 \times 2} & \mathbb{0}_{4 \times 2} & -\frac{1}{L}\mathbb{I}_{4 \times 4} \end{bmatrix} \cdot \mathbf{D}_{\alpha\beta 0} \in \mathbb{R}^{8 \times 9} \quad (2.25b)$$

$$\mathbf{E} = \sqrt{3} \cdot \begin{bmatrix} \frac{1}{L_{in}}\mathbb{I}_{2 \times 2} & \mathbb{0}_{2 \times 2} \\ \mathbb{0}_{2 \times 2} & \frac{k_r}{L_{out}} \left(\frac{1}{\tau_r}\mathbb{I}_{2 \times 2} - \mathbb{J}\omega \right) \\ \mathbb{0}_{4 \times 2} & \mathbb{0}_{4 \times 2} \end{bmatrix} \in \mathbb{R}^{4 \times 8} \quad (2.25c)$$

2.5. Power Flow Analysis

The power components play a crucial role in the M3C-DES. Due to the absence of floating capacitors and the fixed voltage on the SMs DC-Link, the integration of energy storage can be achieved by altering the balance of power flow within the converter. This results in an operation with an unbalanced power flow. Moreover, the use of the assumption of a fixed voltage allows the power flow into the converter, enabling the operational states shown in Table 2.1

Operating Mode	Port power sign			Description
	Grid	Motor	DES	
I	+	-	+	Driving from grid and DES
II	+	-	-	Driving and charging from grid
III	-	+	+	Regenerating and discharging to grid
IV	-	+	-	Regenerating to grid and ES
V	-	-	+	Charging DES from grid and motor
VI	+	+	-	Driving and feeding grid from DES

Table 2.1. Feasible operating modes for the M3C-DES, where input power is positive (+) and output power is negative(-)

The distributed energy storage (DES) operates as a third port, capable of both receiving and delivering energy based on the operational point. Furthermore, as it isn't a singular power source, a seventh (VII) operational point becomes possible. This situation leads to power imbalances within the converter, resulting in power flowing into and out of the DES simultaneously.

The Modular Multilevel Multilevel Matrix Converter is composed of nine arms that forms three sub-converters (Fig. 2.1). By introducing the instantaneous power vector, i.e,

$$\mathbf{p}_{\mathcal{K}} = \begin{bmatrix} p_1 & \dots & p_9 \end{bmatrix}^T \in \mathbb{R}^9 \quad (2.26)$$

the following relationship can be stated:

$$\mathbf{p}_{\mathcal{K}} = \text{diag}\{\mathbf{v}_{\mathcal{K}}\} \mathbf{i}_{\mathcal{K}} \quad (2.27)$$

Working with the previous expressions is challenging due to the strong coupling of internal states, the current and voltage components of the cluster are composed of two frequencies (2.4). However, by applying the Double- $\alpha\beta$ and disregarding the inductance effects, a mathematical power expressions for the vector $\mathbf{p}_{\alpha\beta 0}^{\alpha\beta 0}$ can be derived (Kawamura et al., 2014; Diaz et al., 2017). It is important to highlight that the power is contingent

solely upon the input and output voltage as well as specific components of the state space vector (2.21). The power components are expressed via:

$$p_{\alpha}^{\alpha} = \frac{1}{\sqrt{3}} (v_{\phi}^{\alpha} i_{\alpha}^0 - v_{\chi}^{\alpha} i_0^{\alpha}) + \frac{1}{\sqrt{6}} (v_{\phi}^{\alpha} i_{\alpha}^{\alpha} - v_{\phi}^{\beta} i_{\beta}^{\alpha}) - \frac{1}{\sqrt{6}} (v_{\chi}^{\alpha} i_{\alpha}^{\alpha} - v_{\chi}^{\beta} i_{\alpha}^{\beta}) - v_{n0} i_{\alpha}^{\alpha} \quad (2.28)$$

$$p_{\alpha}^{\beta} = \frac{1}{\sqrt{3}} (v_{\phi}^{\alpha} i_{\beta}^0 - v_{\chi}^{\alpha} i_0^{\beta}) + \frac{1}{\sqrt{6}} (v_{\phi}^{\alpha} i_{\alpha}^{\beta} - v_{\phi}^{\beta} i_{\beta}^{\beta}) + \frac{1}{\sqrt{6}} (v_{\chi}^{\alpha} i_{\alpha}^{\beta} + v_{\chi}^{\beta} i_{\alpha}^{\alpha}) - v_{n0} i_{\alpha}^{\beta} \quad (2.29)$$

$$p_{\beta}^{\alpha} = \frac{1}{\sqrt{3}} (v_{\phi}^{\beta} i_{\alpha}^0 - v_{\chi}^{\alpha} i_0^{\beta}) - \frac{1}{\sqrt{6}} (v_{\phi}^{\alpha} i_{\beta}^{\alpha} + v_{\phi}^{\beta} i_{\alpha}^{\alpha}) - \frac{1}{\sqrt{6}} (v_{\chi}^{\alpha} i_{\beta}^{\alpha} - v_{\chi}^{\beta} i_{\beta}^{\beta}) - v_{n0} i_{\beta}^{\alpha} \quad (2.30)$$

$$p_{\beta}^{\beta} = \frac{1}{\sqrt{3}} (v_{\phi}^{\beta} i_{\alpha}^0 - v_{\chi}^{\alpha} i_0^{\beta}) - \frac{1}{\sqrt{6}} (v_{\phi}^{\alpha} i_{\beta}^{\beta} + v_{\phi}^{\beta} i_{\alpha}^{\alpha}) + \frac{1}{\sqrt{6}} (v_{\chi}^{\alpha} i_{\beta}^{\beta} + v_{\chi}^{\beta} i_{\alpha}^{\alpha}) - v_{n0} i_{\beta}^{\beta} \quad (2.31)$$

$$p_{\alpha}^0 = \frac{1}{\sqrt{6}} (v_{\phi}^{\alpha} i_0^{\alpha} - v_{\phi}^{\beta} i_0^{\beta}) - \frac{1}{\sqrt{3}} (v_{\chi}^{\alpha} i_{\alpha}^{\alpha} + v_{\chi}^{\beta} i_{\alpha}^{\beta}) - v_{n0} i_0^{\alpha} \quad (2.32)$$

$$p_{\beta}^0 = -\frac{1}{\sqrt{6}} (v_{\phi}^{\alpha} i_0^{\beta} + v_{\phi}^{\beta} i_0^{\alpha}) - \frac{1}{\sqrt{3}} (v_{\chi}^{\alpha} i_{\beta}^{\alpha} + v_{\chi}^{\beta} i_{\beta}^{\beta}) - v_{n0} i_0^{\beta} \quad (2.33)$$

$$p_0^{\alpha} = -\frac{1}{\sqrt{6}} (v_{\chi}^{\alpha} i_{\alpha}^0 - v_{\chi}^{\beta} i_{\beta}^0) + \frac{1}{\sqrt{3}} (v_{\phi}^{\alpha} i_{\alpha}^{\alpha} + v_{\phi}^{\beta} i_{\beta}^{\alpha}) - v_{n0} i_{\alpha}^0 \quad (2.34)$$

$$p_0^{\beta} = \frac{1}{\sqrt{6}} (v_{\chi}^{\alpha} i_{\alpha}^0 - v_{\chi}^{\beta} i_{\beta}^0) + \frac{1}{\sqrt{3}} (v_{\phi}^{\alpha} i_{\alpha}^{\beta} + v_{\phi}^{\beta} i_{\beta}^{\beta}) - v_{n0} i_{\beta}^0 \quad (2.35)$$

$$p_0^0 = \frac{1}{\sqrt{3}} \left(v_\phi^\alpha i_0^\alpha + v_\phi^\beta i_0^\beta \right) - \frac{1}{\sqrt{3}} \left(v_\chi^\alpha i_\alpha^0 + v_\chi^\beta i_\beta^0 \right) \quad (2.36)$$

The power components given in (2.28)-(2.36) represent the energy changes in the M3C. For instance, the power flow p_0^0 represent the total amount of power into/from the converter. The components p_α^α , p_β^α , p_α^β and p_β^β refers to the intra-subconverter power flow and the variation of p_0^α , p_0^β , p_α^0 and p_β^0 comprises an inter-subconverter power flow.

Notice that only the terms involving circulating current and common-mode voltage are controllable, while the others are oscillating perturbations that appear in the converter operation. Additionally in the cross components such as p_α^α , p_β^α , p_α^β and p_β^β the common-mode voltage (CMV) depends of the injection of circulating current, on the contrary in the components p_α^0 , p_β^0 , p_0^α and p_0^β the common mode voltage can be used only if the converter is operating. When circulating current and CMV are not injected the average power of the components given in (2.28)-(2.35) are equal to 0.

Fig. 2.3 depicts the power flow across the M3C-DES, highlighting how alterations in the power component values can result in an imbalanced power flow. In Fig. 2.3a, a modification in p_β^0 triggers an inter-sub-converter power flow adjustment. This change induces an unbalanced power flow situation by transferring power between two sub-converters at the output port. A similar phenomenon is observed in Fig. 2.3b, where adjustments in p_0^β can lead to power imbalances between two sub-converters at the input port.

The intra-subconverter imbalance power operation as is shown in Fig. 2.3 can be achieved by altering any of the components in (2.28)-(2.31). Specifically, in this context, Fig. 2.3a and Fig. 2.3b demonstrate the power flow adjustments involving p_β^α and p_α^β , respectively.

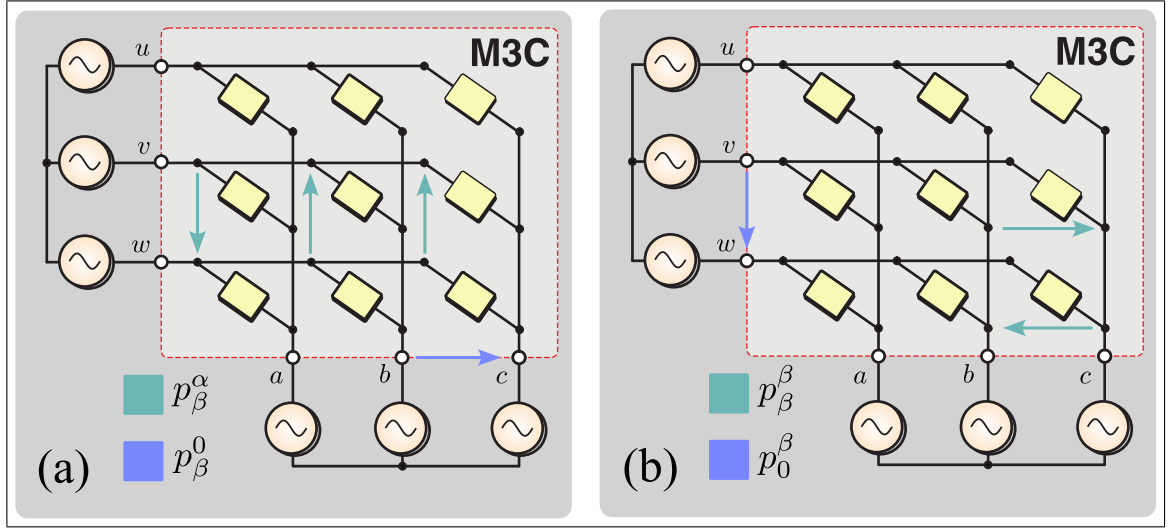


Figure 2.3. Power flow through the M3C: (a) The depiction of power exchange between sub-converters at the output port (purple) and the power flow between clusters (green). (b) The portrayal of power interaction between sub-converters at the input port (purple) and the power transfer between clusters (green).

2.6. Instantaneous Power

The Double- $\alpha\beta 0$ representation is a useful tool for understanding the power flow across the converter. However, the natural oscillations generated by the converter's operation makes it challenging to define common-mode voltage and circulating currents. A large number of filters are necessary to obtain the average power, allowing the development of an average power controller (Rubio, Pereda, Rojas, & Poblete, 2022; Salvadores, Pereda, & Poblete, 2022; Zhang, Tang, Yang, & Gao, 2019).

A direct value for the average power can be obtained in the $\alpha\beta$ framework. In this framework, an expression for the instantaneous power flowing through the converter is derived, as explained in reference (Mora et al., 2021). Using the magnitude invariant Clarke transform, the instantaneous active power for each arm in the M3C-DES can be expressed as follows:

$$2p_\kappa = v_\kappa^\alpha i_\kappa^\alpha + v_\kappa^\beta i_\kappa^\beta \quad (2.37)$$

From the previous equation, it is clear that the single-phase α and β components of voltage and currents of each arm are needed. For this purpose, a rotation of the three-phase vector of voltages and currents is proposed. An example of this method using only the three-phase output voltages is shown in the following equations:

$$\begin{bmatrix} v_a^\alpha \\ v_a^\beta \end{bmatrix} = \mathbf{C}_{\alpha\beta} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \quad \begin{bmatrix} v_c^\alpha \\ v_c^\beta \end{bmatrix} = \mathbf{C}_{\alpha\beta} \begin{bmatrix} v_c \\ v_a \\ v_b \end{bmatrix} \quad (2.38)$$

The vector $v_a^{\alpha\beta}$ contains a component in phase with a and his ninety degrees rotation. The same analysis can be made for the vector $v_c^{\alpha\beta}$ which contains a component in phase with c and his ninety degrees rotation. The graphic representation of this idea is shown in Fig. 2.4.

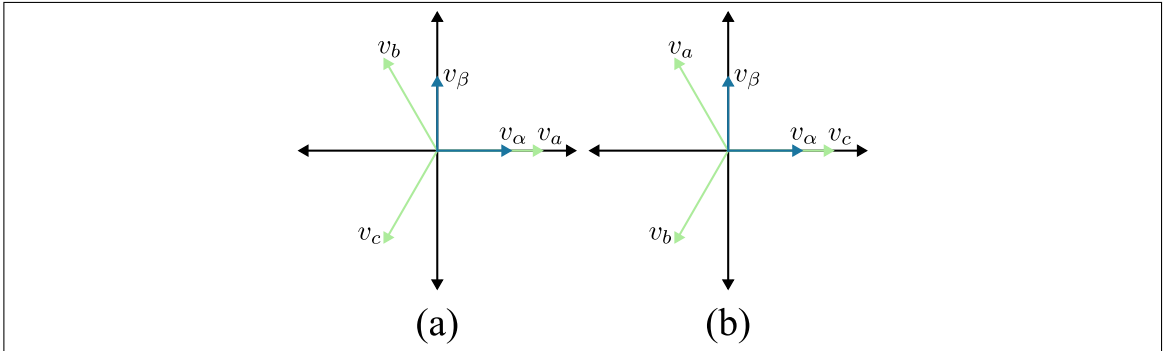


Figure 2.4. Representation of $\alpha\beta$ vectors. (a) shows the traditional Clarke transform, which allows obtaining a component in phase with a and its ninety-degree rotation. In (b) is shown the effect produced by the rotation of the vector abc .

Once we have all the variables and his respective ninety degrees rotation is necessary to understand the composition of one branch of the converter. Analyzing by each frequency

in the clusters of the converter, we can decompose the cluster current into the following statements:

$$i_{\kappa} = \underbrace{\frac{1}{3}(i_{\kappa_{\phi}} + i_{\kappa_{\chi}})}_{i_{\kappa_{\phi\chi}}} + i_{\kappa_z} \quad \forall \kappa \in \mathcal{K} \quad (2.39)$$

where $i_{\kappa_{\phi\chi}}$ is the current flowing in the branch with null circulating currents, i.e. $i_{\kappa_z} = 0$. For instance, the basic current for the first branch is $i_{1_{ua}} = \frac{1}{3}(i_u - i_a)$. Besides, the circulating current is composed of two components, i.e. $i_{\kappa_z} = i_{\kappa_{z\phi}} + i_{\kappa_{z\chi}}$. A similar approach can be made for the cluster voltage, however it will be approximated to:

$$v_{\kappa} = (v_{\kappa_{\phi}} - v_{\kappa_{\chi}}) + v_{n0} \quad \forall \kappa \in \mathcal{K} \quad (2.40)$$

where $v_{\kappa_{\phi}}$ and $v_{\kappa_{\chi}}$ are the values of the instantaneous voltages in the input and output port. For the first branch voltage (v_1) it will be $v_{\kappa_{\phi}} = v_u$ and $v_{\kappa_{\chi}} = v_a$. Moreover, the common-mode voltage is equal for every branch in the converter and it is divided in the two frequencies $v_{n0} = v_{n0_{\phi}} + v_{n0_{\chi}}$. Finally by replacing (2.39) and (2.40) in (2.37) and using the rotated $\alpha\beta$ transform:

$$2p_{\kappa} = (\mathbf{v}_{n0_{\phi}}^{\alpha\beta T} + \mathbf{v}_{\kappa_{\phi}}^{\alpha\beta T})\left(\frac{\mathbf{i}_{\kappa_{\phi}}^{\alpha\beta}}{3} + \mathbf{i}_{\kappa_{z\phi}}^{\alpha\beta}\right) + (\mathbf{v}_{n0_{\chi}}^{\alpha\beta T} - \mathbf{v}_{\kappa_{\chi}}^{\alpha\beta T})\left(\frac{\mathbf{i}_{\kappa_{\chi}}^{\alpha\beta}}{3} + \mathbf{i}_{\kappa_{z\chi}}^{\alpha\beta}\right) \quad \forall \kappa \in \mathcal{K} \quad (2.41)$$

In order to obtain a generalized expression for the instantaneous power, the vector of the average power is defined as $\mathbf{p}_{\mathcal{K}} = [p_1 \dots p_9]^T$ and the vectors $\mathbf{i}_{\mathcal{X}}^{\alpha} \in \mathbb{R}^3$, $\mathbf{i}_{\mathcal{X}}^{\beta} \in \mathbb{R}^3$, $\mathbf{i}_{\Phi}^{\alpha} \in \mathbb{R}^3$ and $\mathbf{i}_{\Phi}^{\beta} \in \mathbb{R}^3$, which contains the currents of the ports in the rotated $\alpha\beta$ framework. In addition, the vectors $\mathbf{i}_{\mathcal{K}_{z\phi}}^{\alpha} \in \mathbb{R}^9$, $\mathbf{i}_{\mathcal{K}_{z\chi}}^{\alpha} \in \mathbb{R}^9$, $\mathbf{i}_{\mathcal{K}_{z\phi}}^{\beta} \in \mathbb{R}^9$ and $\mathbf{i}_{\mathcal{K}_{z\chi}}^{\beta} \in \mathbb{R}^9$ which contains the circulating currents of the nine branches. Finally, with this definitions the following relationship can be derived:

$$2\mathbf{p}_K = \underbrace{\frac{1}{3}\mathbf{V}_{\Phi X}^{\alpha\beta}\mathbf{i}_{\Phi X}^{\alpha\beta}}_{\mathbf{p}_{\Phi X}} + \frac{1}{3}\mathbf{V}_{\Phi X}^0\mathbf{i}_{\Phi X}^{\alpha\beta} + \mathbf{V}_z^{\alpha\beta}\mathbf{i}_z + \mathbf{V}_z^0\mathbf{i}_z \quad (2.42)$$

where,

$$\mathbf{i}_{\Phi X}^{\alpha\beta} = \begin{bmatrix} \mathbf{i}_{\Phi}^{\alpha} & \mathbf{i}_{X}^{\alpha} & \mathbf{i}_{\Phi}^{\beta} & \mathbf{i}_{X}^{\beta} \end{bmatrix}^T \in \mathbb{R}^{12 \times 1} \quad (2.43)$$

$$\mathbf{i}_z = \begin{bmatrix} \mathbf{i}_{K_{z\phi}}^{\alpha} & \mathbf{i}_{K_{z\chi}}^{\alpha} & \mathbf{i}_{K_{z\phi}}^{\beta} & \mathbf{i}_{K_{z\chi}}^{\beta} \end{bmatrix}^T \in \mathbb{R}^{36 \times 1} \quad (2.44)$$

$$\mathbf{V}_{\Phi X}^{\alpha\beta} = \begin{bmatrix} \mathbf{V}_{\Phi}^{\alpha} & v_a^{\alpha}\mathbb{1}_{3 \times 1} & \mathbf{0}_{3 \times 1} & \mathbf{0}_{3 \times 1} & \mathbf{V}_{\Phi}^{\beta} & v_a^{\beta}\mathbb{1}_{3 \times 1} & \mathbf{0}_{3 \times 1} & \mathbf{0}_{3 \times 1} \\ \mathbf{V}_{\Phi}^{\alpha} & \mathbf{0}_{3 \times 1} & v_b^{\alpha}\mathbb{1}_{3 \times 1} & \mathbf{0}_{3 \times 1} & \mathbf{V}_{\Phi}^{\beta} & \mathbf{0}_{3 \times 1} & v_b^{\beta}\mathbb{1}_{3 \times 1} & \mathbf{0}_{3 \times 1} \\ \mathbf{V}_{\Phi}^{\alpha} & \mathbf{0}_{3 \times 1} & \mathbf{0}_{3 \times 1} & v_c^{\alpha}\mathbb{1}_{3 \times 1} & \mathbf{V}_{\Phi}^{\beta} & \mathbf{0}_{3 \times 1} & \mathbf{0}_{3 \times 1} & v_c^{\beta}\mathbb{1}_{3 \times 1} \end{bmatrix} \in \mathbb{R}^{9 \times 12} \quad (2.45)$$

$$\mathbf{V}_z^{\alpha\beta} = \begin{bmatrix} \mathbf{V}_{\Phi}^{\alpha} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & -v_a^{\alpha}\mathbb{I}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \dots \\ \mathbf{0}_{3 \times 3} & \mathbf{V}_{\Phi}^{\alpha} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & -v_b^{\alpha}\mathbb{I}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \dots \\ \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{V}_{\Phi}^{\alpha} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & -v_c^{\alpha}\mathbb{I}_{3 \times 3} & \dots \\ \dots & \mathbf{V}_{\Phi}^{\beta} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & -v_a^{\beta}\mathbb{I}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} \\ \dots & \mathbf{0}_{3 \times 3} & \mathbf{V}_{\Phi}^{\beta} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & -v_b^{\beta}\mathbb{I}_{3 \times 3} & \mathbf{0}_{3 \times 3} \\ \dots & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{V}_{\Phi}^{\beta} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & -v_c^{\beta}\mathbb{I}_{3 \times 3} \end{bmatrix} \in \mathbb{R}^{9 \times 36} \quad (2.46)$$

$$\mathbf{V}_{\Phi X}^0 = \begin{bmatrix} v_{n0\Phi}^{\alpha}\mathbb{I}_{3 \times 3} & v_{n0X}^{\alpha}\mathbb{1}_{3 \times 1} & \mathbf{0}_{3 \times 2} & v_{n0\Phi}^{\beta}\mathbb{I}_{3 \times 3} & v_{n0X}^{\beta}\mathbb{1}_{3 \times 1} & \mathbf{0}_{3 \times 2} \\ v_{n0\Phi}^{\alpha}\mathbb{I}_{3 \times 3} & v_{n0X}^{\alpha}\mathbb{1}_{3 \times 1} & \mathbf{0}_{3 \times 2} & v_{n0\Phi}^{\beta}\mathbb{I}_{3 \times 3} & v_{n0X}^{\beta}\mathbb{1}_{3 \times 1} & \mathbf{0}_{3 \times 2} \\ v_{n0\Phi}^{\alpha}\mathbb{I}_{3 \times 3} & v_{n0X}^{\alpha}\mathbb{1}_{3 \times 1} & \mathbf{0}_{3 \times 2} & v_{n0\Phi}^{\beta}\mathbb{I}_{3 \times 3} & v_{n0X}^{\beta}\mathbb{1}_{3 \times 1} & \mathbf{0}_{3 \times 2} \end{bmatrix} \in \mathbb{R}^{9 \times 12} \quad (2.47)$$

$$\mathbf{V}_z^0 = \begin{bmatrix} v_{n0\Phi}^\alpha \mathbb{I}_{9 \times 9} & v_{n0X}^\alpha \mathbb{I}_{9 \times 9} & v_{n0\Phi}^\beta \mathbb{I}_{9 \times 9} & v_{n0X}^\beta \mathbb{I}_{9 \times 9} \end{bmatrix} \in \mathbb{R}^{9 \times 36} \quad (2.48)$$

with,

$$\mathbf{V}_\Phi^\alpha = \text{diag}\{\mathbf{v}_\Phi^\alpha\} = \text{diag}\{[v_u^\alpha \ v_v^\alpha \ v_w^\alpha]^\text{T}\} \in \mathbb{R}^{3 \times 3} \quad (2.49)$$

$$\mathbf{V}_\Phi^\beta = \text{diag}\{\mathbf{v}_\Phi^\beta\} = \text{diag}\{[v_u^\beta \ v_v^\beta \ v_w^\beta]^\text{T}\} \in \mathbb{R}^{3 \times 3} \quad (2.50)$$

Notice that the equation is non-linear due the multiplication of the common-mode voltage ($\mathbf{V}_z^0$) and circulating current ($\mathbf{i}_z$), which can be defined in order to control the internal power flow of the converter (2.28)-(2.35). Moreover, when the circulating current and common-model voltage are null ($\mathbf{V}_{\Phi X}^0 = \mathbb{0}_{9 \times 12}$, $\mathbf{V}_z^0 = \mathbb{0}_{9 \times 36}$ and $\mathbf{i}_z = \mathbb{0}_{36 \times 1}$) the internal power flowing into/from the converter is given by $\mathbf{p}_{\Phi X}$ and has the same value for every component in the vector $\mathbf{p}_K$.

2.7. Summary

This chapter introduces the essential equations for developing the proposed controller in the subsequent section. The primary approach employed for the Modular Multilevel Matrix Converter involves utilizing the Double- $\alpha\beta 0$ transformation to decouple the internal state of the converter.

Additionally, an examination of power within the Double- $\alpha\beta 0$ framework is conducted to illustrate the potential combinations of power flow inside the converter. Nevertheless, a more straightforward expression for instantaneous power is established using the rotation of vectors in the Clarke transform introduced at the beginning of the chapter.

3. PROPOSED OPTIMAL CONTROLLER

This Section explore the proposed controller that enables the distributed energy storage integration in the M3C. The overall control strategy is shown in Fig. 3.1, where a Continuous Control Set Model Predictive Controller (CCS-MPC) is designed as internal controller and the reference for this controller are generated independently in one reference generator for the input side and two external controllers. Besides, in order to obtain the final PWM applied to the converter a modulation stage is required. Since the modulation stage is not a focus of this work, a typical Level-Shifted PWM (LS-PWM) modulation stage is chosen, which could be used for addressing intra-cluster power imbalances (Rubio et al., 2022).

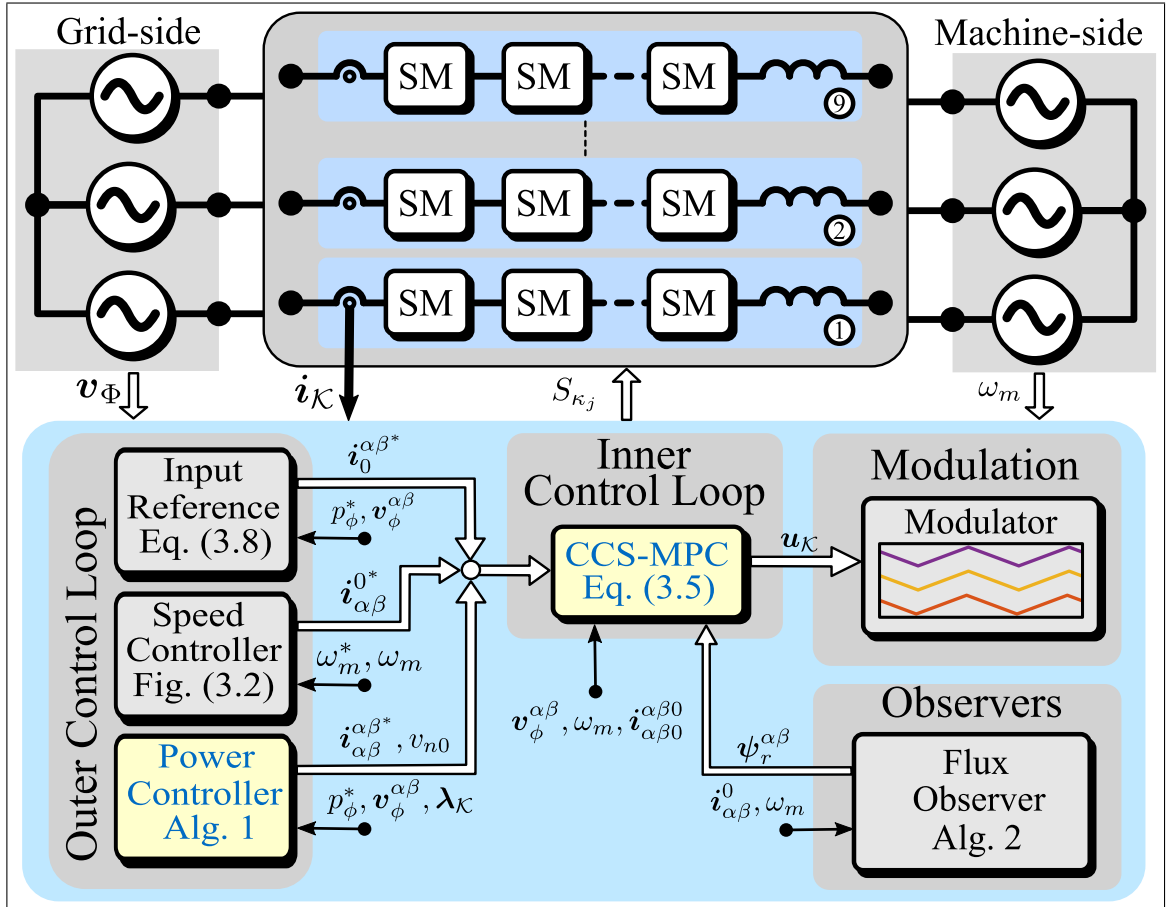


Figure 3.1. Overall Control Strategy

3.1. Inner Control Loop (CCS-MPC)

The internal controller or the inner control loop is the key to generate the control input (2.22). Several control strategies are studied in order to govern the M3C. A standard way to control is by designing and implementing multiple control loops based on proportional-integral (PI) controllers (Kawamura et al., 2014; Diaz et al., 2017). Recent works show that Model Predictive Control can tackle the controls objective in the M3C (Urrutia et al., 2021; R. H. Cuzmar, Mora, Pereda, Poblete, & Aguilera, 2023; R. H. Cuzmar, Mora, Pereda, Aguilera, et al., 2023). However most of those works propose a variable frequency operation but the direct control of an electrical machine is missing.

Continuous Control Set MPC has gained relevance during recent years because it offers a constant switching frequency and the external modulation stage reduces the computational burden, also the output of the control stage can be used for one cluster and reduce the number of variables in the control system (Arias-Esquivel et al., 2023). This work proposes a CCS-MPC that covers the control of the electrical machine by using the dynamic model of the active load.

In order to obtain the control input the model predictive controller requires the discrete time state space model. Applying the Forward Euler discretization to (2.23) using a sampling period T_s , the following discrete time model can be obtained:

$$\mathbf{x}_{k+1} = \mathbf{A}_d \mathbf{x}_k + \mathbf{B}_d \mathbf{u}_k + \mathbf{E}_d \mathbf{v}_k \quad (3.1)$$

where,

$$\begin{aligned} \mathbf{A}_d &= \mathbb{I}_{8 \times 8} + T_s \mathbf{A} \\ \mathbf{B}_d &= T_s \mathbf{B} \\ \mathbf{E}_d &= T_s \mathbf{E} \end{aligned} \quad (3.2)$$

Here, the variable $\mathbf{x}_k$, $\mathbf{u}_k$ and $\mathbf{v}_k$ are the discrete-time version of the continuous-time variables defined in previous section, i.e. (2.21), (2.22) and (2.24). The main control targets are the eight current in the Double- $\alpha\beta 0$ framework. Thus, the reference vector can be expressed by $\mathbf{x}_k^*$. Finally, the CCS-MPC optimization problem is formulated as:

$$\begin{aligned} \min_{\mathbf{x}_{k+1}, \mathbf{u}_k} J_k &= \|\mathbf{x}_{k+1} - \mathbf{x}_{k+1}^*\|_{\mathbf{Q}}^2 + \|\mathbf{u}_k - \mathbf{u}_k^*\|_{\mathbf{\Lambda}}^2 \\ s.t. \quad \mathbf{x}_{k+1} &= \mathbf{A}_d \mathbf{x}_k + \mathbf{B}_d \mathbf{u}_k + \mathbf{E}_d \mathbf{v}_k \\ -\mathbb{1}_{9 \times 1} &\leq \mathbf{u}_k \leq \mathbb{1}_{9 \times 1} \end{aligned} \tag{3.3}$$

where $\mathbf{u}_k^*$ is the steady-state control input to keep the current in the desired steady-state reference $\mathbf{x}_{k+1}^*$, which computation will be described on next sections. The matrices $\mathbf{Q}$ and $\mathbf{\Lambda}$ are used to adjust the MPC, regulating the relationship between error tracking and control action, allowing the dynamic response of the closed-loop system to be modified (Aguilera & Quevedo, 2015). Finally, the control signals are those that are solution of (3.3).

Notice that each value in $\mathbf{u}_k \in [-\mathbb{1}_{9 \times 1}, \mathbb{1}_{9 \times 1}]$, this makes (3.3) a constrained quadratic optimization problem. In order to obtain a state-feedback expression for the control law, the saturated unconstrained optimal solution is proposed and can be obtained by performing the partial derivative into the Lagrangian function, which details can be found on Appendix A:

$$\frac{\partial J_k}{\partial \mathbf{u}_k} = 0 \tag{3.4}$$

The solution of (3.4) lead us to:

$$\mathbf{u}_k^{\text{opt}} = -\mathbf{W}^{-1} \mathbf{F}_k \tag{3.5}$$

where,

$$\begin{aligned}
\mathbf{W} &= \mathbf{B}_d^T \mathbf{Q} \mathbf{B}_d + \mathbf{\Lambda} \\
\mathbf{F}_k &= \mathbf{B}_d^T \mathbf{Q} (\mathbf{A}_d \mathbf{x}_k + \mathbf{E}_d \mathbf{v}_k - \mathbf{x}_{k+1}^*) - \mathbf{\Lambda} \mathbf{u}_k^*
\end{aligned} \tag{3.6}$$

Notice that (2.23) is a time varying system, so the matrix $\mathbf{F}_k$ must be computed at each iteration and the inverse of $\mathbf{W}$ is ensured because $\mathbf{B}_d$ is a diagonal matrix multiplied by a linear transformation whose columns are independent, so this matrix is square and has full rank. Additionally, if the steady-state reference ($\mathbf{u}_k^*$) is well designed (see Section 3.3.4), the optimal solution of the unconstrained problem will be the solution of the constrained problem. However, during transients, the relaxed form of the optimization problem could violate the constraints. A quadratic programming optimization problem with box constraints can be formulated (R. H. Cuzmar, Mora, Pereda, Aguilera, et al., 2023). However, in this work in order to provide feasible solutions a saturation scheme is used, this can produce sub-optimal solutions during transients but is sufficient to handle the reference tracking. Finally the control input at each sampling time is given by:

$$\mathbf{u}_k = \min \left(\mathbb{1}_{9 \times 1}, \max \left(-\mathbb{1}_{9 \times 1}, \mathbf{u}_k^{\text{opt}} \right) \right) \tag{3.7}$$

3.2. Outer Control Loop

The Outer control loop is composed of three independent controllers that operate in parallel and deliver different reference currents already decoupled to the Inner control loop. The three outer controllers are detailed below.

3.2.1. Input Reference Generator

The main proposal of this thesis is grid feeding. Considering a balance electrical grid in the input port, the reference in the input side is obtained by applying an active power reference, for a symmetric current it can be defined by:

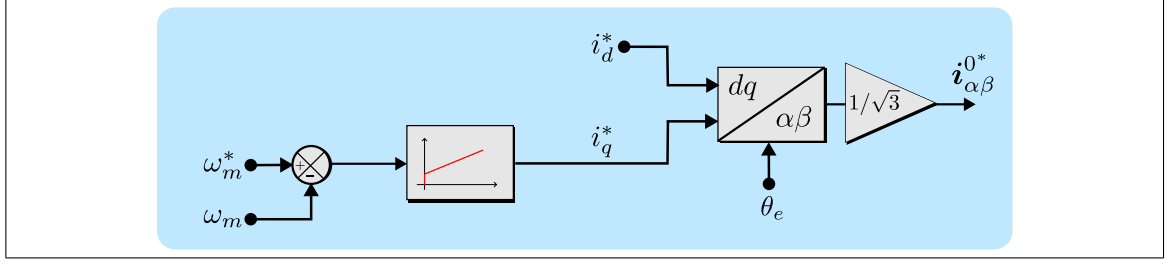


Figure 3.2. Reference design controller for the machine side: Mechanical Speed controller.

$$\mathbf{i}_0^{\alpha\beta*} = \frac{1}{\sqrt{3}} \frac{1}{v_\phi^{\alpha^2} + v_\phi^{\beta^2}} \begin{bmatrix} v_\phi^\alpha & v_\phi^\beta \\ v_\phi^\beta & -v_\phi^\alpha \end{bmatrix} \begin{bmatrix} p_\phi^* \\ q_\phi^* \end{bmatrix} \quad (3.8)$$

where p_ϕ^* and q_ϕ^* are the active and reactive power reference. This simple term allows to regulate the power injected to the grid by controlling the input current. However, non reactive power is injected or considered in this work, i.e. $q_\phi^* = 0$.

3.2.2. Speed Controller

A PI controller with anti-windup is chosen for speed control. Initially, a reference is established in the dq framework using the proposed machine controller and an external reference that magnetizes the squirrel cage rotor (Arias-Esquivel et al., 2023) as depicted in Fig. 3.2. Subsequently, the generated reference ($\mathbf{i}_{\alpha\beta}^{0*}$) is directly sent to the internal current controller (Fig. 3.1). In order to get the $\alpha\beta$ representation from the rotating synchronous frame dq , the phase of the estimated rotor flux ($\psi_r^{\alpha\beta}$) is used:

$$\theta_e = \arctan \left(\frac{\psi_r^\beta}{\psi_r^\alpha} \right) \quad (3.9)$$

3.2.3. Power Controller

There are several ways to achieve unbalanced power operation. However, to the knowledge of the author this has not been a topic of interest for the M3C yet. Recent works, are

focused in energy balancing on a modified Modular Multilevel Matrix Converter, using dual-active bridge as SMs (Kasari & Bhattacharjee, 2023; Das et al., 2020).

Unlike the State Of Charge (SOC) of a battery, the power has not dynamics (Zhang et al., 2019; Neira, Poblete, Pereda, & Nuñez, 2020), so just knowing the average power and developing an expression for the currents and voltages is enough to achieve the propose objective (Yu et al., 2017b), and this is one of the contributions if this work.

From the previous section, the instantaneous power of the nine M3C branches is non-linear (2.42). For the reference generator is necessary to obtain the common-mode voltage (v_{n0}^*) and circulating currents (i_z^*) capable to achieve the desire power reference, i.e. $p_K^* = [p_1^* \dots p_9^*]^T \in \mathbb{R}^9$. This variables can be obtain by solving:

$$2p_K^* = \frac{1}{3}V_{\Phi X}^{\alpha\beta}i_{\Phi X}^{\alpha\beta} + \frac{1}{3}V_{\Phi X}^{0*}i_{\Phi X}^{\alpha\beta} + V_z^{\alpha\beta}i_z^* + V_z^{0*}i_z^* \quad (3.10)$$

In order to solve this non-linear equations a sequential quadratic programming (QP) algorithm based on the Block Coordinate Descent method (BCD) (Tseng, 2001) is generated. The optimization problem is solve by iterating the solution of two linear sub-problems, that can be obtain from the original equation.

3.2.4. Circulating Current Optimization

For the first linear sub-problem the circulating current is chosen. We assume that the (CMV) is constants during the optimization interval, so (2.42) is linear and can be rewritten as:

$$p_K = \underbrace{V_{\Phi X}^{\alpha\beta 0}i_{\Phi X}^{\alpha\beta}}_{p'_{\Phi X}} + V_z^{\alpha\beta 0}i_z \quad (3.11)$$

where,

$$\mathbf{V}_{\Phi X}^{\alpha\beta 0} = \frac{1}{6}\mathbf{V}_{\Phi X}^{\alpha\beta} + \frac{1}{6}\mathbf{V}_{\Phi X}^0 \quad (3.12)$$

$$\mathbf{V}_z^{\alpha\beta 0} = \frac{1}{2}\mathbf{V}_z^{\alpha\beta} + \frac{1}{2}\mathbf{V}_z^0 \quad (3.13)$$

Then, a quadratic constraints programming problem is used in order to minimize the error of between the reference and the actual instantaneous power. However, the circulating current must satisfy the null-space condition, i.e. $\mathbf{P}(\mathbf{i}_{\mathcal{K}_{z\phi}}^\alpha + \mathbf{i}_{\mathcal{K}_{z\chi}}^\alpha) = \mathbf{0}_{6 \times 1}$, to be considered as circulating current components, which means that the sum of the circulating current must be zero at every node of the converter. From equation (2.9), there are four circulating currents, so $\mathbf{i}_\epsilon \in \mathbb{R}^{16}$ is defined as the path for the circulating currents. This approach will help us to reduce the number of variables of the original equations, by rewriting $\mathbf{i}_z$ in terms of $\mathbf{i}_\epsilon$.

$$\mathbf{i}_z = \mathbf{H}\mathbf{i}_\epsilon = \text{diag}\{[\mathbf{N} \ \mathbf{N} \ \mathbf{N} \ \mathbf{N}]^T\}\mathbf{i}_\epsilon \in \mathbb{R}^{36 \times 16} \quad (3.14)$$

where, $\mathbf{N}$ is the null-space matrix of $\mathbf{P}$ and is not unique (R. Cuzmar et al., 2023) and is defined in (3.15). Besides, $\mathbf{i}_\epsilon$ is composed of $\mathbf{i}_{\epsilon_\phi}^\alpha \in \mathbb{R}^4$, $\mathbf{i}_{\epsilon_\chi}^\alpha \in \mathbb{R}^4$, $\mathbf{i}_{\epsilon_\phi}^\beta \in \mathbb{R}^4$ and $\mathbf{i}_{\epsilon_\chi}^\beta \in \mathbb{R}^4$, i.e. $\mathbf{i}_\epsilon = [\mathbf{i}_{\epsilon_\phi}^\alpha \ \mathbf{i}_{\epsilon_\chi}^\alpha \ \mathbf{i}_{\epsilon_\phi}^\beta \ \mathbf{i}_{\epsilon_\chi}^\beta]^T \in \mathbb{R}^{16}$

$$\mathbf{N} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & -1 & 0 & 0 \\ 0 & 0 & -1 & -1 \\ -1 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \in \mathbb{R}^{9 \times 4} \quad (3.15)$$

Finally the optimization problem is formulated as follows:

$$\begin{aligned}
\min_{\mathbf{i}_\epsilon^*} \quad & ||\mathbf{p}_\mathcal{K}^* - \mathbf{p}_\mathcal{K}||^2 + \lambda_z ||\mathbf{i}_\epsilon^*||^2 \\
s.t. \quad & \mathbf{p}_\mathcal{K} = \mathbf{p}'_{\Phi X} + \mathbf{V}_\epsilon^{\alpha\beta 0} \mathbf{i}_\epsilon^* \\
& \mathbf{i}_{min} \leq \mathbf{i}_z^* \leq \mathbf{i}_{max}
\end{aligned} \tag{3.16}$$

where, $\mathbf{i}_{min}$ and $\mathbf{i}_{max}$ are the lower and upper boundaries of the optimization variable, which can be represented in terms of the the dynamic of the system or can be fixed in a predefined value (R. Cuzmar et al., 2023), in this work the current restrictions are not considered; $\mathbf{V}_\epsilon^{\alpha\beta 0} = \mathbf{V}_z^{\alpha\beta 0} \mathbf{H}$. Moreover, in order to limit the current injection a second term is added to the objective function, where λ_u allows the tuning of the reference generator. The optimization problem can be rewritten as:

$$\begin{aligned}
\min_{\mathbf{i}_\epsilon^*} \quad & \mathbf{i}_\epsilon^{*\top} \mathbf{G}_\epsilon \mathbf{i}_\epsilon^* + \mathbf{i}_\epsilon^{*\top} \mathbf{f}_\epsilon \\
s.t. \quad & \mathbf{i}_{min} \leq \mathbf{i}_z^* \leq \mathbf{i}_{max}
\end{aligned} \tag{3.17}$$

where,

$$\mathbf{G}_\epsilon = \begin{bmatrix} \mathbf{V}_\epsilon^{\alpha\beta 0} \\ \sqrt{\lambda_z} \mathbb{I}_{16 \times 16} \end{bmatrix}^\top \begin{bmatrix} \mathbf{V}_\epsilon^{\alpha\beta 0} \\ \sqrt{\lambda_z} \mathbb{I}_{16 \times 16} \end{bmatrix} \in \mathbb{R}^{16 \times 16} \tag{3.18}$$

$$\mathbf{f}_\epsilon = \begin{bmatrix} \mathbf{V}_\epsilon^{\alpha\beta 0} \\ \sqrt{\lambda_z} \mathbb{I}_{16 \times 16} \end{bmatrix}^\top \begin{bmatrix} \mathbf{p}_\mathcal{K}^* - \mathbf{p}'_{\Phi X} \\ \mathbf{0}_{16 \times 1} \end{bmatrix} \in \mathbb{R}^{16 \times 1} \tag{3.19}$$

If we avoid the box-constraints the optimization problem in (3.17) has unique solution if $\mathbf{G}_\epsilon$ is symmetric positive definite (Boyd & Vanderberghe, 2004), which is guaranteed if $\lambda_z > 0$.

3.2.5. Common-Mode Voltage Optimization

The second linear sub-problem can be written assuming that the circulating current is known during the optimization interval. Thus, the common-mode voltage is the output variable. However, (2.42) contains a matrix with the CMV, in order to optimize the vector $\mathbf{v}_0 = [v_{n0\Phi}^\alpha \ v_{n0X}^\alpha \ v_{n0\Phi}^\beta \ v_{n0X}^\beta]^\text{T} \in \mathbb{R}^4$, lead us to the following relationship.

$$2\mathbf{p}_K = \underbrace{\frac{1}{3}\mathbf{V}_{\Phi X}^{\alpha\beta} \mathbf{i}_{\Phi X}^{\alpha\beta} + \mathbf{V}_z^{\alpha\beta} \mathbf{i}_z^{\alpha\beta}}_{2\mathbf{p}'_{\Phi X z}} + \underbrace{\left(\frac{1}{3}\mathbf{I}_{\Phi X} + \mathbf{I}_z\right)}_{2\mathbf{I}_{\Phi X z}} \mathbf{v}_0 \quad (3.20)$$

where,

$$\mathbf{I}_{\Phi X} = \begin{bmatrix} \mathbf{i}_\Phi^\alpha & i_a^\alpha \mathbb{1}_{3 \times 1} & \mathbf{i}_\Phi^\beta & i_a^\beta \mathbb{1}_{3 \times 1} \\ \mathbf{i}_\Phi^\alpha & i_b^\alpha \mathbb{1}_{3 \times 1} & \mathbf{i}_\Phi^\beta & i_b^\beta \mathbb{1}_{3 \times 1} \\ \mathbf{i}_\Phi^\alpha & i_c^\alpha \mathbb{1}_{3 \times 1} & \mathbf{i}_\Phi^\beta & i_c^\beta \mathbb{1}_{3 \times 1} \end{bmatrix} \in \mathbb{R}^{9 \times 4} \quad (3.21)$$

$$\mathbf{I}_z = \begin{bmatrix} \mathbf{i}_{\mathcal{K}z\phi}^\alpha & \mathbf{i}_{\mathcal{K}z\chi}^\alpha & \mathbf{i}_{\mathcal{K}z\phi}^\beta & \mathbf{i}_{\mathcal{K}z\chi}^\beta \end{bmatrix} \in \mathbb{R}^{9 \times 4} \quad (3.22)$$

with, $\mathbf{i}_\Phi^\alpha = [i_u^\alpha \ i_v^\alpha \ i_w^\alpha]^\text{T}$ and $\mathbf{i}_\Phi^\beta = [i_u^\beta \ i_v^\beta \ i_w^\beta]^\text{T}$. Then, a quadratic programming optimization problem is used in order to obtain the optimal value of $\mathbf{v}_0$.

$$\begin{aligned} \min_{\mathbf{v}_0^*} \quad & ||\mathbf{p}_K^* - \mathbf{p}_K||^2 + \lambda_0 ||\mathbf{v}_0^*||^2 \\ s.t. \quad & \mathbf{p}_K = \mathbf{p}'_{\Phi X z} + \mathbf{I}_{\Phi X z} \mathbf{v}_0^* \\ & \mathbf{v}_{min} \leq \mathbf{v}_0^* \leq \mathbf{v}_{max} \end{aligned} \quad (3.23)$$

Similarly to the circulating current sub-problem (3.16), the second term of the objective functions allows us to tune the reference generator and limits the injection of the

common-mode voltage, through λ_0 . Additionally, the problem can be reformulated as in (3.17),

$$\begin{aligned} \min_{\mathbf{v}_0^*} \quad & \mathbf{v}_0^{*\top} \mathbf{G}_0 \mathbf{v}_0^* + \mathbf{v}_0^{*\top} \mathbf{f}_0 \\ \text{s.t.} \quad & \mathbf{v}_{min} \leq \mathbf{v}_0^* \leq \mathbf{v}_{max} \end{aligned} \quad (3.24)$$

where,

$$\mathbf{G}_0 = \begin{bmatrix} \mathbf{I}_{\Phi Xz} \\ \sqrt{\lambda_0} \mathbb{I}_{4 \times 4} \end{bmatrix}^\top \begin{bmatrix} \mathbf{I}_{\Phi Xz} \\ \sqrt{\lambda_0} \mathbb{I}_{4 \times 4} \end{bmatrix} \in \mathbb{R}^{4 \times 4} \quad (3.25)$$

$$\mathbf{f}_0 = \begin{bmatrix} \mathbf{I}_{\Phi Xz} \\ \sqrt{\lambda_0} \mathbb{I}_{4 \times 4} \end{bmatrix}^\top \begin{bmatrix} \mathbf{p}_{\mathcal{K}}^* - \mathbf{p}'_{\Phi Xz} \\ \mathbb{0}_{4 \times 1} \end{bmatrix} \in \mathbb{R}^{4 \times 1} \quad (3.26)$$

One key property of the problem in (3.24) is that $\mathbf{v}_0 = \mathbb{0}_{4 \times 1}$ if the converter is not operating, i.e. $\mathbf{I}_{\Phi Xz} = \mathbb{0}_{9 \times 4}$. In particular, the problem has unique solution if $\mathbf{G}_0$ is symmetric positive definite (Boyd & Vanderberghe, 2004), which requires $\lambda_0 > 0$.

3.2.6. Optimization Algorithm

There are several ways to solve an optimization problem (Boyd & Vanderberghe, 2004). In order to implement the reference generator in a controller we adopt an exterior point active set method based on a Lagrangian function and the Karush–Kuhn–Tucker (KKT) conditions. This approach has been shown effective managing box-constraints in QP problems, which details can be found in (Baidya et al., 2020), and will be referred as *QPS* (Quadratic Programming Solver).

Solving (3.17) and (3.24) independently may result in sub-optimal solutions due to the coupling of the two problems, as described by the nonlinear equation (3.10). An iterative optimization method is proposed, the main idea initialize the variable and matrices and solve (3.17), then using the solution $\mathbf{i}_e^*$, formulate and solve (3.24) to obtain $\mathbf{v}_0^*$. Then

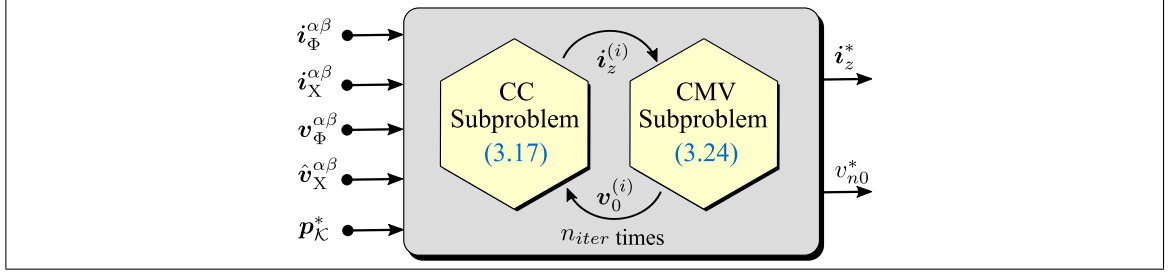


Figure 3.3. Two Steps Sequential Optimization Method

$$\mathbf{i}_{\alpha\beta}^{\alpha\beta*} = \mathbf{D}_{\alpha\beta 0}^z (\mathbf{i}_{\mathcal{K}_{z\phi}}^{\alpha*} + \mathbf{i}_{\mathcal{K}_{z\chi}}^{\alpha*}) \quad (3.27)$$

The fundamental property of this method is that the solution remains in the number of iterations (n_{iter}), more iterations bring the problem closer to its optimally. However, the computational burden increases, making its implementation difficult. In the last section, we will see that a great number of iterations are not required to achieve the proposed objective.

3.3. Practical Issues

This section introduce the critical issues regarding the practical implementation of the proposed controller. In particular, the CCS-MPC for the speed drives requires the rotor flux ($\psi_r^{\alpha\beta}$) estimation and the steady-state control input ($\mathbf{u}_k^*$). Besides, a grid observer is implemented in order to reject grid disturbances and correctly generate the input current reference.

3.3.1. Flux Observer

From (2.24), we understand that the rotor flux is essential for accurately generating the control input ($\mathbf{u}_k$) in the CCS-MPC. However, the rotor flux is not a directly measurable state. To determine its value, estimators or observers are required (Cárdenas, 2011). In this work a Kalman Filter is used for the complete output states estimations.

In order to implement the Kalman Filter (KF) the dynamic model of the system is required, which is given by (2.19) and (2.18). Then, like any system, the state space arises:

$$\begin{aligned}\dot{\mathbf{x}} &= \mathcal{G}\mathbf{x} + \mathcal{F}\mathbf{u} \\ \mathbf{y} &= \mathcal{H}\mathbf{x}\end{aligned}\tag{3.28}$$

where,

$$\mathbf{x} = \begin{bmatrix} i_{\alpha\beta}^0 & \psi_r^{\alpha\beta} \end{bmatrix}^T \in \mathbb{R}^4\tag{3.29}$$

$$\mathcal{G} = \begin{bmatrix} -\frac{r_{out}}{L_{out}}\mathbb{I}_{2\times 2} & -\sqrt{3}k_r(\mathbb{J}\omega - \frac{1}{\tau_r}\mathbb{I}_{2\times 2}) \\ \sqrt{3}\frac{L_m}{\tau_r}\mathbb{I}_{2\times 2} & -(\frac{1}{\tau_r}\mathbb{I}_{2\times 2} - \mathbb{J}\omega) \end{bmatrix} \in \mathbb{R}^{4\times 4}\tag{3.30}$$

$$\mathcal{F} = \begin{bmatrix} -\frac{1}{L_{out}}\mathbb{I}_{2\times 2} \\ \mathbb{0}_{2\times 2} \end{bmatrix} \in \mathbb{R}^{4\times 2}\tag{3.31}$$

$$\mathcal{H} = \begin{bmatrix} \mathbb{I}_{2\times 2} & \mathbb{0}_{2\times 2} \end{bmatrix} \in \mathbb{R}^{2\times 4}\tag{3.32}$$

As we can see in Equation (3.28), it represents a time-varying system. To ensure accuracy in observing the states, the Kalman gain is computed online. For this purpose, the discrete-time model is required. In this case we use the zero-order-hold approximation with a sample time T_k , which leads us to the following matrices:

$$\begin{aligned}
\mathcal{G}_d &= e^{\mathcal{G}T_k} \\
\mathcal{F}_d &= \mathcal{G}^{(-1)}(\mathcal{G}_d - \mathbb{I}_{4 \times 4})\mathcal{F} \\
\mathcal{H}_d &= \mathcal{H}
\end{aligned} \tag{3.33}$$

Once we get the discrete-time dynamic system, the input and output are defined as $\mathbf{y}_k = \mathbf{i}_{\alpha\beta}^0$ and $\mathbf{u}_k = \mathbf{v}_{\alpha\beta}^0$. Then, the state space for the prediction is given by:

$$\hat{\mathbf{x}}_{k+1,k} = \mathcal{G}_d \hat{\mathbf{x}}_{k,k} + \mathcal{F}_d \mathbf{u}_k \tag{3.34}$$

with, $\hat{\mathbf{x}}_{k+1,k}$ the predicted state at the time step k . Then, based on (Becker, 2023) the Algorithm 2 shows the implementation of the kalman filter.

It is important to highlight that the step **Initialize** only happen once, at the first iteration of the algorithm, and the algorithm iterates between **Step 1** and **Step 2** every sample period T_k . The matrix $\mathcal{Q} \in \mathbb{R}^{4 \times 4}$ is known as process noise covariance, which allows the kalman filter to decide how much the variables can change in one period. On the other hand, the matrix $\mathcal{R} \in \mathbb{R}^{2 \times 2}$ is the measurement covariance, both matrices allow us to tune the kalman gain ($\mathbf{K}_k$). The output vector of the implemented kalman filter is the rotor flux $\psi_r^{\alpha\beta}$ in the period $k + 1$.

3.3.2. Grid Observer

The input current reference is computed directly with the grid voltage (3.8). To avoid disturbances that could distort the desire reference a kalman filter is used, for that purpose the rotation matrix in the $\alpha\beta$ framework is used, i.e.:

Algorithm 2: Kalman Filter

Input: $\mathbf{y}_k$ and $\mathbf{u}_k$

Output: $\psi_r^{\alpha\beta}$

Define: $\mathcal{Q}$ and $\mathcal{R}$

Initialize: $\hat{\mathbf{x}}_{k,k}, \mathcal{P}_{k,k}$

Step 1: Predict

1. Extrapolate the state: $\hat{\mathbf{x}}_{k+1,k} = \mathcal{G}_d \hat{\mathbf{x}}_{k,k} + \mathcal{F}_d \mathbf{u}_k$
 2. Extrapolate uncertainty: $\mathcal{P}_{k+1,k} = \mathcal{G}_d \mathcal{P}_{k,k} \mathcal{G}_d^T + \mathcal{Q}$
-

Step 2: Correct

1. Compute the Kalman Gain: $\mathbf{K}_k = \mathcal{P}_{k,k-1} \mathcal{H}_d^T (\mathcal{H}_d \mathcal{P}_{k,k-1} \mathcal{H}_d^T + \mathcal{R}_k)^{-1}$
 2. Update estimate with measurement: $\hat{\mathbf{x}}_{k,k} = \hat{\mathbf{x}}_{k,k-1} + \mathbf{K}_k (\mathbf{y}_k - \mathcal{H}_d \hat{\mathbf{x}}_{k,k-1})$
 3. Update the estimate uncertainty:
 $\mathcal{P}_{k,k} = (\mathcal{I}_{4 \times 4} - \mathbf{K}_k \mathcal{H}) \mathcal{P}_{k,k-1} (\mathcal{I}_{4 \times 4} - \mathbf{K}_k \mathcal{H})^T + \mathbf{K}_k \mathcal{R}_k \mathbf{K}_k$
-

Update: $\psi_r^{\alpha\beta}$

$$\begin{aligned} \mathbf{v}_{k+1}^{\alpha\beta} &= \Theta_g \mathbf{v}_k^{\alpha\beta} \\ \mathbf{y}_k &= \mathbf{v}_\phi = \mathcal{C} \mathbf{v}_k^{\alpha\beta} \end{aligned} \tag{3.35}$$

where,

$$\Theta_g = \begin{bmatrix} \cos(\omega_g T_s) & -\sin(\omega_g T_s) & 0 & 0 \\ \sin(\omega_g T_s) & \cos(\omega_g T_s) & 0 & 0 \\ 0 & 0 & \cos(10\omega_g T_s) & -\sin(10\omega_g T_s) \\ 0 & 0 & \sin(10\omega_g T_s) & \cos(10\omega_g T_s) \end{bmatrix} \in \mathbb{R}^{2 \times 2} \tag{3.36}$$

$$\mathcal{C} = \begin{bmatrix} \mathbb{I}_{2 \times 2} & \mathbb{I}_{2 \times 2} \end{bmatrix} \in \mathbb{R}^{2 \times 4} \tag{3.37}$$

with, ω_g the frequency of the input voltage ($v_\phi^{\alpha\beta}$) and T_s the sample period. The states vector is composed of the voltage and a perturbation of high frequency, i.e. $\hat{\mathbf{v}}_k^{\alpha\beta} = [\hat{v}_\phi^{\alpha\beta} \ \hat{v}_p]^T \in \mathbb{R}^4$. The implementation is similar to the Algorithm 2. However, the system is time invariant, so the implementation requires only to extrapolate the state and update the estimation with measurements (3.38) with a kalman gain ($\mathbf{K}_\phi$), which can be computed offline.

$$\hat{\mathbf{v}}_{k+1}^{\alpha\beta} = \Theta_g \hat{\mathbf{v}}_k^{\alpha\beta} + \mathbf{K}_\phi (v_\phi^{\alpha\beta} - \mathbf{C} \hat{\mathbf{v}}_k^{\alpha\beta}) \quad (3.38)$$

It is worth noting that (3.38) results in voltage prediction ($\hat{v}_\phi^{\alpha\beta}$). Thus, can be directly used to compensate the inherent one sampling period delay of digital platforms. A similar conclusion is valid for the observer flux observer described on Algorithm 2.

3.3.3. Power Reference

To tackle the unbalance power operation a reference must be generated. There are many methods to achieve this objective, but in this work we use something similar to the described in (R. H. Cuzmar et al., 2021). Firstly, we calculate the power reference in the balanced case. From the input port is enough to use the reference shown in (3.8) and from the output side we estimate the voltage, that can be obtained from the following relationship.

$$\hat{\mathbf{v}}_x^{\alpha\beta} = \sqrt{3}\sigma L_s \begin{bmatrix} 0 & -\omega_e \\ \omega_e & 0 \end{bmatrix} \mathbf{i}_{\alpha\beta}^{0*} + \sqrt{3}r_\sigma \mathbf{i}_{\alpha\beta}^{0*} - k_r \left(\frac{1}{\tau_r} \mathbb{I}_{2 \times 2} - \mathbb{J}\omega \right) \psi_r^{\alpha\beta} \quad (3.39)$$

where, ω_e is the electrical frequency of the stator and is calculated with the following least-squares problem.

$$\begin{bmatrix} \sin(\omega_e T_k) \\ \cos(\omega_e T_k) \end{bmatrix} = (\mathbf{L}^T \mathbf{L})^{(-1)} \mathbf{L}^T \mathbf{s} \quad (3.40)$$

where,

$$\begin{aligned} \mathbf{L} &= \begin{bmatrix} -\psi_{r_k}^\beta & \psi_{r_k}^\alpha \\ \psi_{r_k}^\alpha & \psi_{r_k}^\beta \end{bmatrix} \in \mathbb{R}^{2 \times 2} \\ \mathbf{s} &= \begin{bmatrix} \psi_{r_{k+1}}^\alpha \\ \psi_{r_{k+1}}^\beta \end{bmatrix} \in \mathbb{R}^{2 \times 1} \end{aligned} \quad (3.41)$$

Finally, the electrical frequency ω_e is obtained by the following equation.

$$\omega_e = \arctan \left(\frac{\sin(\omega_e T_k)}{\cos(\omega_e T_k)} \right) / T_k \quad (3.42)$$

Once we get the estimated voltage the active power at the machine side (P_m^*) is equivalent to:

$$p_X^* = \sqrt{3} \hat{v}_X^\alpha i_\alpha^{0*} + \sqrt{3} \hat{v}_X^\beta i_\beta^{0*} \quad (3.43)$$

We define λ_κ from the vector $\boldsymbol{\lambda}_\mathcal{K} = [\lambda_1 \dots \lambda_9]^T$ as the percentage of power that can be extracted from the branch κ . Then the reference power that is used in the circulating current and CMV generator, is defined as:

$$p_\kappa^* = \lambda_\kappa \frac{(p_\Phi^* + p_X^*)}{9} \quad \forall \kappa \in \mathcal{K} \quad (3.44)$$

where, the following relationship must be fulfilled:

$$\sum_{\kappa \in \mathcal{K}} \lambda_\kappa = 1 \quad (3.45)$$

3.3.4. Steady-State Control Input Reference

The steady-state control input reference ($\mathbf{u}_{\mathcal{K}}^*$) is generated using (2.3). With some modifications, the desired reference voltage is expressed as:

$$\mathbf{v}_{\mathcal{K}}^* = \mathbf{P}^T \begin{bmatrix} \mathbf{v}_{\Phi} \\ \hat{\mathbf{v}}_{\chi} \end{bmatrix} - L \frac{d\mathbf{i}_{\mathcal{K}}^*}{dt} - r\mathbf{i}_{\mathcal{K}}^* - \mathbb{1}_{9 \times 1} v_{n0}^* \quad (3.46)$$

However, for a digital implementation, $\mathbf{i}_{\mathcal{K}}^*$ cannot be derived in continuous time. Therefore, the relationship can be rewritten as follows:

$$\mathbf{v}_{\mathcal{K}}^* = \mathbf{P}^T \begin{bmatrix} \mathbf{C}_{\alpha\beta}^T & \mathbb{0}_{3 \times 2} \\ \mathbb{0}_{3 \times 2} & \mathbf{C}_{\alpha\beta}^T \end{bmatrix} \begin{bmatrix} \mathbf{v}_{\phi}^{\alpha\beta} \\ \hat{\mathbf{v}}_{\chi}^{\alpha\beta} \end{bmatrix} - \Upsilon (L\Gamma \mathbf{i}_{\Phi Xz}^{\alpha\beta*} - r\mathbf{i}_{\Phi Xz}^{\alpha\beta*}) - \mathbb{1}_{9 \times 1} v_{n0}^* \quad (3.47)$$

where,

$$\Upsilon = \begin{bmatrix} \mathbf{P}^\dagger & \mathbb{0}_{9 \times 6} & \mathbb{I}_{9 \times 9} & \mathbb{I}_{9 \times 9} & \mathbb{0}_{9 \times 16} \end{bmatrix} \in \mathbb{R}^{9 \times 48} \quad (3.48)$$

$$\mathbf{\Gamma} = \begin{bmatrix}
\mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & -\omega_g \mathbb{I}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \dots \\
\mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & -\omega_e \mathbb{I}_{3 \times 3} & \dots \\
\omega_g \mathbb{I}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \dots \\
\mathbf{0}_{3 \times 3} & \omega_e \mathbb{I}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \dots \\
\mathbf{0}_{9 \times 3} & \mathbf{0}_{9 \times 3} & \mathbf{0}_{9 \times 3} & \mathbf{0}_{9 \times 3} & \dots \\
\mathbf{0}_{9 \times 3} & \mathbf{0}_{9 \times 3} & \mathbf{0}_{9 \times 3} & \mathbf{0}_{9 \times 3} & \dots \\
\mathbf{0}_{9 \times 3} & \mathbf{0}_{9 \times 3} & \mathbf{0}_{9 \times 3} & \mathbf{0}_{9 \times 3} & \dots \\
\mathbf{0}_{9 \times 3} & \mathbf{0}_{9 \times 3} & \mathbf{0}_{9 \times 3} & \mathbf{0}_{9 \times 3} & \dots \\
\dots & \mathbf{0}_{3 \times 9} & \mathbf{0}_{3 \times 9} & \mathbf{0}_{3 \times 9} & \mathbf{0}_{3 \times 9} \\
\dots & \mathbf{0}_{3 \times 9} & \mathbf{0}_{3 \times 9} & \mathbf{0}_{3 \times 9} & \mathbf{0}_{3 \times 9} \\
\dots & \mathbf{0}_{3 \times 9} & \mathbf{0}_{3 \times 9} & \mathbf{0}_{3 \times 9} & \mathbf{0}_{3 \times 9} \\
\dots & \mathbf{0}_{3 \times 9} & \mathbf{0}_{3 \times 9} & \mathbf{0}_{3 \times 9} & \mathbf{0}_{3 \times 9} \\
\dots & \mathbf{0}_{9 \times 9} & \mathbf{0}_{9 \times 9} & -\omega_g \mathbb{I}_{9 \times 9} & \mathbf{0}_{9 \times 9} \\
\dots & \mathbf{0}_{9 \times 9} & \mathbf{0}_{9 \times 9} & \mathbf{0}_{9 \times 9} & -\omega_e \mathbb{I}_{9 \times 9} \\
\dots & \omega_g \mathbb{I}_{9 \times 9} & \mathbf{0}_{9 \times 9} & \mathbf{0}_{9 \times 9} & \mathbf{0}_{9 \times 9} \\
\dots & \mathbf{0}_{9 \times 9} & \omega_e \mathbb{I}_{9 \times 9} & \mathbf{0}_{9 \times 9} & \mathbf{0}_{9 \times 9}
\end{bmatrix} \in \mathbb{R}^{48 \times 48} \quad (3.49)$$

$$\mathbf{i}_{\Phi X z}^{\alpha \beta *} = \begin{bmatrix} \mathbf{i}_{\Phi}^{\alpha *} & -\mathbf{i}_X^{\alpha *} & \mathbf{i}_{\Phi}^{\beta *} & -\mathbf{i}_X^{\beta *} & \mathbf{i}_{\mathcal{K}_{z\phi}}^{\alpha *} & \mathbf{i}_{\mathcal{K}_{z\chi}}^{\alpha *} & \mathbf{i}_{\mathcal{K}_{z\phi}}^{\beta *} & \mathbf{i}_{\mathcal{K}_{z\chi}}^{\beta *} \end{bmatrix}^T \in \mathbb{R}^{48 \times 1} \quad (3.50)$$

with, $\mathbf{P}^\dagger$ the Moore–Penrose inverse matrix of $\mathbf{P}$. Notice, that $\mathbf{\Gamma}$ is just a extended formulation of the ninety degrees rotation matrix in the $\alpha\beta$ representation. Finally, the $\mathbf{v}_{\mathcal{K}}^*$ is normalized using the total amount of voltage available in the clusters, lead us to:

$$\mathbf{u}_{\mathcal{K}}^* = \frac{\mathbf{v}_{\mathcal{K}}^*}{NV_{dc}} \quad (3.51)$$

3.4. Summary

This chapter has presented the proposed control strategy for both the internal controller and the unbalanced power reference generator for the M3C-DES.

On one hand, the internal loop controls the injected current, composed of input, output, and circulating currents, through a CCS-MPC. Additionally, the control strategy incorporates consideration of the machine model at the output port, providing the branches' duty cycles.

On the other hand, a preceding stage generates the circulating currents and common-mode voltage references. Utilizing the equations outlined in previous sections, a two-step optimization algorithm is formulated to minimize the cost functions, enabling the converter's operation with unbalanced power.

Finally, implementation issues are discussed concerning the estimation of the rotor flux, the generation of power references, and the steady-state control input reference. Furthermore, a grid voltage observer is introduced to enhance the controller's performance.

4. SIMULATION AND EXPERIMENTAL RESULTS

The previous chapter introduced a control scheme. The derived equations were implemented in a simulation environment to validate the proposal under various operating modes, as described in Chapter 2 (refer to Table 2.1). Furthermore, to not saturate this sections with graphics the operational modes IV, and V, are not considered. Subsequently, the proposal was executed in a digital controller and tested in an experimental setup, showcasing the effectiveness and high-quality performance of the proposed strategy.

4.1. Experimental Setup

The experimental setup used in this work, along with a brief description of each of its components, is shown in Fig. 4.1. This consists of a modular multilevel matrix converter, an induction machine, and a programmable grid emulator. For the control system a real-time controller (Opal-RT OP4510 and OP4520) was used.

The converter used is formed by 9 clusters, each of which contains 3 Full-Bridge SMs with floating capacitors. A manual switch system allows the connection/disconnection of 48 [V] DC sources to the DC-link of each SM. In this manner, the converter can emulate distributed energy storage. The parameters of the M3C configuration used are shown in Table 4.1. As mentioned earlier, a 3 kW induction machine (IM) is connected to the output port, and the parameters are obtained from (Lizana, 2023), where the test for the parameter estimation are detailed.

4.2. Simulation Results

Before experimental verification, a simulation environment was developed to validate the proposal. The primary aim was to achieve operational points that the experimental setup couldn't replicate, such as injecting power into the branches due to the presence of unidirectional DC sources. This was achieved using the software MATLAB/Simulink

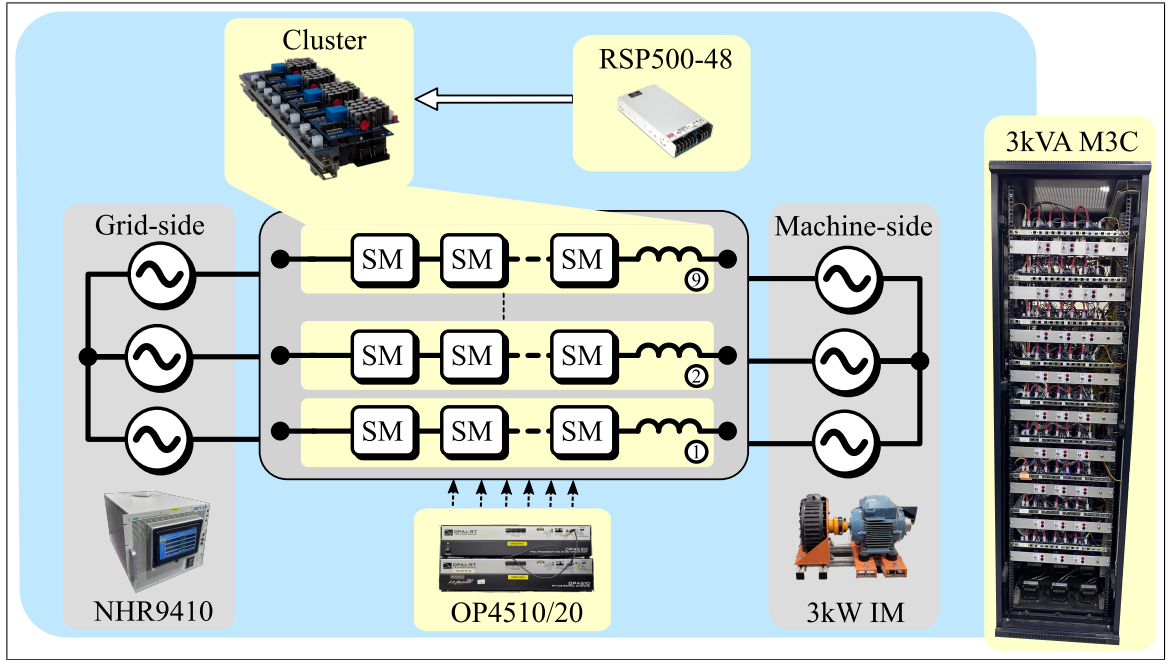


Figure 4.1. Experimental Setup.

Table 4.1. Converter Parameters.

Description	Parameter	Value
Number of SMs	N	3
SMs DC Voltage	v_{dc}	48 [V]
SMs Capacitance	C_{sm}	987 μ [H]
Branch Inductance	L	2.5 m[H]
Branch Resistance	r	0.1 [Ω]
Grid Voltage	V_{ϕ}^{rms-ff}	80 [V]
Grid Frequency	f_{in}	60 [Hz]
Input Inductance	L_g	1 m[H]
Pole Pairs	ρ	2
Stator Resistance	R_s	1.8 [Ω]
Rotor Resistance	R_r	1.8 [Ω]
Stator Leakage Inductance	L_{ls}	26 m[H]
Rotor Leakage Inductance	L_{lr}	26 m[H]
Magnetizing Inductance	L_m	0.1 [H]

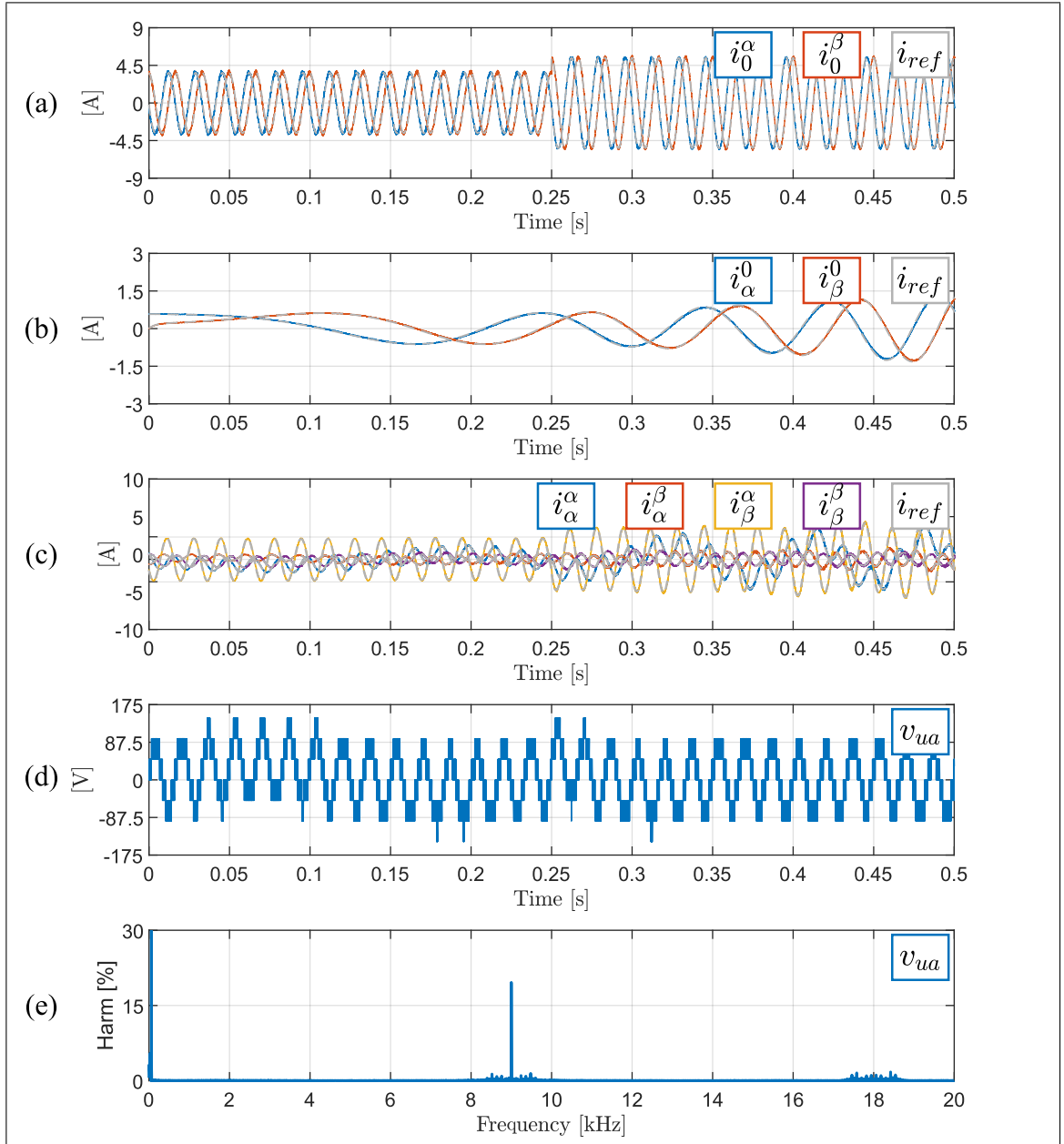


Figure 4.2. Simulation: Current controller performance (a) Input current (b) Output current (c) Circulating current (d) Modulated voltage (e) Voltage spectrum

and PLECS Blockset. The simulation utilizes the same parameters as the original converter and enables the representation of multiple operational points (refer to Table 2.1) beyond those achievable with the physical prototype.

4.2.1. Current Controller

The results illustrated in Fig.4.2 play a crucial role in the experimental verification. While the primary objective focuses on achieving unbalanced power operation through optimal power control, the efficacy of the current controller and precise modulation ensures the safe operation of the converter. Additionally, using the Double- $\alpha\beta 0$ representation allows for a step change in both the input and circulating current without disturbing the output performance. This is attributed to the decoupled mathematical representation. Furthermore, the fast dynamics of the MPC enable accurate tracking of instantaneous changes in the references.

Fig. 4.2 illustrates the performance of the current controller, also known as the inner controller (CCS-MPC). The first three graphs display the currents in the Double- $\alpha\beta 0$ framework along with their respective references. In Fig. 4.2a, the input current tracking is depicted, showcasing a fast response to a step change within 0.25 seconds, facilitated by the model predictive control features. Conversely, Fig. 4.2b portrays the current tracking for the output current during variable frequency operation. In this case, the dynamics are slower due to the operation of the induction machine. Finally, Fig. 4.2c demonstrates the tracking of circulating current, highlighting a step change at 0.25 seconds caused by a modification in the input power. This alteration forces the current reference to attain a higher value due the optimization in the power controller.

Additionally, Fig. 4.2d illustrates the modulated voltage applied in the cluster ua . This graph demonstrates the presence of the $2N + 1 = 7$ levels of the branch converter, confirming the accurate application of the control action in the simulation environment. Fig. 4.2e shows the voltage spectrum with the fundamental frequencies at the origin (input and output), and a 9 kHz component from the level-shifted pulse width modulation (LS-PWM) with six carriers.

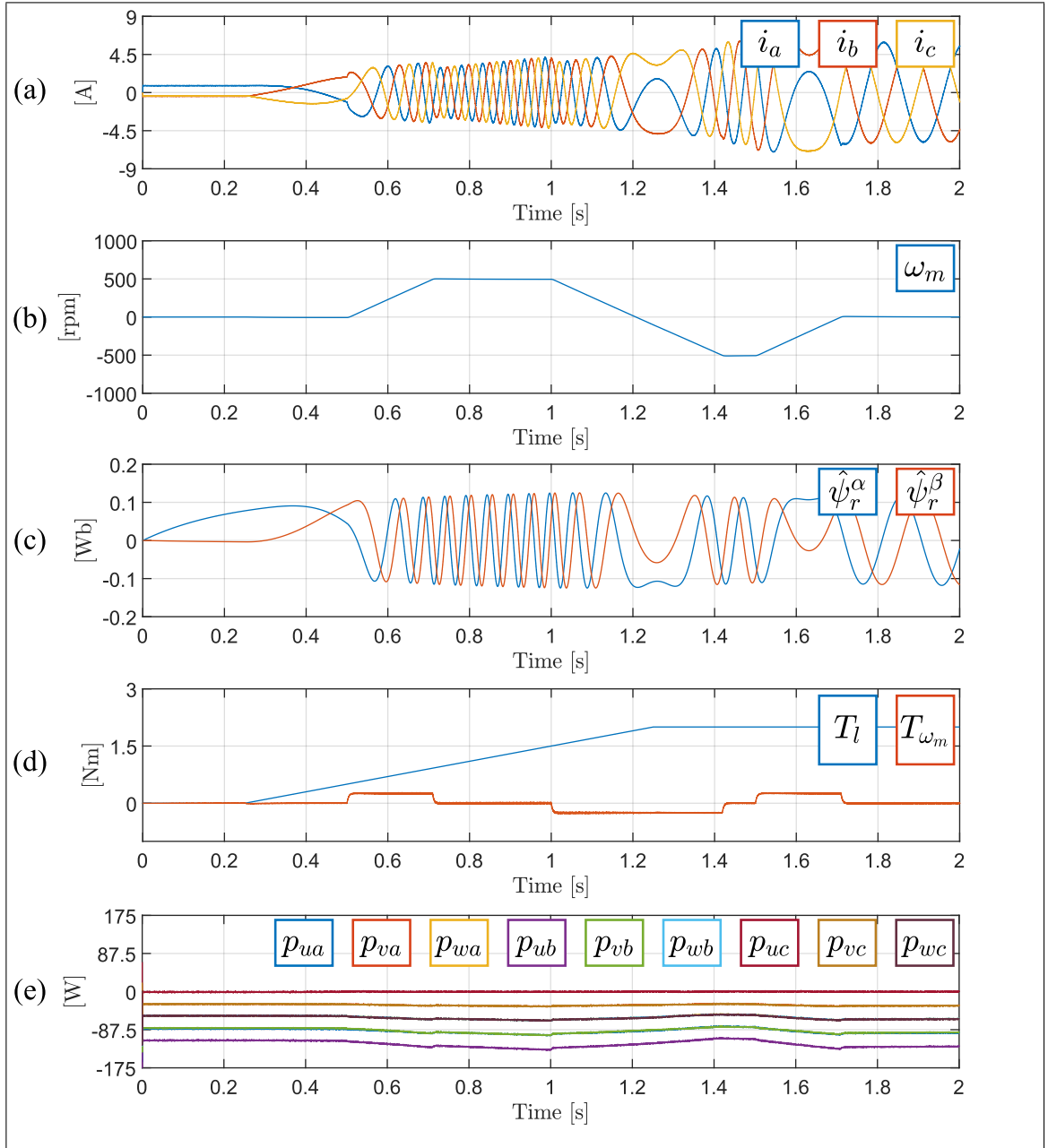


Figure 4.3. Simulation: Machine control performance (a) Machine current (b) Mechanical speed (c) Estimated Rotor flux (d) Electrical torque (e) Power in the branches

4.2.2. Machine Control

Fig. 4.3 displays the dynamic behavior of the machine side during converter operation over a 2 [s] interval. Fig. 4.3a is an extended version of the output current shown in 4.2b, where the initial DC value is noticeable due to the magnetization of the induction machine. As outlined in Section 3.2.2, the speed controller is a PI controller responsible for regulating the mechanical speed of the system. The controller's performance is depicted in Fig. 4.3b, initiating from zero mechanical speed and reaching a set-point of 500 [rpm] at 0.5 [s] and -500 [rpm] at 1 [s] to test the external control loop. Furthermore, to demonstrate an accurate speed and current tracking, a torque ramp load ranging from 0 to 2 [Nm] is applied to the mechanical system in the induction machine. This effect is observable in Fig. 4.3d, where the electrical torque of the machine can be observed. The figure shows the decomposition of the electrical torque into the load torque (T_l) and the torque applied to achieve the speed control objective (T_{ω_m}). The application of positive torque leads to motor and generator operation of the machine during the change in mechanical speed direction, demonstrating operating modes III and VI (refer to Table 2.1). This effect is illustrated in Fig. 4.3e, where the variation of power in the branches under an unbalanced operation indicates reduced power from the DES when the mechanical speed is negative.

The rotor flux plays a critical role in the overall control scheme as it is necessary for the current controller, the speed controller, and the reference generator in the power controller. This variable is estimated using a Kalman Filter, and the result is depicted in Fig. 4.3c, where the rotor flux is observed and constrained to a constant value with the inclusion of a rotor flux magnitude controller added to the original speed controller (refer to Fig. 3.2). It is notable that the rotor flux exhibits the same frequency behavior as the current due to their direct relationship (2.18).

4.2.3. Optimal Power Control

The primary focus of this work is the optimal power control, which is evaluated through simulations presented in Fig. 4.4. Fig. 4.4a illustrates the power levels at the

converter's three ports: the grid, the machine, and the distributed energy storage. The time interval enables the display of bidirectional power at the input port, depicting operational modes I and VI. In this scenario, the power of the induction machine is restricted to prevent modulator saturation.

Fig. 4.4b depicts the three-phase input current injected into the grid, notably showcasing a change in direction within a 0.1 [s] time interval. Additionally, it demonstrates a non-evidential overshoot during the grid current step change at 1 [s]. On the other hand, Fig. 4.4c displays the three-phase machine current, illustrating the magnetization at the onset and the frequency ramp resulting from variable speed and load operation.

The most relevant graphics are the last three. Fig. 4.4 (d) and (e) illustrate the outcome of the optimization problem formulated in (3.17) after employing the iterative BCD algorithm. Notably, it shows the input's presence accompanied by variations in the output port frequency in the circulating currents. Similarly, the conclusion applies to the CMV (Fig. 4.4)e, where an initial injection of a DC offset is apparent to achieve unbalanced power while adhering to system conditions. It is important to note that only fundamental frequencies are observable in both variables, omitting the injection of harmonics, which is assumed to represent the optimal operating point due to the formulation by separating the fundamental frequencies on Chapter 2.6. Moreover, in areas where the machine has sufficient voltage, the optimal value converges to almost purely injecting circulating current, potentially compromising the system's efficiency. As a future work, it is proposed to conduct a detailed analysis by adjusting the weights and adding constraints to ensure safe operation of the converter.

Finally, Fig. 4.4f illustrates the unbalanced power distribution among the nine branches of the converter. At 0.5 [s], one branch exhibits zero power injection while the others show a doubled injection to compensate for this branch's consumption. Similarly, at 1.5 [s], one branch changes the direction of its power flow, indicating the feasibility of operating mode VII. However, a change in the input current disrupts the unbalanced power operation due to

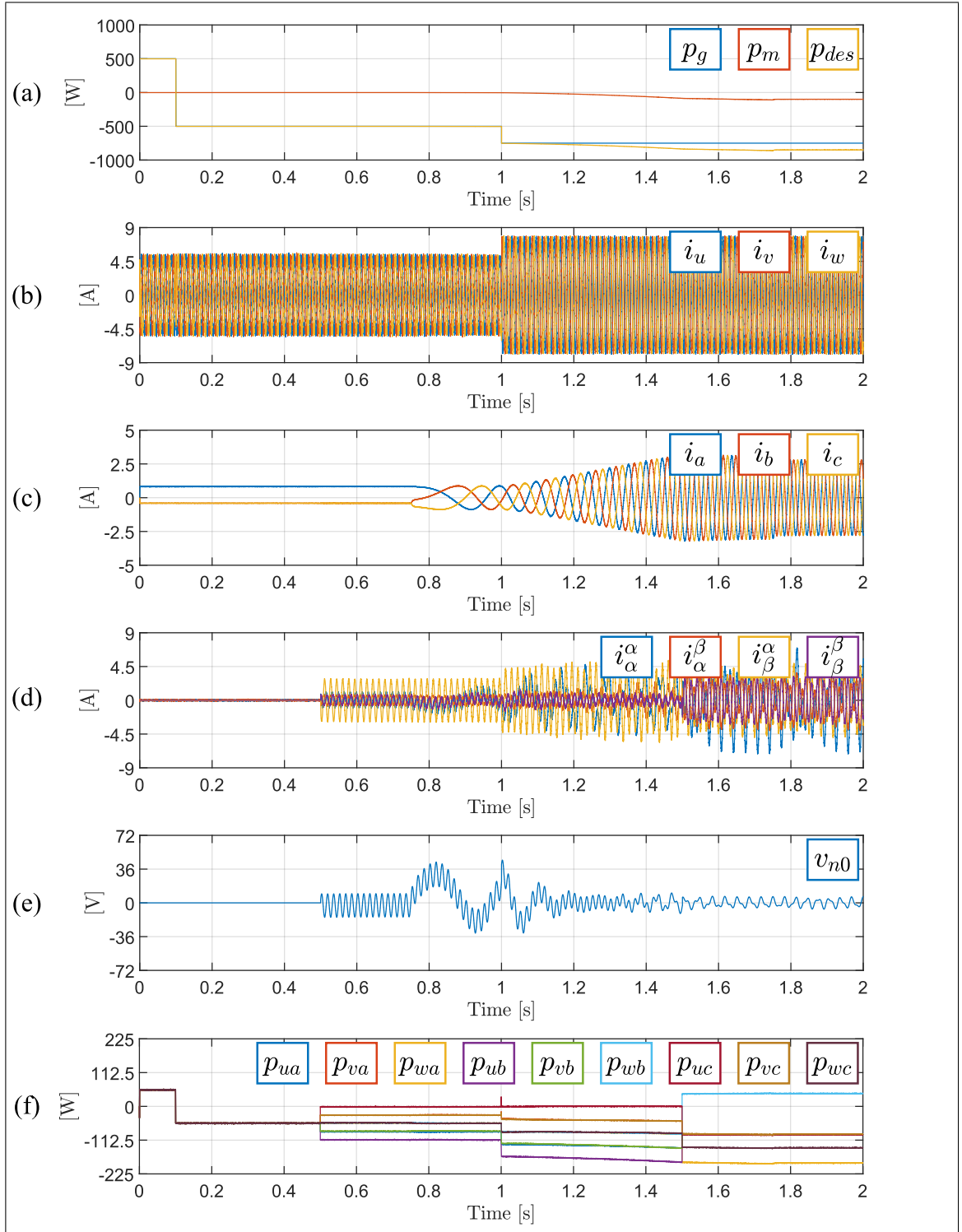


Figure 4.4. Simulation: Unbalanced power operation (a) Total power at the converter ports (b) Input current (c) Output current (d) Circulating current (e) Common-mode voltage (f) Power in the branches

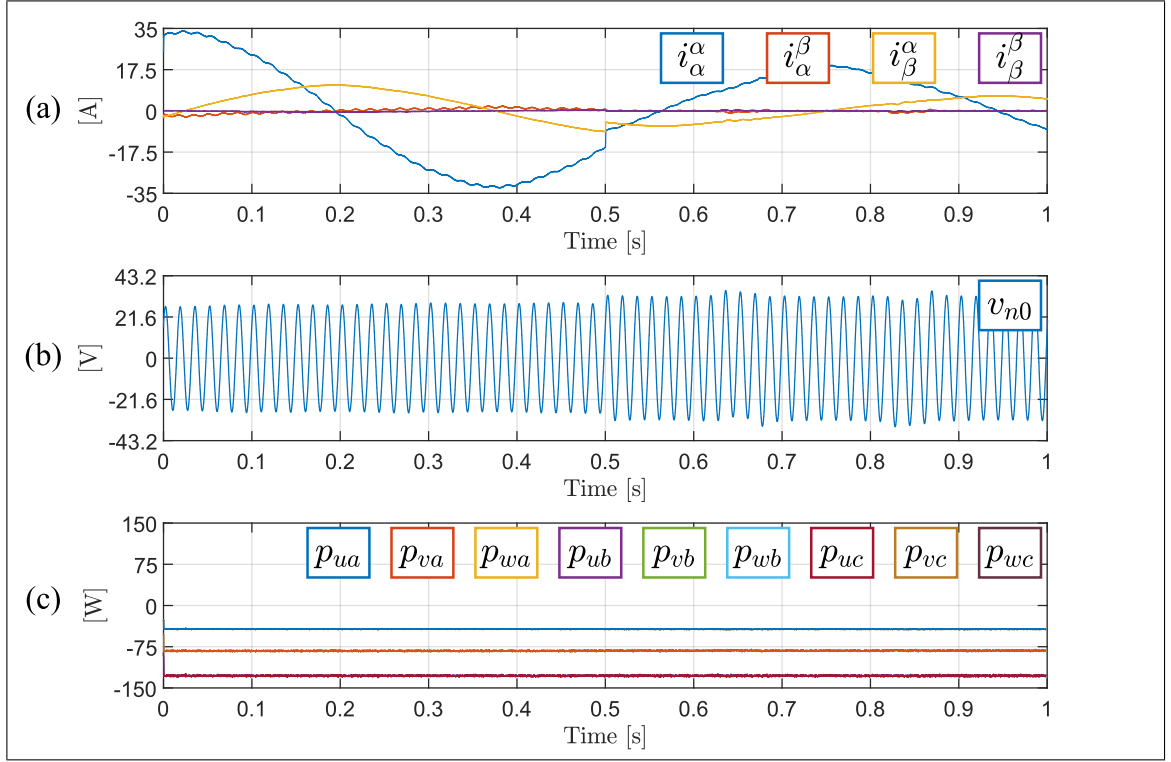


Figure 4.5. Simulation: Unbalanced power operation (a) Circulating current (b) Common-mode voltage (c) Power in the branches

modulator saturation at a particular sample time. This effect can be mitigated by reducing the response time of the reference generator.

In the previous section, two box constrained optimization problems were formulated (3.17) (3.24). However, in this study, we solely consider constraints for the CMV to prevent saturation of the modulator. Nevertheless, at high speeds or torques, the CMV naturally decreases. Fig. 4.5 indicates a change in CMV constraints at 0.5 seconds. It's important to note that these restrictions alter the waveforms for this variable while maintaining power reference tracking. However, in unbalanced power operations, current saturation could distort branch power, potentially leading to suboptimal responses. Conversely, the CMV is predominantly necessary at low speeds, reducing the circulating current magnitude. This might cause saturation at higher speeds, so the CMV must tend to zero at this operational point, which aligns naturally with the optimizer (Fig 4.4)e.

Table 4.2. Control Parameters

Description	Parameter	Value
Sample frequency	f_s	9 k[Hz]
MPC error gain matrix	$\mathbf{Q}$	$100 \cdot \mathbb{I}_{8 \times 8}$
MPC steady-state gain matrix	$\mathbf{\Lambda}$	$\mathbb{I}_{9 \times 9}$
KF covariance of the process noise matrix	$\mathbf{Q}$	$\text{diag}\{[10 \ 10 \ 10^{-5} \ 10^{-5}]\}$
KF covariance of the observation noise matrix	$\mathbf{R}$	$\mathbb{I}_{9 \times 9}$
Machine control proportional gain	K_p	0.03
Machine control integral gain	K_i	0.007
Circulating current generator factor	λ_z	10
CMV generator factor	λ_0	10
Number of Iterations	n_{iter}	5

This section serves to validate the control strategy before experimental implementation. The results presented herein demonstrate the effectiveness of the proposed approach, minimizing implementation issues within the experimental setup. In unbalanced power operations, the utilization of constraints often results in saturated responses in current and common-mode voltage, as commonly observed in this operation mode (Rivera, Angulo, Acuna, & Mora, 2023). Nonetheless, certain specific operating points imply non-saturated responses, akin to the scenario depicted in Fig. 4.5.

4.3. Experimental Results

Experimental results are provided to validate the proposal. All control loops and observers were tuned heuristically (Zanchetta, 2011), and the parameters used, including the sampling frequency chosen to generate a single update per carrier period, are mentioned in Table 4.2.

4.3.1. Current Controller

In order to validate the entire proposal, the current controller plays a pivotal role. Fig. 4.6 shows the behavior of the internal controller, which is capable of controlling the input current (Fig. 4.6a), the output current (Fig. 4.6b), and the circulating currents (Fig. 4.6c). The results do not differ significantly from the simulations presented in the previous section. However, some differences are observed, which can be attributed to tuning parameters, the commissioning of the converter, and variations in the dynamic system model.

The input power experiences a step reference change from 250 [W] to 500 [W] at 0.2 [s] (Fig. 4.6a). The controller adeptly achieves steady-state reference tracking, showcasing its responsiveness to changes in input power. Despite a fast dynamic response, a minor overshoot is noticeable. This overshoot can be linked to the elevated error matrix gain within the MPC objective function, as outlined in Table 4.2. While the overshoot is present, the controller maintains stability, quickly settling to the new steady-state. Fine-tuning of controller parameters and the correct model fit may further optimize this response.

Fig. 4.6b displays the machine-side current in the Double- $\alpha\beta 0$ framework. The proposed controller adeptly achieves reference tracking at the desired machine speed, with a detailed analysis of its behavior to follow. Furthermore, Fig. 4.6c illustrates circulating current reference tracking, where measured values closely match the references despite the presence of input and output frequency.

At 0.2 [s], a step change occurs in the circulating current reference, met with a prompt response from the controller. However, minor oscillations are observable in the current waveform. These oscillations may be attributed to perturbations in measurements and resonances among parasitic elements of the converter. As a result, the current controller is validated to demonstrate the behavior of the entire proposal.

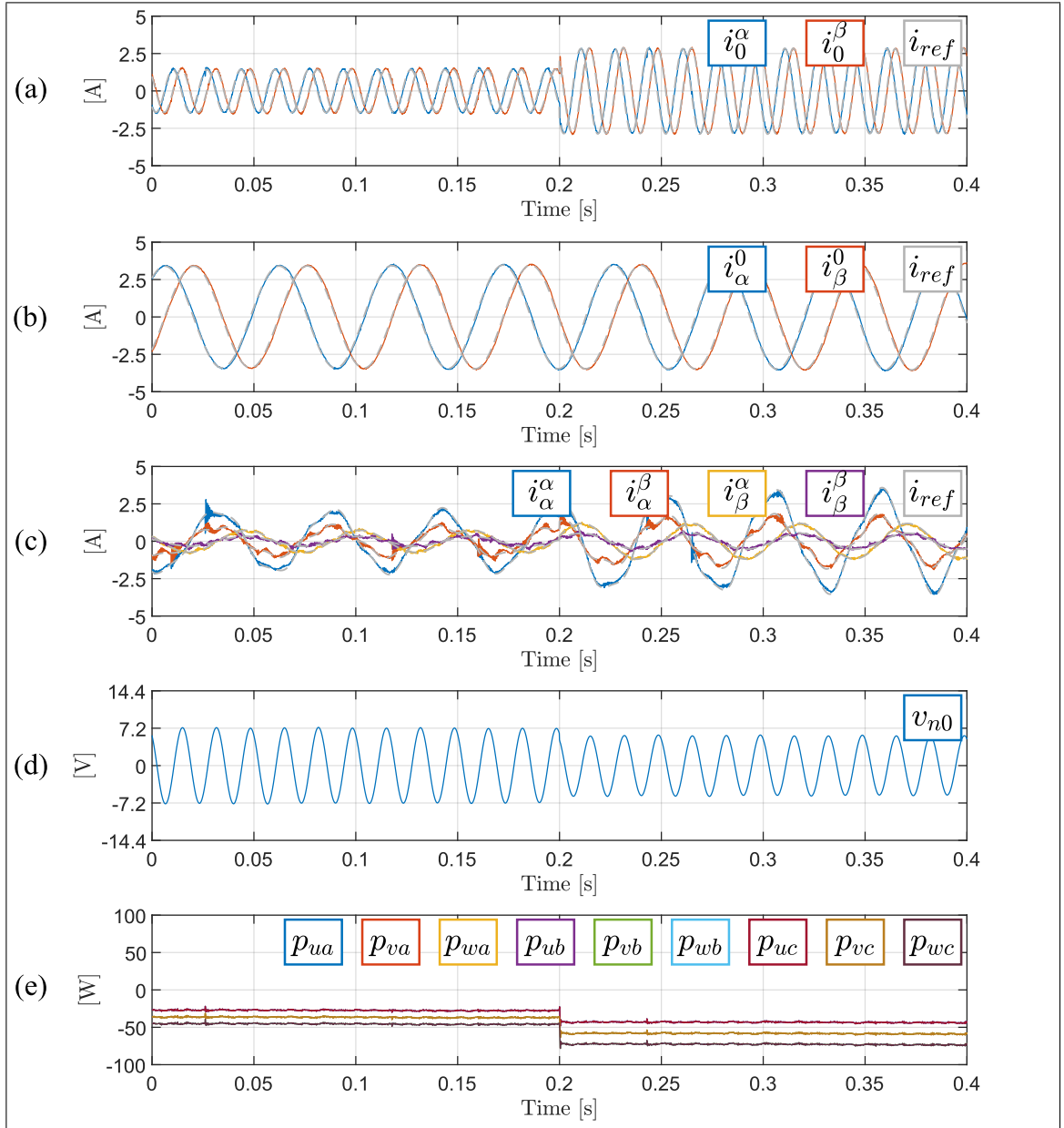


Figure 4.6. Experimental: Current controller performance (a) Input current (b) Output current (c) Circulating current (d) Common-mode voltage (e) Power in the branches

4.3.2. Machine Control

The machine control was previously analyzed in the simulation sections; however, the experimental implementation required overcoming several challenges, with the measurement of speed being fundamental. To measure the mechanical speed of the machine, an incremental encoder with 4096 pulses per revolution (ppr) is attached to the rotor of the induction machine. Fig. 4.7 shows the experimental results under varying mechanical speeds, ranging from low speed (10 [rpm]) to a higher value (900 [rpm]), which is equivalent to the curve shown in Fig.4.7c.

Fig. 4.7a shows the current in the Double- $\alpha\beta 0$ framework, under frequency variation. A low frequency component can be seen at low speed, converging to a higher frequency due to the outer loop which is capable to control the mechanical speed (Fig. 4.7c). From Fig. 4.6b we can see that the current follows the reference correctly and the machine parameters act like a filter, avoiding high ripple at this specific sample time.

Finally, it was mentioned that the rotor flux value is essential for both the current controller and power controller. Figure 4.7b shows the observed rotor flux, which has low noise due to the Kalman Filter, and its behavior is similar to the one described in the simulation section. That is, it has the same frequency as the stator current, but in this case, the magnitude is not part of a control loop. As a result, it undergoes variations when the mechanical speed tends to increase.

4.3.3. Optimal Power Control

To emulate distributed energy storage, each SM DC-link is connected to a unidirectional power source from the RSP-500 series. Consequently, the system cannot inject energy into the third port of the converter, restricting feasible operating modes to I, III and VI (Refer to Table 2.1). However, due to limitations in the speed measurements, the electrical machine will operate only as a motor.

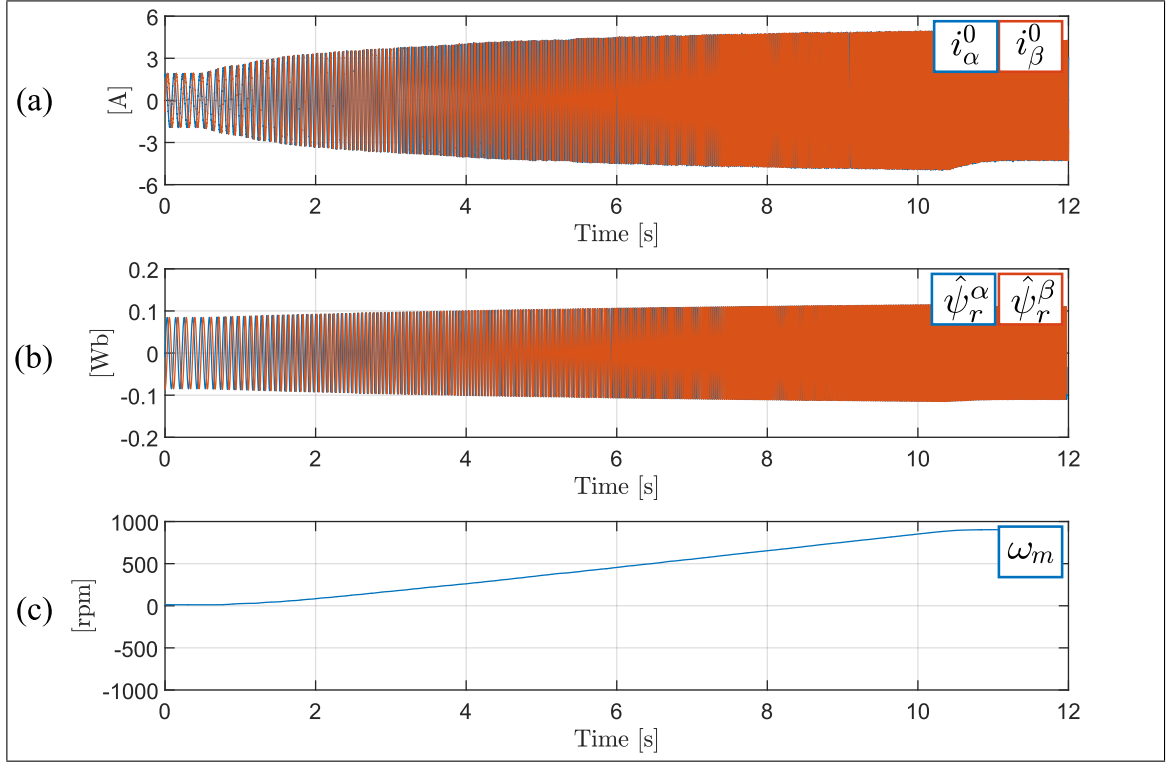


Figure 4.7. Experimental: Machine control performance (a) Machine current (d) Rotor flux (c) Mechanical speed

Fig. 4.8 shows the experimental results obtained from the setup under different operational modes. Figs. 4.8a, 4.8b, and 4.8c display the input, output, and circulating current, respectively. The waveform of these currents are better observed in the first three graphs of Fig. 4.6. It is noteworthy that, despite variations in the magnitude of current and voltage on the machine side, the circulating current does not vary significantly. However, the common-mode voltage tends to decrease proportionally with the increase in machine current (See (2.35), (2.34)).

The first unbalanced operation point occurs at 0.25 [s] of the time step, as shown in Fig. 4.8e. Fig. 4.6 serves as zoom-in of 4.8, so in Fig. 4.6d, it is evident that the CMV injected at this operational point is predominantly composed of the grid-side frequency. This phenomenon is explained by the power imbalance occurring between the sub-converters

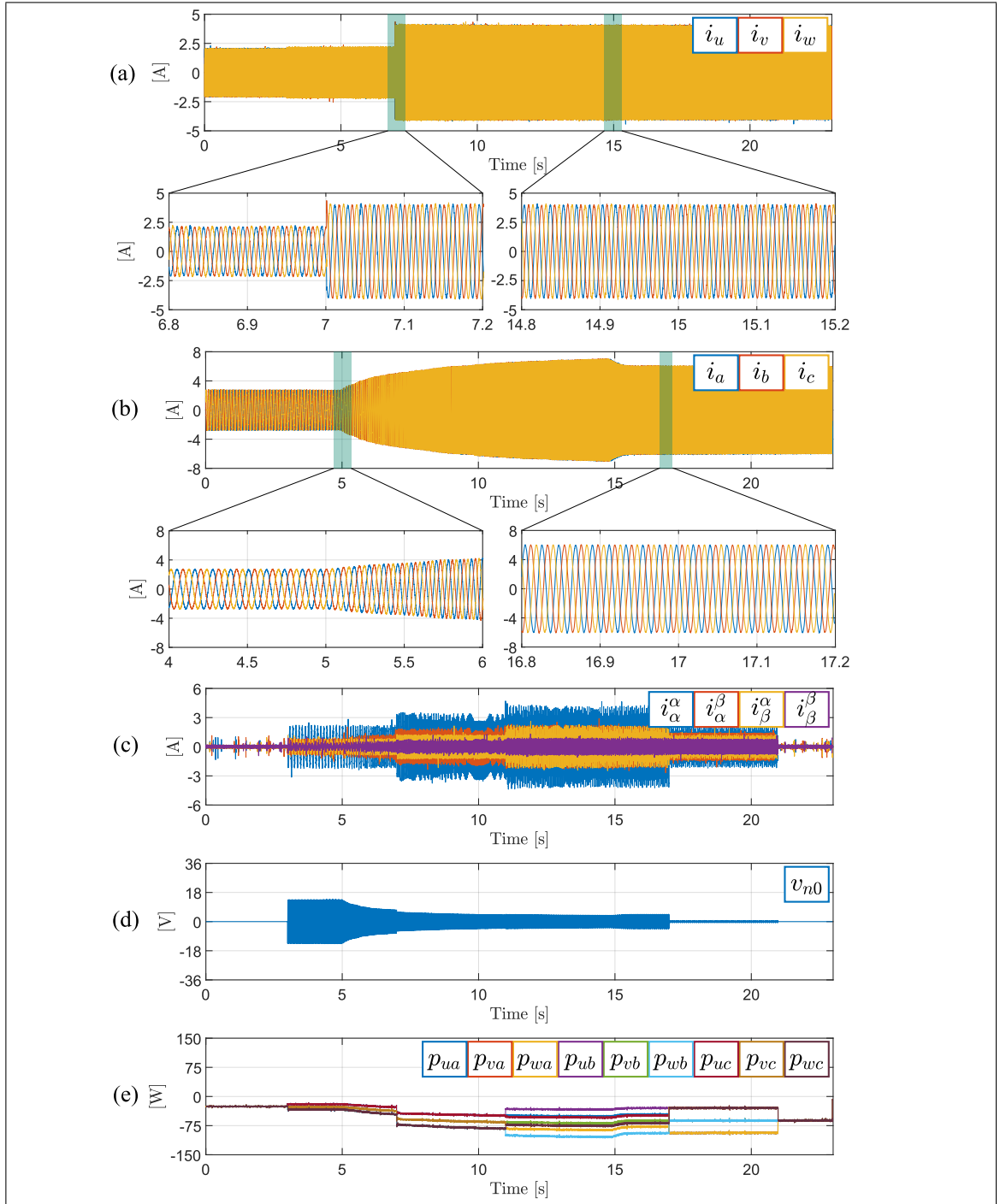


Figure 4.8. Experimental: Unbalanced power operation (a) Input current (b) Output current (c) Circulating current (d) Common-mode voltage (e) Power in the branches

connected to the output port. Consequently, a variation in the common-mode voltage becomes necessary and is in phase with the grid current (See (2.35), (2.34), and Fig. 2.3).

Fig. 4.6e provides a zoom-in on the branches' power, revealing that a variation in the input power induces a rearrangement of the variables, ensuring minimal impact on the power imbalance. Additionally, in Fig. 4.8e, the full horizon of the branches' power is depicted, presenting various operational points that lead to changes in the circulating current and common-mode voltage (4.8c and 4.8d).

4.3.4. Low Speed Operation

The primary operational point for evaluating the effectiveness of the proposed controller is low voltage operation. In the experimental setup depicted in Fig. 4.1, achieving low voltage involves running the induction motor at low speed. The oscilloscope results in Fig. 4.9 display various currents, including i_u , i_a , i_{ua} , and i_{wa} . Additionally, the voltages v_{ua} and v_{n0} are presented for a comprehensive view of the experimental outcomes. Moreover, three independent zoom-ins are presented to facilitate a more detailed analysis.

In the first row of Fig. 4.9, the speed is depicted, ranging from 10 [rpm] to 120 [rpm], demonstrating the system's capability to operate in variable speed drives. The second row illustrates unbalanced power operation across the nine branches over the full 10-second horizon of the oscilloscope capture. Two instances of imbalance are evident, highlighting variations in branch currents and common-mode voltage without causing interferences in the input or output currents.

The initial imbalance occurred at 1 [s], showcasing a notable alteration in the branch currents. In this instance, i_{ua} increases, while i_{wa} remains nearly constant. A slight modification in the common-mode voltage is observed at this operational point, attributed to the injection of both input and output components. Following this change in branch reference power, the machine begins to accelerate, leading to variations in current magnitudes due

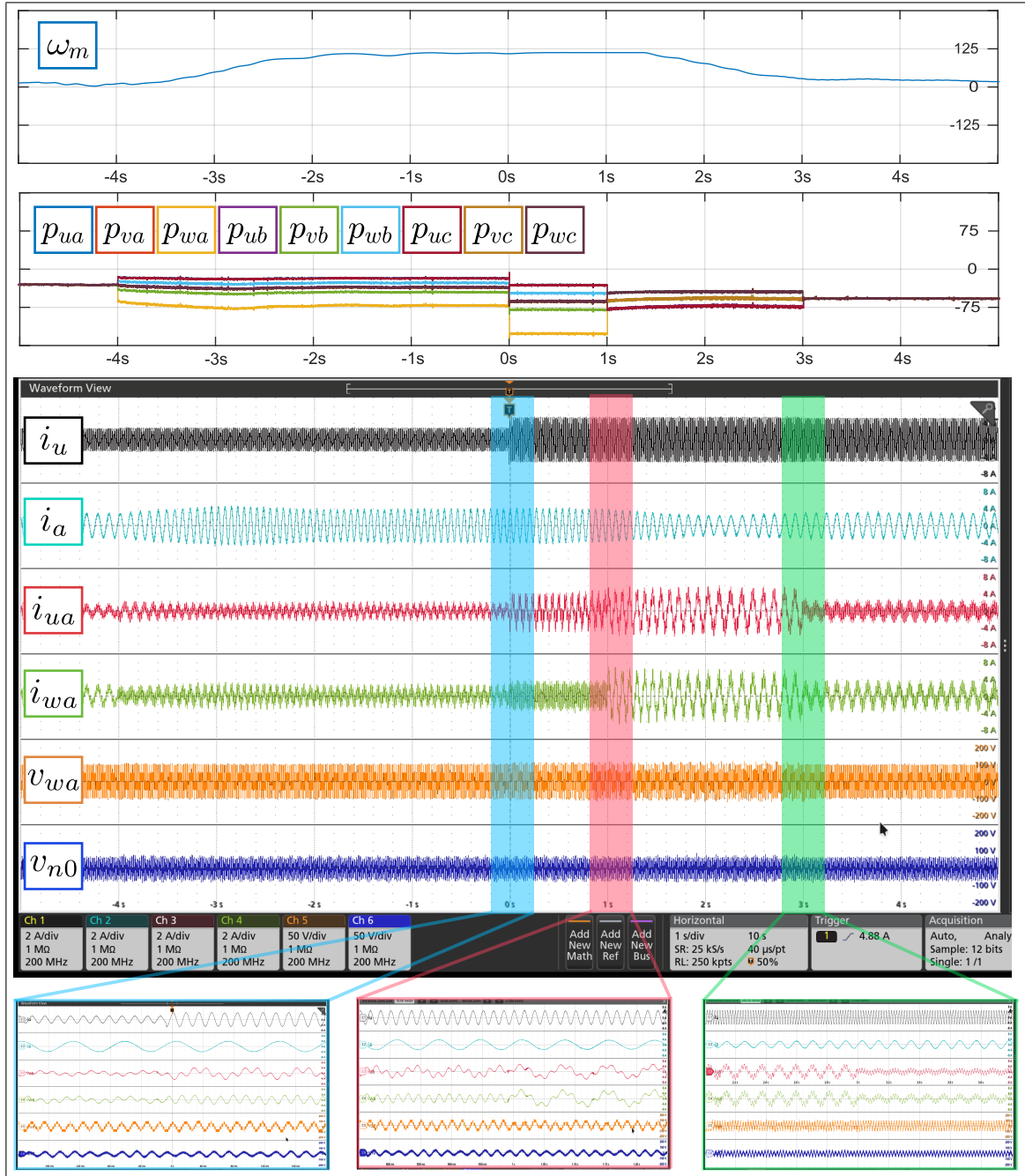


Figure 4.9. Experimental: Overall control strategy, considering low speed speed operation (Ch 1) Input current i_u (Ch 2) Output current i_a (Ch 3) Branch current i_{ua} (Ch 4) Branch current i_{wa} (Ch 5) Cluster voltage v_{ua} (Ch 6) Common-mode voltage v_{n0}

to the alteration in machine voltage. Despite these changes, the branches' power remains close to their respective references.

At 5 [s], a reference change is initiated on the input side, accompanied by a fast dynamic response from the controller. Simultaneously, the power controller adjusts the references of the circulating currents, resulting in an increase in both branch currents, as depicted in Fig. 4.9. A detailed zoom-in highlighted in blue provides additional insights into this reference change, revealing the presence of both input and output frequencies in the cluster voltage and the common-mode voltage.

Furthermore, at 6 [s], a second power imbalance is introduced, resulting in a significant alteration in branch currents. The specifics of this change are illustrated in the zoom-in highlighted in red. Subsequently, the system is compelled to operate without unbalanced power between branches, resulting in a reduction of branch currents and a modification in the common-mode voltage waveform. For additional details, refer to the zoom-in highlighted in green.

4.4. Summary

This chapter introduced the experimental setup and the parameters employed for the digital implementation of the proposed controller.

The simulation results demonstrated the capability of the proposed controller to operate under various operating modes as outlined in Table 2.1. Consequently, the controller was utilized to validate the control strategy in scenarios involving variable speed drives and power injection to the grid, replicating the conditions of the experimental implementation.

Finally, the proposed controller is successfully implemented in a digital controller, validating its effectiveness in operating a real prototype across various operational points. These include scenarios such as input side reference changes, machine control, and unbalanced power operation.

5. CONCLUSIONS AND FUTURE WORK

The M3C is capable of integrating distributed energy storage, performing three simultaneous conversion stages. However, the control strategies associated with this type of application are still under development. The main challenges lie in the external control loop's capability to define currents and voltages to balance the power distribution of the DES. The dynamic nature of DES necessitates ongoing research to develop effective control strategies for the M3C, aiming to enhance its adaptability and responsiveness in real-world scenarios. As these control strategies evolve, the M3C holds the potential to play a crucial role in creating resilient and efficient power distribution systems.

Nevertheless, the literature predominantly focuses on distributed energy storage for converters with fewer branches. Numerous publications delve into topics such as CHB and M2C, emphasizing energy storage in the context of PV, battery energy storage, and HVDC applications. In recent years, the M3C has gained prominence for its direct AC/AC conversion capabilities. However, only modified topologies of M3C using dual active bridge as sub-modules, include capabilities for integrating energy storage.

This work introduced an independent formulation of the instantaneous power for each branch, utilizing the three-phase components at the input and output ports. The formulated approach enables the computation of power for each branch based on the currents and voltages of the system, irrespective of the loads connected to the two three-phase systems.

Furthermore, the implementation of a null space matrix in the circulating currents reduce the number of variables that the controller needs to compute. This unique approach allows for the formulation of an external control loop that remains independent of the number of sub-modules within the converter. It fixes the size of both optimization problems, although the computing time exhibits a strong dependence on the number of iterations employed in the Block Coordinate Descent method utilized to obtain the solution.

At a more profound level, the formulated internal controller assumes a fundamental role owing to its fast response to step changes. The design of the current controller facilitates the integration of machine control on the output side of the converter. This integration is achieved through the direct inclusion of the current model of the induction motor in the dynamic equations that describe the system. However, this approach exhibits a notable dependence on external perturbations, which can prove detrimental if the estimation of the rotor flux is imprecise or if the measurement of the grid contains significant noise components.

Experimental and simulation results have been conducted to successfully validate the proposed control scheme. The algorithm enables the system to operate with inter-cluster unbalanced power using the optimal reference generator. Simulations provide insights into the system's behavior under various operating modes across different speed ranges. Meanwhile, experimental results demonstrate that the proposed controller is capable of operating in a digital implementation, effectively driving the functionalities of an experimental setup.

Even though the system works, the dynamic response has a strong dependence on the system parameters. Therefore, the control must vary its parameters to establish an optimal operational point. Despite this challenge, the conducted research validates the hypothesis, demonstrating that:

- *H* An optimal control strategy, based on the theory of instantaneous power and complemented by an internal predictive current control, effectively governs a modular multilevel matrix converter with distributed energy storage (M3C-DES). This control scheme manages both network supply and machine drive operations, generating optimal references for circulating currents and common mode voltage, and showing robust performance, particularly during unbalanced power operation scenarios.

Therefore, this thesis has successfully developed an M3C-DES model, incorporating the electrical machine dynamics. It includes the design of a control loop based on model predictive control for the primary currents (input, output, and circulating current). Additionally, it proposes an external controller using the instantaneous power theory capable of generating an optimal combination of common-mode voltage and circulating current to induce intra-cluster power imbalance. These achievements fulfill the main objectives of this work.

The main proposition of this work is grounded in the instantaneous power theory and is currently limited to inter-cluster unbalanced operation. Further research is essential to identify a suitable controller for the M3C-DES. This may involve comparing control methods and developing metrics for a thorough analysis of converter design and the capabilities it might require under this type of operation—questions that remain open in the context of M3C.

In addition, the developed theory could potentially be employed to devise a direct power control strategy for the nine branches, streamlining the overall control strategy by reducing the number of modules that must be implemented. Additionally, for a fully distributed energy storage integration, an active intra-cluster unbalanced power approach must be implemented. Furthermore, the implementation of a reference generator for the power controller, based on an energy management system, could be intriguing.

Lastly, exploring more advanced internal control strategies, such as implementing low switching frequency or space vector modulation, holds the potential to enhance the overall system efficiency. These avenues represent exciting directions for future research and development in the M3C-DES domain.

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APPENDIX

A. SOLUTION TO THE PREDICTIVE CONTROL PROBLEM

The model predictive control optimization problem involves constraints. Nevertheless, it is possible to circumvent the box constraints by formulating the following optimization:

$$\begin{aligned}
 \min_{\mathbf{x}_{k+1}, \mathbf{u}_k} J_k &= \|\mathbf{x}_{k+1} - \mathbf{x}_{k+1}^*\|_Q^2 + \|\mathbf{u}_k - \mathbf{u}_k^*\|_\Lambda^2 \\
 s.t. \quad \mathbf{x}_{k+1} &= \mathbf{A}_d \mathbf{x}_k + \mathbf{B}_d \mathbf{u}_k + \mathbf{E}_d \mathbf{v}_k \\
 -\mathbf{1}_{9 \times 1} &\leq \mathbf{u}_k \leq \mathbf{1}_{9 \times 1}
 \end{aligned} \tag{A.1}$$

Using the objective function and replacing with the discrete-time dynamic model:

$$\begin{aligned}
 J_k &= (\mathbf{A}_d \mathbf{x}_k + \mathbf{B}_d \mathbf{u}_k + \mathbf{E}_d \mathbf{v}_k - \mathbf{x}_{k+1}^*)^\top \mathbf{Q} (\mathbf{A}_d \mathbf{x}_k + \mathbf{B}_d \mathbf{u}_k + \mathbf{E}_d \mathbf{v}_k - \mathbf{x}_{k+1}^*) + \dots \\
 &\quad \dots (\mathbf{u}_k - \mathbf{u}_k^*)^\top \mathbf{\Lambda} (\mathbf{u}_k - \mathbf{u}_k^*) \tag{A.2}
 \end{aligned}$$

where the optimum can be found making the partial derivative with respect to $\mathbf{u}_k$ equal to zero

$$\frac{\partial J_k}{\partial \mathbf{u}_k} = 2(\mathbf{B}_d^\top \mathbf{Q} (\mathbf{A}_d \mathbf{x}_k + \mathbf{B}_d \mathbf{u}_k + \mathbf{E}_d \mathbf{v}_k - \mathbf{x}_{k+1}^*)) + 2\mathbf{\Lambda} (\mathbf{u}_k - \mathbf{u}_k^*) = 0 \tag{A.3}$$

and reordering:

$$-(\mathbf{B}_d^\top \mathbf{Q} \mathbf{B}_d) \mathbf{u}_k = \mathbf{B}_d^\top \mathbf{Q} (\mathbf{A}_d \mathbf{x}_k + \mathbf{E}_d \mathbf{v}_k - \mathbf{x}_{k+1}^*) - \mathbf{\Lambda} \mathbf{u}_k^* \tag{A.4}$$

Consequently, the optimal control input is:

$$\mathbf{u}_k^{\text{opt}} = -\mathbf{W}^{-1}\mathbf{F}_k \quad (\text{A.5})$$

where,

$$\begin{aligned} \mathbf{W} &= \mathbf{B}_d^{\text{T}}\mathbf{Q}\mathbf{B}_d + \Lambda \\ \mathbf{F}_k &= \mathbf{B}_d^{\text{T}}\mathbf{Q}(\mathbf{A}_d\mathbf{x}_k + \mathbf{E}_d\mathbf{v}_k - \mathbf{x}_{k+1}^*) - \Lambda\mathbf{u}_k^* \end{aligned} \quad (\text{A.6})$$

B. GLOSSARY

B.1. Abbreviations

NPC	Neutral Point Campled
MMC	Modular Multilevel Converter
CHB	Cascaded H-Bridge
M2C	Modular Multilevel Converter
M3C	Modular Multilevel Matrix Converter
TSCB	Triple Star Cells Bridge
DES	Distributed Energy Storage System
AC	Alternating Current
DC	Direct Current
SM	Sub-module
HB	Half-Bridge
FB	Full-Bridge
DAB	Dual Active Bridge
PV	Photovoltaic
BESS	Battery Energy Storage System
WECS	Wind Energy Conversion System
FACTS	Flexible AC Transmission System
LFAC	Low-Frequency AC Transmission Systems
THD	Total Harmonic Distortion
P	Proportional Controller
PI	Proportional Integral Controller
PR	Proportional Resonant Controller
MPC	Model Predictive Controller
CCS	Continuous Control Set
FCS	Finite Control Set

SISO	Single Input Single Output
MIMO	Multiple Inputs Multiple Output
CMV	Common-Mode Voltage
N	Number of Sub-Modules
SOC	State of Charge
PWM	Pulse Width Modulation
LS	Level Shifted
KF	Kalman Filter
KKT	Karush-Kuhn Tucker
QP	Quadratic Programming
BCD	Block Coordinate Descent Method
QPS	Quadratic Programming Solver

B.2. Set and Indexes

$\mathcal{K}$	Branch Index $\in \{1 \dots 9\}$
Φ	Input Port Index $\in \{u, v, w\}$
X	Output Port Index $\in \{a, b, c\}$
j	Sub-Module Index $\in [1, N]$
g	Grid Side
m	Machine Side
$\alpha\beta 0$	Orthogonal Components or Clarke Transform Index
t	Continuous Time
k	Discrete Time
z	Circulating Current
$n0$	Common-Mode

B.3. Variables

v_κ	Voltage of the Branch $\kappa \in \mathcal{K}$
v_ϕ	Voltage of the Phase $\phi \in \Phi$
v_χ	Voltage of the Phase $\chi \in \mathbf{x}$
v_{n0}	Common-Mode Voltage
v_κ^α	Voltage of the Branch $\kappa \in \mathcal{K}$
v_κ^β	Ninety Degree Component of the Voltage of the Branch $\kappa \in \mathcal{K}$
v_ϕ^β	Ninety Degree Component of the Voltage of the Phase $\phi \in \Phi$
v_χ^β	Ninety Degree Component of the Voltage of the Phase $\chi \in \mathbf{x}$
i_κ	Current of the Branch $\kappa \in \mathcal{K}$
i_ϕ	Current of the Phase $\phi \in \Phi$
i_χ	Current of the Phase $\chi \in \mathbf{x}$
i_κ^α	Current of the Branch $\kappa \in \mathcal{K}$
i_ϕ^α	Current of the Phase $\phi \in \Phi$
i_χ^α	Current of the Phase $\chi \in \mathbf{x}$
i_κ^β	Ninety Degree Component of the Current of the Branch $\kappa \in \mathcal{K}$
i_ϕ^β	Ninety Degree Component of the Current of the Phase $\phi \in \Phi$
i_χ^β	Ninety Degree Component of the Current of the Phase $\chi \in \mathbf{x}$
i_z^α	Circulating Current
i_ϵ^α	Circulating Current
i_z^β	Ninety Degree Component of the Circulating Current
i_ϵ^β	Ninety Degree Component of the Circulating Current
p_κ	Power of the Branch $\kappa \in \mathcal{K}$
p_g^*	Reference Power of the Grid Side
ω_m	Mechanical Speed

B.4. Vectors

$\mathbf{x}$	State Space Vector
$\mathbf{u}_{\mathcal{K}}$	Control Input Vector
$\mathbf{i}_{\alpha\beta 0}^{\alpha\beta 0}$	Double- $\alpha\beta 0$ Currents Vector
$\mathbf{v}_{\alpha\beta 0}^{\alpha\beta 0}$	Double- $\alpha\beta 0$ Voltages Vector
$\mathbf{p}_{\alpha\beta 0}^{\alpha\beta 0}$	Double- $\alpha\beta 0$ Voltages Vector
$\boldsymbol{\psi}_r^{\alpha\beta}$	Rotor Flux Vector
$\mathbf{v}_{\phi}^{\alpha\beta}$	Input Side Voltage Vector
$\mathbf{v}_{\chi}^{\alpha\beta}$	Output Side Voltage Vector
$\lambda_{\mathcal{K}}$	Unbalanced Power Factor

B.5. Matrices

$\mathbf{C}_{\alpha\beta 0}$	Clarke Transform Matrix
$\mathbf{D}_{\alpha\beta 0}$	Double- $\alpha\beta 0$ Transformation Matrix
$\mathbf{P}$	Incidence Matrix
$\mathbf{P}_{\alpha\beta 0}$	Port to Double- $\alpha\beta 0$ Transformation Matrix
$\mathbf{N}$	Nullspace Matrix of $\mathbf{P}$
$\mathbf{A}$	State Matrix
$\mathbf{B}$	Control Input Matrix
$\mathbf{E}$	External Perturbance Matrix