



PONTIFICIA UNIVERSIDAD CATÓLICA DE CHILE
ESCUELA DE INGENIERÍA

MODULAR MULTILEVEL CONVERTER UNDER UNBALANCED INTER-CLUSTER POWER OPERATION: A COMPREHENSIVE ANALYSIS OF CURRENT AND VOLTAGE REQUIREMENTS

TOMÁS SALVADORES VIERTTEL

Thesis submitted to the Office of Research and Graduate Studies
in partial fulfillment of the requirements for the degree of
Master of Science in Engineering

Advisor:

JAVIER PEREDA TORRES

Santiago de Chile, July 2023

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*To my beloved aunt Aurora, whom I
deeply cherish*

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ABSTRACT

Power converters perform crucial tasks that enable and stimulate the electrification of modern society. Among them, the Modular Multilevel Converter (MMC) has recently made significant contributions in fields like energy transmission, and has much more to offer. The MMC can be integrated with a Distributed Energy System (DES) to expand its functionalities and carry out multiple simultaneous tasks. For instance, a MMC integrated with a Battery Energy Storage System (BESS) can interface ac and dc grids while providing services like countering the variability and intermittency of renewable energy sources. However, a DES can induce power imbalances within the MMC, which affects the operating current and voltages of the converter. Research has raised concerns about this phenomenon, but its full extent, behavior, and implications in different MMC-DES applications are yet to be uncovered.

This thesis formulates equations that model the effects of inter-cluster power imbalances in MMCs. To do so, it introduces a set of metrics that represent imbalances and their effects. It proposes a mathematical representation of DESs based on their expected imbalance capabilities and analyses the strain they could exert on a MMC. These examinations are carried out for multiple possible operating modes of a MMC-DES.

The equations and metrics developed in this thesis allow the quick evaluation of possible MMC-DES applications regarding the current and voltage requirements of MMC hardware. It provides tools to compare the convenience of MMC and similar contending topologies for applications with DES integration. The developed imbalance model equations are validated through experiments and simulations that test them under diverse conditions and operating points.

Keywords: Modular Multilevel Converter (MMC), Distributed Energy System (DES), Power Imbalance, Battery Energy Storage System (BESS), Current Overrating, Voltage Overrating.

RESUMEN

Los convertidores de potencia realizan labores cruciales que permiten y estimulan la electrificación de la sociedad moderna. Entre ellos, el Conversor Modular Multinivel (MMC) ha realizado recientemente importantes aportes en campos como la transmisión de energía, y tiene mucho más que ofrecer. El MMC puede integrarse con un Sistema de Energía Distribuido (DES) para ampliar sus funcionalidades y llevar a cabo múltiples tareas simultáneas. Por ejemplo, un MMC integrado con un sistema de almacenamiento de energía en baterías (BESS) puede interconectar redes ac y dc prestando servicios como contrarrestar la variabilidad e intermitencia de fuentes de energía renovables. Sin embargo, un DES puede inducir desbalances de potencia dentro del MMC, lo que afecta a las corrientes y voltajes internos del convertidor. Investigaciones han suscitado preocupación por este fenómeno, pero aún no se ha desvelado todo su alcance, comportamiento e implicaciones en diferentes aplicaciones MMC-DES.

Esta tesis formula ecuaciones que modelan los efectos de desbalances de potencia inter-cluster en MMCs. Para ello, introduce un conjunto de métricas que representan los desbalances y sus efectos. Propone una representación matemática de los DES basada en sus capacidades de desbalance y analiza el estrés que podrían ejercer a un MMC. Esto se lleva a cabo para múltiples modos de operación de un MMC-DES.

Las ecuaciones y métricas desarrolladas en esta tesis permiten evaluar rápidamente posibles aplicaciones de MMC-DES en cuanto a los requisitos de corriente y voltaje del MMC. Provee herramientas para comparar la conveniencia del MMC y topologías similares para aplicaciones con integración de DES. Las ecuaciones del modelo de desbalance desarrolladas son validadas mediante experimentos y simulaciones que las evalúan en diversas condiciones y puntos de operación.

Palabras Claves: Conversor Modular Multinivel (MMC), Sistema de Energía Distribuido (DES), Desbalance de Potencia, Sistema de Almacenamiento de Energía en Baterías (BESS), Sobredimensionamiento de Corriente, Sobredimensionamiento de Voltaje.

1. INTRODUCTION

Power converters are one of the main enablers of the level of electrification of modern society. They allow an efficient transfer and conversion of electrical energy from one form to another. A wide range of modern devices and processes depend on them, from phone chargers to wind energy generation plants. Power converters are continuously adopted in new functions of our society, enabling higher energy productivity in industrial processes and better quality products (van Wyk & Lee, 2013).

There are many topologies of power converters. Their variety in characteristics makes them suitable for different use cases. Among them are the multilevel converters, which have multiple voltage levels. They can operate at high voltage magnitudes and produce cleaner currents than conventional two-level converters, thus reducing the filtering inductors size. However, they are harder to build and control and occasionally more expensive.

In the last 20 years, the Modular Multilevel Converter (MMC), also known as Double Star Bridge Cell (DSBC), has become a relevant topology. Its modularity makes it convenient for medium to high-voltage use cases and easy to maintain and repair. Although it is currently used in fields like high voltage dc (HVDC) transmission and medium voltage motor drives, researchers continuously explore new applications to exploit its potential fully. Its attributes are continuously being investigated in new fields through ongoing research.

The MMC can perform three conversion stages simultaneously by attaching a distributed energy system (DES) to the submodules (SMs), forming a MMC-DES as shown in Fig. 1.1. The SMs contain a dc voltage capacitor that must be fixed at a voltage level much lower than the main dc link. These capacitors can be floating or act as an interface or secondary dc link by connecting them to another device of the same voltage. Such devices can be the distributed components of an energy system. These DES components can be directly connected to the SM capacitor or through a dc-dc conversion stage. A MMC-DES

has the capacity to exchange energy in any direction between the two main ports and the DES.

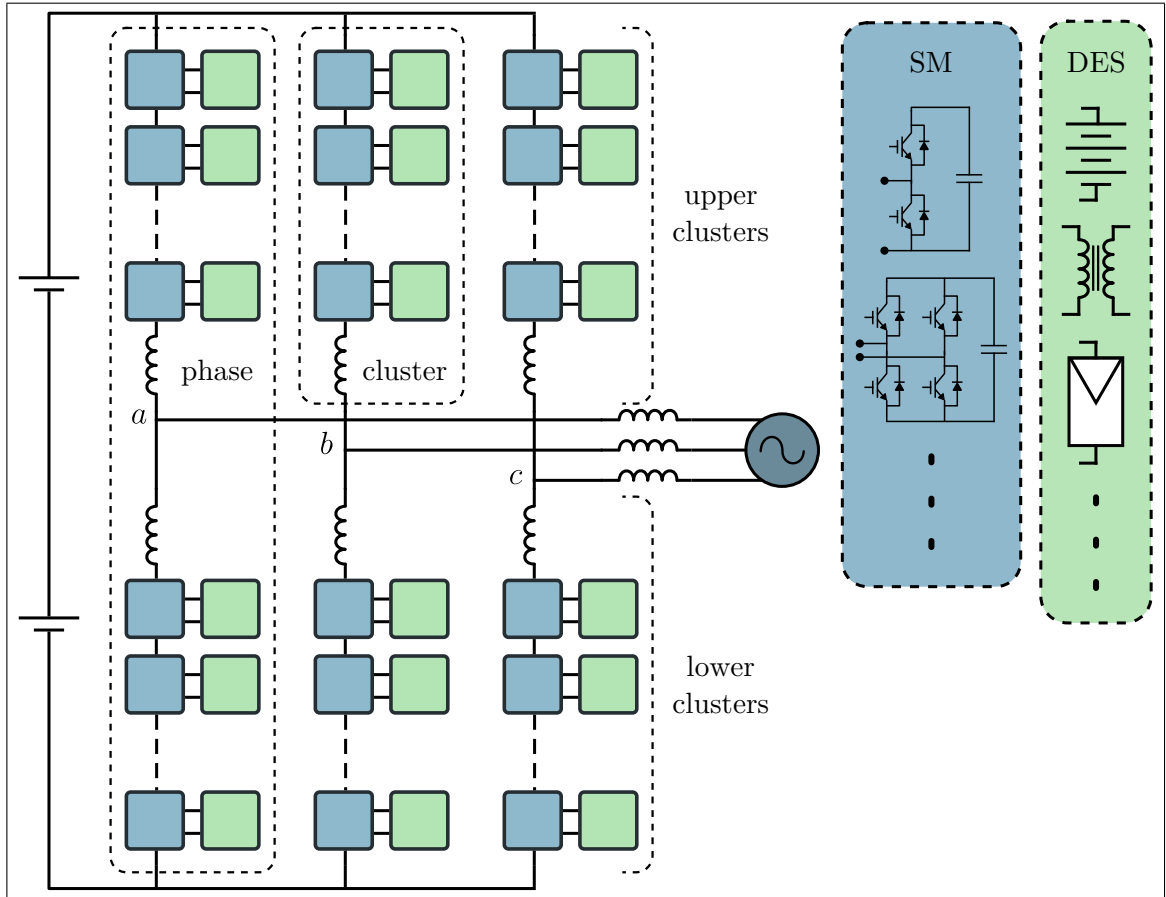


Figure 1.1. MMC-DES topology.

Integrating a third conversion stage presents additional challenges to the converter design. Its control algorithm must first be conditioned to perform an additional task without disturbing the original port control, while having the same actuator space available. Traditionally, the control of these DES is performed by injecting new currents into the converter. The injected currents can grow considerably if the DES does not provide or consume an even amount of energy across all the SMs. Depending on the operating point, the new aggregated currents can be higher or lower than the standard currents. Therefore, it is

essential to determine the size of these currents during the design stage of the converter to select electrical components that can withstand them.

1.1. Modular multilevel converter with integrated distributed energy system (MMC-DES)

The term “distributed energy system” is hereby used to refer to a collection of electrical devices with a dc voltage terminal that can be connected to one or more SMs of a MMC to exchange energy in one or two directions. There have been several proposals for MMC-DES integration in the literature, with varying degrees of practical applicability. Nonetheless, this remains an open area of research, and new kinds of MMC-DES may be proposed in the future.

1.1.1. Battery energy storage system (BESS)

Battery energy storage systems (BESS) are devices designed to store and deliver energy in batteries for a specific application. They typically consist of batteries, a battery management system, an energy management system, and a power conversion system. BESS enables the large-scale use of batteries by providing both storage capacity and an interface for utilizing that capacity (ADB, 2018).

BESS have a wide range of applications. They can perform Peak Shaving, absorbing excess grid energy demand and generation when the network capacity is strained. This function is a significant enabler of large-scale adoption of renewable energy, helping to counter the variability that characterizes solar and wind generation (X. Li, Hui, & Lai, 2013). Similarly, they can be used for Load Leveling, reallocating consumer demand to off-peak hours. BESS have an extremely fast time response capability that makes them well-suited for providing complementary services to the grid. In addition to these services, they can also be used for other grid applications, such as uninterruptible power supplies (UPS) or microgrid support (Dunn, Kamath, & Tarascon, 2011). Moreover, they are a

crucial component of electric vehicles (EVs), providing the necessary power for these vehicles to operate.

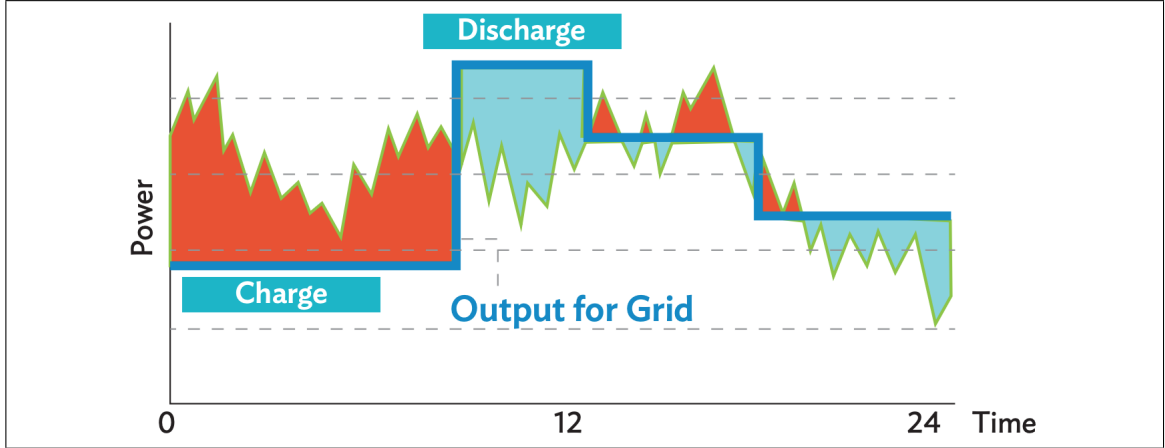


Figure 1.2. Consumer demand without and with Load Leveling (ADB, 2018).

A MMC-BESS is a MMC with batteries connected to its submodules. The batteries can be directly connected to the SM capacitor or interfaced through a dc-dc conversion stage. With only minor adjustments to its control algorithms, the converter can handle the batteries and perform the battery management system (BMS) tasks (Vasiladiotis & Rufer, 2015). As a result, the MMC-BESS can function as both an ac-dc converter and a BESS without requiring additional semiconductors. However, in certain situations it may be convenient to use additional semiconductors to interface the batteries to the SM or to implement more complex structures like full-bridge SMs, as noted in previous studies (Trintis, Munk-Nielsen, & Teodorescu, 2011)(Ali, Ling, Tian, & Huang, 2021). The converter can also be configured for partial battery integration when less storage capacity is needed.

The MMC-BESS is an expensive BESS topology (Wang et al., 2016), but it can be an attractive solution for use cases that benefit from its distinctive characteristics. First and foremost, is its ability to integrate low-voltage battery cells into medium-to-high voltage applications. Modular multilevel BESS topologies like the MMC and the Cascaded

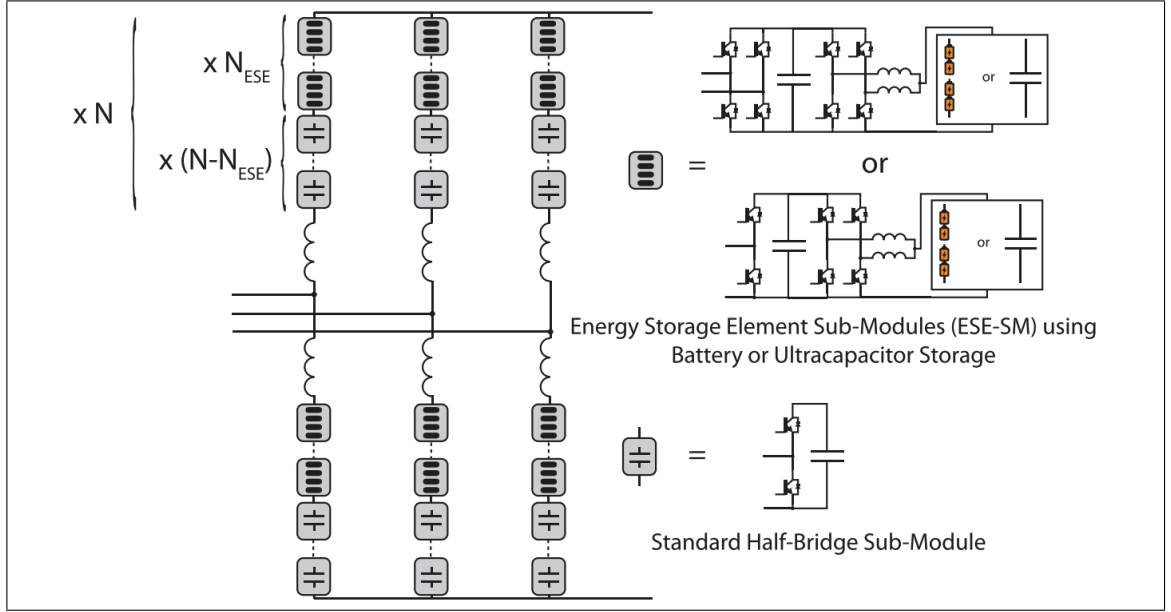


Figure 1.3. MMC with partially integrated energy storage system (Judge & Green, 2019).

H-Bridge (CHB) have also demonstrated greater efficiency than conventional two-level converters (Wang et al., 2016). Moreover, the distributed integration of the BESS into the MMC has proven to be more efficient than centralized integration (Soong & Lehn, 2014a). Another significant advantage of the MMC-BESS is its capability to perform three energy conversion processes interchangeably at the same time. This capability is a crucial differentiator between MMC-BESS and CHB-BESS, with which it shares most of its distinctive features. Furthermore, the modularity of MMC also makes it robust against battery failures. Additional benefits of MMC-BESS include the use of low-voltage semiconductors for high-voltage applications and high-quality power output.

MMC-BESS has been proposed for several applications, such as providing grid services (Judge & Green, 2019) (Wang et al., 2016) (Blatsi, Judge, Finney, & Merlin, 2023) (Ali et al., 2021), serving as the primary EV power conversion system (Quraan, Yeo, & Tricoli, 2016) (Ronanki & Williamson, 2018) (Quraan, Tricoli, D'Arco, & Piegari, 2017), countering variability in wind generation and providing fault ride-through capability (Novakovic & Nasiri, 2017), and functioning as a STATCOM with active and reactive

power provision (Cupertino et al., 2020)(Chaudhary, Cupertino, Teodorescu, & Svensson, 2020), all of this while maintaining robustness to failures (Soong & Lehn, 2016) (Chen, Li, & Cai, 2017).

MMC-BESS is by far the most studied and relevant of the MMC-DES topologies. It has been analyzed under different configurations and conditions, and experimentally verified multiple times. The multiple application proposals and analysis validate to some degree its potential and convenience and put in evidence its flexibility as a topology.

1.1.2. Solid state transformer (SST)

A solid-state transformer (SST), also known as a power electronic transformer (PET), is an isolated ac/ac power converter designed to replace conventional grid transformers. It consists of three main stages: first, a low-frequency to high-frequency ac conversion stage; second, an up/down voltage step through a high-frequency transformer; and finally, a high-frequency to low-frequency ac conversion stage. SSTs have a significantly reduced size and weight compared to traditional transformers because of the use of a high-frequency transformer for the magnitude step. They also provide a higher level of controllability in the conversion process, enabling functionalities like voltage sag compensation, power flow control, or fault current limitation. However, SSTs are more complex and expensive than conventional transformers because they are power electronics circuits (Ronan, Sudhoff, Glover, & Galloway, 2002) (She, Burgos, Wang, Wang, & Huang, 2012).

In a MMC-SST, each of the SMs is connected to a dual active bridge (DAB) that performs an isolated dc voltage step. These DABs are then mutually connected and fed into a dc/ac conversion stage, as described in (Briz, Lopez, Rodriguez, & Arias, 2016)(Shojaei & Joós, 2013). It is worth noting that the junction of the DAB outputs provides a low-to-medium voltage power source that can be easily accessed. Research has shown that a MMC-SST can also function as a four-port converter, with high/medium voltage ac and dc ports and medium/low voltage ac and dc ports (Zhou et al., 2021)(D. Li, Ji, Sun, Zhao, & Zhang, 2020). This configuration allows the integration of many functionalities into one

converter, which can be very convenient, but at the cost of requiring a large number of semiconductors.

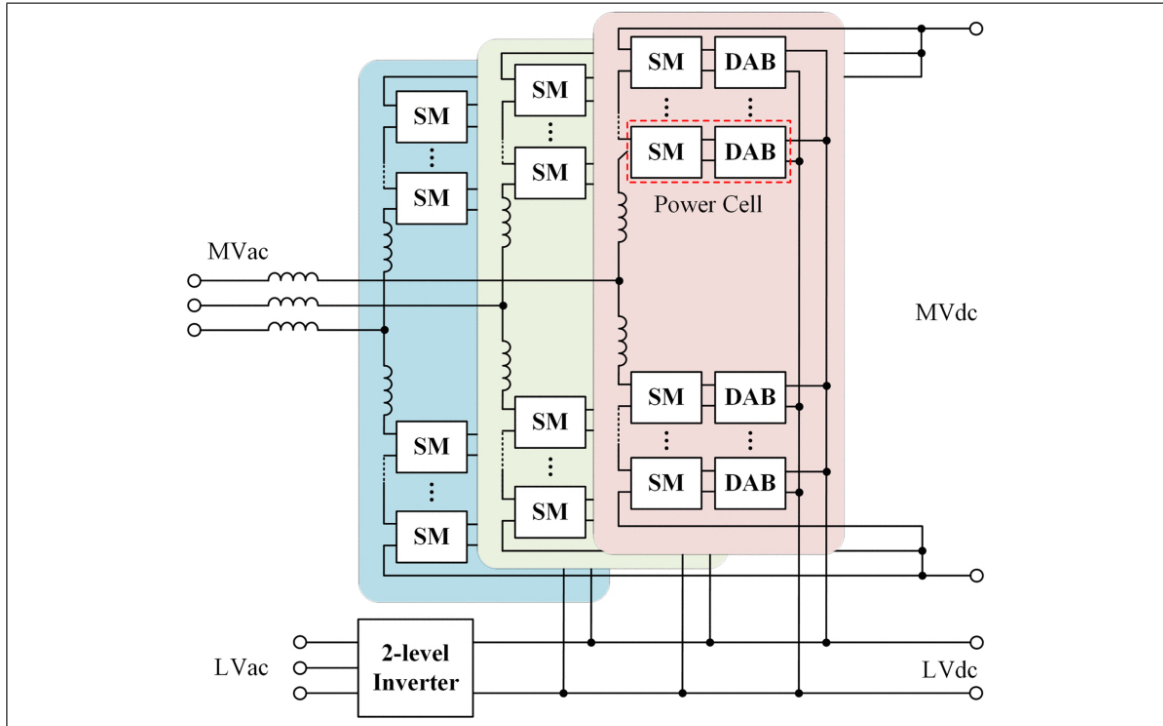


Figure 1.4. MMC with integrated SST and additional dc port (Zhou et al., 2021).

1.1.3. Photovoltaic (PV) systems

Solar power converters are devices used to convert the electrical energy generated by solar panels into a form that the electrical grid can use. There are many converter topologies that differ mainly depending on the characteristics and power level of the solar system being designed. Small module inverters convert the energy of a single solar module, while big solar generation plants use central inverters to connect large solar module networks to the grid.

In a MMC-PV configuration, single solar modules or arrays are connected to the SMs. This can be achieved through a dc-dc conversion stage (Rivera et al., 2013) or by directly

connecting them (Acharya, Ricco, Sera, Teodorescu, & Norum, 2019). Each SM can perform a separate maximum power point tracking (MPPT), and if a dc-dc stage is used, the MPPT can be separated from the MMC control. However, if the solar modules are directly connected, the MMC control must regulate the SMs at different capacitor voltages to perform separate MPPTs for each module (Ali et al., 2021). Using a dc-dc stage also provides the benefit of isolating the panels from the rest of the SM, which increases safety by allowing for the grounding of the panels. However, this topology can be more expensive due to the additional cost of the isolation circuitry.

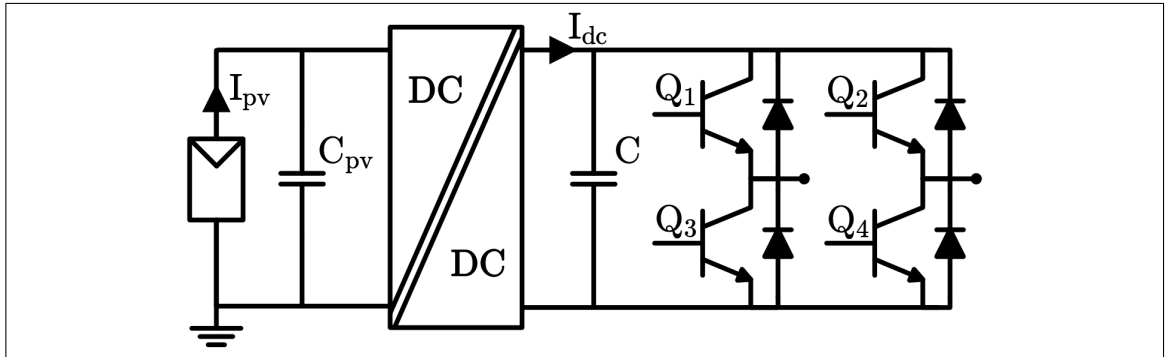


Figure 1.5. Isolated MMC SM with integrated PV system (Perez et al., 2013).

The MMC is typically better suited for high-power applications, making it a direct competitor to central inverters in the PV sector. Its modular design offers increased reliability as an energy provider to the grid and allows for more efficient MPPT. By incorporating an isolated dc-dc stage, the MMC-PV can function as a transformerless solar inverter, handling the isolation in the SMs. Additionally, the MMC-PV can be connected to a HVDC grid and an ac grid simultaneously, allowing for real-time energy dispatch based on the requirements of the grid. However, it should be noted that this scenario is atypical. Despite its many advantages, the MMC-PV has the drawback of being a far more complex and presumably more expensive system than a central inverter.

1.1.4. Other energy systems

As seen in the previous cases, the MMC-DES is very flexible regarding the kinds of DES it can integrate. The converter SMs can accommodate virtually any device connected to a dc voltage port, provided that the voltage magnitude is similar to that of the SM capacitor and operates within the power transfer capability of the converter SMs (Liang et al., 2021). A creative example of such adaptability is an MMC with EV charging functionality, but it has not drawn much attention (Mao et al., 2019). With this level of adaptability, new DESs could be proposed in the future for MMC integration, like Fuel Cell systems (FC).

A MMC-DES can also integrate a hybrid DES, which combines devices of different natures to complement their strengths or serve different purposes. For instance, both a distributed PV system and BESS can be integrated into a MMC simultaneously (Perez et al., 2013), enabling a PV plant to provide energy throughout the day and night. This system has been recognized as a possible future trend for the MMC (Dekka, Wu, Fuentes, Perez, & Zargari, 2017) (Perez, Bernet, Rodriguez, Kouro, & Lizana, 2015) (Ali et al., 2021). Moreover, the MMC can accommodate other hybrid DES that could be proposed in the future. Proof-of-concept studies showcase the flexibility of possible future hybrid DES (Briz et al., 2016) (Schroeder & Jaeger, 2019), emphasizing the MMC-DESs versatility in integrating different types of DES.

The appropriate deployment of the presented MMC-DES applications entails critical challenges, primarily concerning the control and optimal design of the converter. While the control aspect of the MMC-DES has been addressed multiple times in the literature, the proposed schemes are typically tied to the specific application of each research study and lack a more general perspective. This situation has resulted in a pool of algorithm options without clear relationships, making it difficult to determine the most appropriate choice for a particular use case, or to compare them in terms of robustness and performance. Regarding the converter design, no work addresses in depth the impact of the

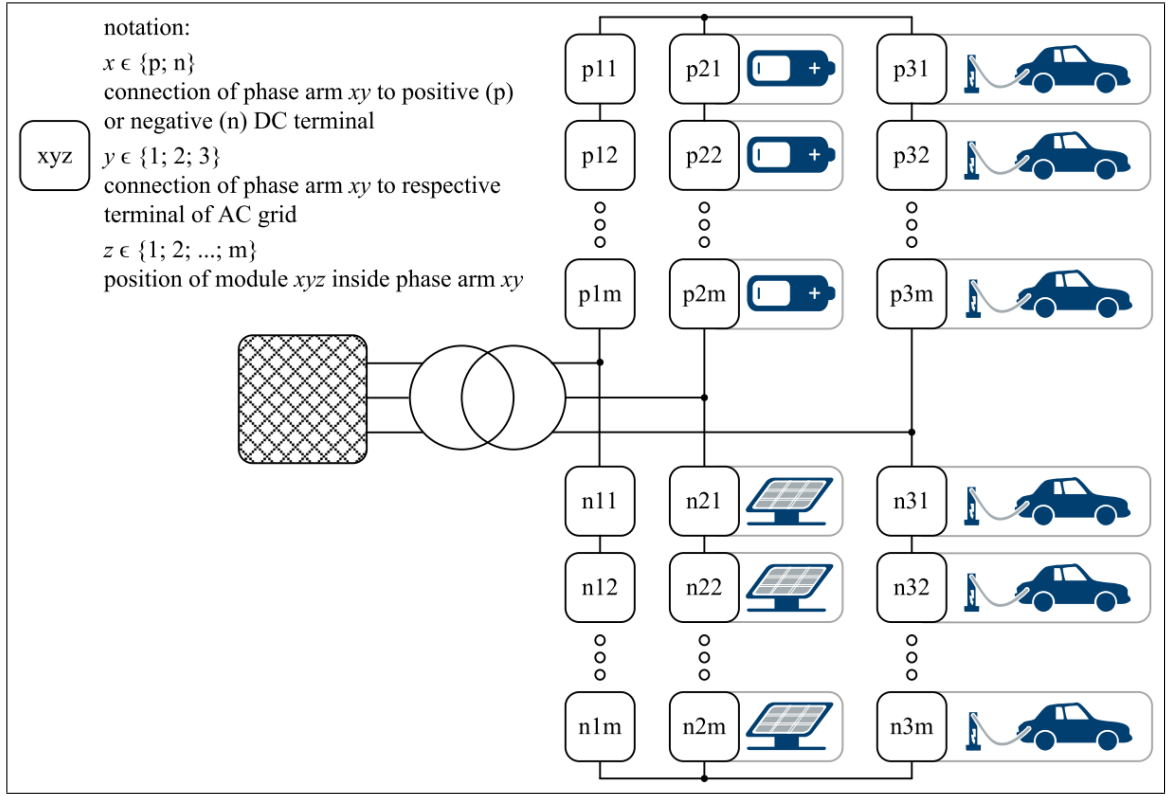


Figure 1.6. MMC with integrated hybrid DES (Schroeder & Jaeger, 2019).

imbalanced operation on the converter voltages and currents. Consequently, there is currently no practical way to discern the operating limitations of a MMC-DES design or to determine the optimal design capable of withstanding a predetermined imbalanced operating point. These factors emphasize the need for a more comprehensive understanding of MMC-DES systems and their operation.

1.2. Issues and challenges

The standard MMC is a fairly complex converter, and as such, it encompasses a highly diverse suite of issues and challenges. This is rightfully corresponded with a substantial amount of research. The MMC-DES carries these challenges, augmenting some of them and adding new ones due to the increased complexity.

Researchers have proposed several SM topologies tackling different factors, such as cost, losses, fault robustness, capacitor voltage ripple, control complexity, design complexity, footprint size, etc. There is no ideal SM, so there always will be a degree of compromise on some of the factors. By far the most utilized topology is the half-bridge SM (HBSM), because it tops the cost and control complexity factors, while generally having a sufficient performance on the other factors (Ali et al., 2021). However in a MMC-DES, new requirements can arise and former factors can gain importance, making the HBSM a suboptimal or inadequate choice. For example the high capacitor voltage ripple characteristic of the HBSM can inject second order currents into a BESS, which could have varying damaging effects depending on the battery chemistry (Wang et al., 2016). Or the parasitic capacitance of a determined DES could cause leakage currents if the SM topology has no insulation, which can have different relevance depending on the type of DES. Therefore, while the SM selection debate is somewhat settled in the MMC, this topic is uncharted regarding the MMC-DES. Every DES type has unique properties and requirements, and different SM topologies could prove to be suitable for different DES types.

Another common concern of the MMC is reliability. Faults are likely to occur due to the high component count of the topology. One of the virtues of the MMC is that most of the times it can be easily repaired by replacing a faulty SM. There are still concerns nonetheless, such as the control response to the faults, or the immediate effects of fault on the converter operation and the hardware integrity. A first major challenge is fault detection. Multiple algorithms have been proposed and reviewed for the MMC (Rojas et al., 2022), but it is not clear how they would perform in an MMC-DES. On top of that, adding a DES brings more points of failure, creating more failure types than the described in the existing methods. The methods would need to be modified to be able to identify and classify the new faults. Another important topic that needs research is the effects and dangers of MMC-side faults on DES hardware and vice versa. This issues are of course related to the SM topology, as they respond differently to component and system failures. For example the full-bridge SM (FBSM) can block dc fault currents, whereas HBSM cannot (Dekka et al., 2017).

A MMC with a high SM count also requires a high number of voltage sensors, current sensors and communication channels, all of which add cost and hardware complexity. This issue can be far more relevant in a MMC-DES than in a standalone MMC, because having a DES component in each SM can potentially multiply by two the number of voltage sensors of the overall system, and multiply by a much greater factor the number of current sensors. The final number of extra sensors that a MMC-DES would necessitate depends on the measurements that the chosen control system would require to ensure a stable and reliable operation. Also having so many sensors increases the chance of a sensor malfunction, which can disturb the converter control. Furthermore, adding so many more sensors would demand much more bandwidth from the controller and communication systems, which can already be a complication in large MMCs (Perez et al., 2015). The research and development of robust and precise observers and sensorless control methods for the MMC-DES can help mitigate this issues (Poblete et al., 2022) (Pizarro, Poblete, Droguett, Pereda, & Núñez, 2023).

The control design of a MMC-DES can be quite challenging as it entails multiple control objectives: output terminal control (power, current, torque or voltage), DES output control (energy/SoC, power or current), SM capacitor voltage control, circulating current control and SM capacitor voltage ripple delimitation. While the MMC-DES control has been addressed multiple times in the literature, the proposed schemes are typically tied to the specific application of each research study, and lack a more general perspective. This situation has resulted in a pool of control algorithm options without clear relationships, making it difficult to determine the most appropriate choice for a particular use case, or to compare them in terms of robustness and performance. Research is needed to determine which are the most appropriate control design choices for the different types of DES.

The operation of an MMC-DES can entail increases in the converter current and capacitor voltage ripple, with respect to a similarly sized MMC. These issues have been partially addressed in the literature, but no work studies in depth the impact of the imbalanced operation on the converter voltages and currents. Some DES may be prone to small

imbalances, like a brand new BESS. But others may need to generate and withstand high imbalances, like a BESS composed of heterogeneous second-life batteries, a PV system exposed to shading on one of its arrays, or a hybrid DES. Despite the direct high impact that imbalances can have on the converter operation, there is currently no practical way to discern the operating limitations of a MMC-DES design, or to determine the optimal design capable of withstanding a predetermined imbalanced operating point. These and the above mentioned issues showcase the need for a more comprehensive understanding of MMC-DES systems and their operation.

1.3. Thesis contribution

The main contribution of this thesis is a comprehensive understanding of the effects that inter-cluster power imbalances have on the MMC, along with design equations that allow the quick evaluation of MMC-DES applications regarding hardware overrating due to power imbalances. This thesis also provides a general view of the implementation of MMC-DES control. It reviews the proposed control schemes for different MMC-DES and defines a classification for them, bringing order and conceptual coherence to previously isolated work.

This work introduces a new system of metrics that can represent inter-cluster power imbalances in a MMC. The metrics are used to determine analytical expressions that show the impact of inter-cluster imbalances on the converter current and voltage. To the best of the author knowledge, existing work encompasses only numerical analysis and simulations for a bounded operating point and specific application (Judge & Green, 2019). This approach lacks the generality that an analytical expression provides for both the converter operating point and type of DES.

Previous analytical research has been conducted on two other modular converters, specifically the delta and star CHBs (Yu, Konstantinou, Hredzak, & Agelidis, 2016) (Yu, Konstantinou, Townsend, Aguilera, & Agelidis, 2017); however, the MMC has not been

studied from this approach. The earlier work cannot be directly applied to the MMC, since the differences in topology and control are significant enough to require a substantially different modeling of the problem. Former research has examined limitations on MMC intra-cluster imbalances (Liang et al., 2021) (Liang et al., 2022) (Pan, Sun, Zhao, Li, & Cai, 2022), but these studies encompass SM and DES design, which are out of the scope of this thesis.

The presented analysis is extended to study the operating limitations imposed by the converter hardware on imbalances, ultimately providing tools for the MMC-DES hardware design. These tools will allow the precise and reliable hardware design of a MMC-DES regarding current and voltage. Furthermore, the developed equations will enable the generation of current magnitude, efficiency and temperature maps across the imbalance range. The equations and subsequent analysis were verified through simulations and partially validated using experiments, subject to the capabilities of the laboratory hardware.

This thesis also provides secondary contributions that were hereby developed as means to analyze and evaluate MMC inter-cluster power imbalances, but can be utilized with other purposes. It derives a new control algorithm for the MMC-DES, which allows the management of the DES and grids powers in an unconventional but promising way. It also proposes a new metric to evaluate the suitability of the MMC for DES integration, which can be easily extended for other converters to evaluate them and compare them. However, the present work does not elaborate much on those contributions, as they are not the focus this thesis.

1.3.1. Hypothesis

The hypothesis that drives this work is:

H An analytical expression can be derived to accurately determine the current and voltage rating requirements of a MMC with inter-cluster power imbalances.

1.3.2. Objectives

1.3.2.1. General objective

The general objective of this thesis is:

- O* To develop equations that accurately model the current and voltage requirements associated with MMC power imbalances, providing solid evidence for the validity of the model, to generate tools for MMC-DES hardware design and a deeper understanding of MMC power imbalances.

1.3.2.2. Specific objectives

- O*₁ To classify and organize the existing control schemes for MMC-DES and then to select the most appropriate scheme for showcasing and analyzing the behavior of the converters operating variables (currents and voltages) under power imbalances.
- O*₂ To derive a thorough control algorithm capable of handling inter-cluster power imbalances.
- O*₃ To establish a metric to quantify the inter-cluster power imbalances of a MMC-DES.
- O*₄ To determine the mathematical relationship between the defined metric and the converters operating variables.
- O*₅ To implement the developed control algorithm in simulations and experiments to operationally verify the discovered mathematical relationship between imbalances and variables.

1.3.3. Thesis structure

Chapter 1 introduces the concept of a MMC-DES along with its applications and associated challenges. It also defines the scope and objectives of this work.

Chapter 2 reviews and classifies the control schemes for MMC-DES proposed in the literature, then selects the most appropriate scheme for the objectives of this thesis.

Chapter 3 describes the complete mathematical modeling of the converter, demonstrating the usefulness of the $\Sigma\Delta$ transform to decouple the converter state equations and for the power equations analysis.

Chapter 4 details the control schemes used to operate the converter in its different modes while maintaining control over inter-cluster power imbalances.

Chapter 5 establishes imbalance metrics and derives the mathematical relationship between converter overrating requirements and inter-cluster power imbalances.

Chapter 6 analyzes various converter operating points and ranges, exploring how different possible MMC-DES imbalance ranges relate to the overrating requirements. It evaluates the overall capacity of the MMC to operate under imbalances.

Chapter 7 showcases the experimental and simulated results that validate the previous mathematical derivation and analyses.

Chapter 8 presents the conclusions of the thesis.

2. STATE OF THE ART ON MMC-DES CONTROL

A MMC-DES control algorithm must achieve the typical MMC control objectives, while managing the energy flow between the converter ports and the DES. This flow behaves differently depending on the type of DES, its characteristics, and how it is integrated into the MMC. For instance, in a fully rated MMC-SST, each SM provides or consumes the same power as the others, resulting in a balanced operation. However, in a partially rated MMC-BESS, the SMs without batteries must maintain a net zero energy flow to maintain the capacitor at a stable voltage, whereas the SMs with batteries will have non-zero energy flows, leading to an unbalanced operation. Various physical mechanisms can produce imbalances, and there are multiple ways to incorporate these mechanisms into a control scheme that seamlessly integrates the control of a MMC and a DES.

2.1. Power imbalances

All MMC-DES systems must handle power imbalances in a controlled manner, as a single SM failure can cause an unbalanced operation due to different operational constraints on the faulty SM compared to the others. The significance of imbalances varies among different DES types. For instance, a MMC-PV exposed to partial shading would have to sustain large power imbalances, in contrast to the MMC-SST. Imbalance magnitudes depend not only on the type of DES, but also on its characteristics. For example, a fully rated MMC-BESS with brand-new identical batteries would experience low imbalances. However, if the BESS comprises second-life batteries, their differences in health and overall characteristics would result in larger imbalances.

Imbalances can be classified into two main categories according to the classic cluster segmentation of the MMC components: intra-cluster and inter-cluster. Intra-cluster imbalances occur among SMs within a single cluster, whose aggregated power is referred to as cluster power. On the other hand, inter-cluster imbalances occur between different clusters within the MMC and can be further subdivided into *horizontal* and *vertical* imbalances.

Horizontal refers to an imbalance between the aggregated power of different phases, while Vertical refers to imbalances between the two clusters of a single phase.

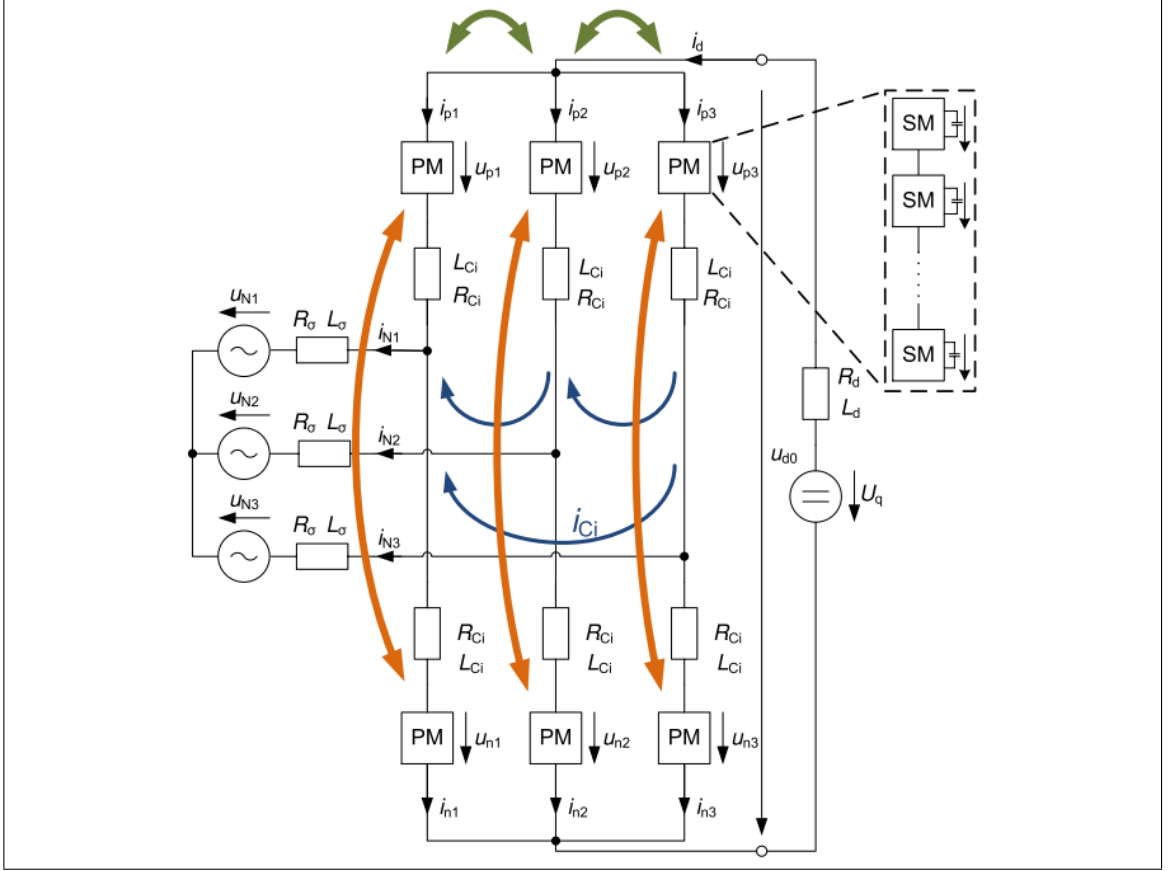


Figure 2.1. Types of MMC inter-cluster imbalances. Vertical imbalances in orange and horizontal imbalances in green (Münch et al., 2010).

Intra-cluster imbalances are handled in the converter modulation, and there are two main mechanisms to manage them. The first one uses a PS-PWM algorithm with adjustments in the modulating voltages of each SM, which allows them to contribute different-sized portions of the cluster modulating voltage (Vasiladiotis & Rufer, 2015) (Zhang, Tang, Yang, & Gao, 2019) (Chen et al., 2017). Given they share the same current, they contribute different amounts of power or energy through the modulation of uneven voltages. A proportional (P) controller determines these voltage alterations. The second mechanism employs a sorting algorithm that decides which SMs actively participate in

the power flow. This mechanism works in conjunction with LS-PWM (Liang et al., 2021) (Darus, Pou, Konstantinou, Ceballos, & Agelidis, 2014) or NLC modulation (Judge & Green, 2019), where the sorting algorithm assigns each SM a voltage level. Both of these mechanisms are adaptations of former SM capacitor voltage balance methods, suggesting that additional MMC intra-cluster balance methods could be adapted for MMC-DES imbalances, such as PS-MPC (Poblete, Neira, Aguilera, Pereda, & Pou, 2021), hybrid sorting (Qin & Saeedifard, 2013) (Tu, Xu, & Xu, 2011), or model predictive sorting (Ilves, Harnefors, Norrga, & Nee, 2015) (Townsend et al., 2016).

Inter-cluster power can also be managed in a couple different ways (Milovanović & Dujić, 2021), but all of them use the same underlying mechanisms, dc circulating current to manage horizontal power flow, and fundamental frequency ac circulating current to manage vertical power. The reason for this can be seen in the equations of Chapter 3 and will be further explained in Chapter 4. The most widespread way to manage inter-cluster imbalances is the one derived from (Münch et al., 2010), which was also the first to be proposed and arguably the simplest. It is important to note that the minimal amount of circulating current occurs when all the power is evenly balanced among the clusters, whereas an unbalanced power distribution requires the injection of additional currents.

Fundamental frequency common mode voltage can also be used for inter-cluster imbalances (Vasiladiotis & Rufer, 2015) (Chen et al., 2017), but it is not convenient for two main reasons. The injection of common mode voltage requires modulator space, which is normally scarce, thus heavily limiting the achievable imbalances. Also, the use of common mode voltage for horizontal imbalances disturbs vertical imbalances and vice versa. This means that using it for one type of imbalance will affect the control performance of the other and require the injection of additional currents to counter the disturbances. Hence, common mode voltage is often not even mentioned as a tool to control inter-cluster imbalances (Debnath, Qin, Bahrani, Saeedifard, & Barbosa, 2015) (Milovanović & Dujić, 2021), and this work will hereafter follow that trend. Fig. 2.2 summarizes the types of MMC-DES imbalances and the mechanisms to produce them.

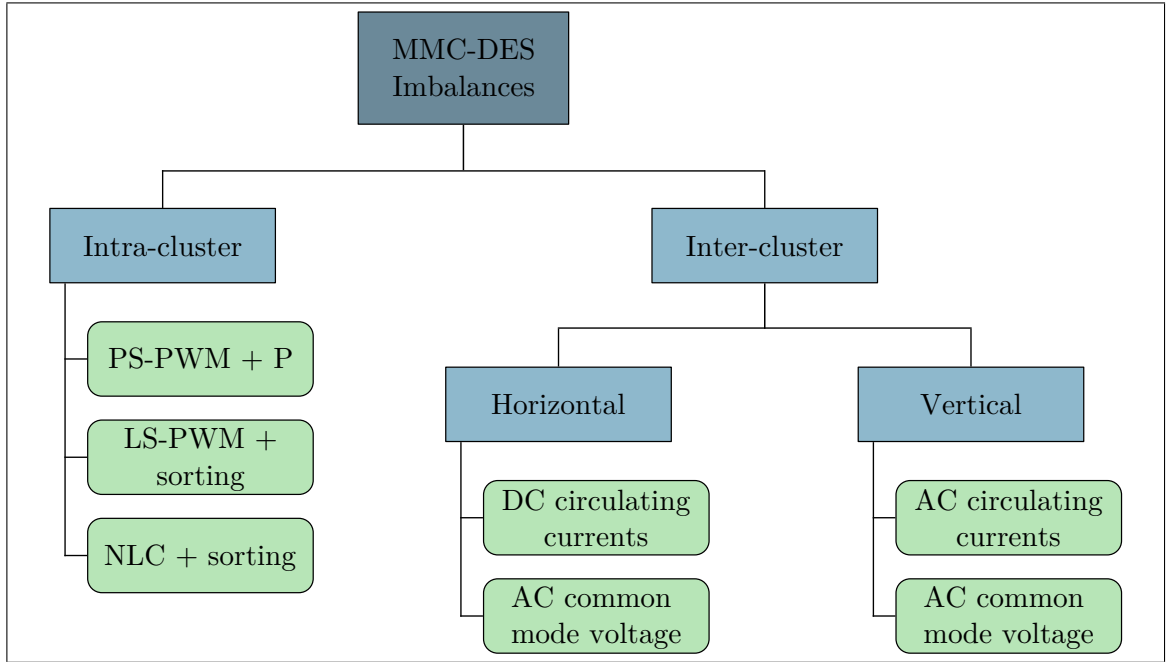


Figure 2.2. MMC-DES imbalances. Categories in blue, mechanisms in green.

2.2. Control integration of a MMC and a DES

Having the tools to perform power imbalances is necessary but not sufficient to integrate an DES into the MMC. The control scheme of an MMC-DES must also address hierarchies on decision making and information flow paths. Consequently, different approaches for the control integration arise, which are mainly classified in Fig. 2.3. This is a broad classification, MMC-DES control schemes can be mainly fitted into one of these categories, but sometimes contain elements from more than one category.

In a MMC-DES, the converter and the DES are connected at the SM level, through the SM capacitors. For there to be a stable power flow, one of the two must ensure that the SM capacitor stays at a stable voltage, acting as a controlled voltage source. Meanwhile the other acts as a controlled current source, effectively controlling the power flow between the two systems.

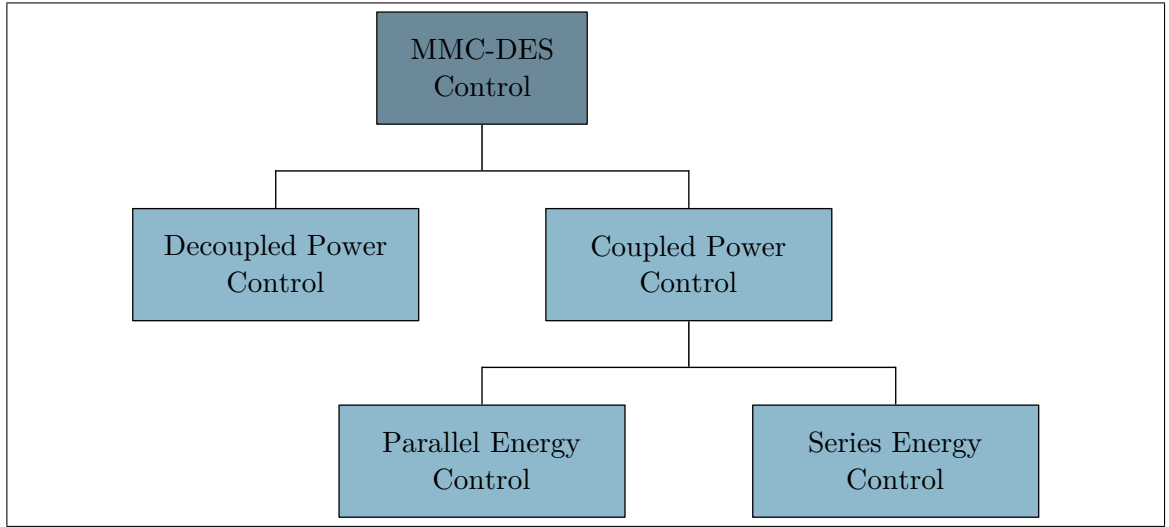


Figure 2.3. MMC-DES control classification.

2.2.1. Decoupled power control (DPC)

Operating under DPC (Fig. 2.4), the MMC acts as the controlled voltage source while the DES forces the power flow (Briz et al., 2016) (Soong & Lehn, 2014b) (Schroeder & Jaeger, 2019). By doing so, the MMC control scheme must only take care of the voltage or energy present in the SM capacitors. This resembles the operation of the standard MMC, where the control scheme must address the internal balance to guarantee that the SM capacitors stay at the reference voltage. However in this case, the DES will heavily disturb the capacitor voltages by injecting active power, and the MMC must establish internal power flows to counter the disturbances. Due to the active presence of large disturbances, the internal balance must be far more robust than in a standalone MMC, but with a similar scheme nonetheless.

There are multiple aspects to take into consideration for a DPC scheme. The DES must take care of necessary balance actions for its components. For example in case of a BESS, the DES must have a SoC balance scheme incorporated. This is because the DES is free to inject varying power among the SMs while the MMC only ensures the stability of the joint system. The DES must also take account for the limitations of the MMC. As

the power flow between both is controlled through a dc current, it can have a very fast dynamic response. However any step in the power flow requires a complementary response from the MMC balance algorithm. Some portions of the MMC balance are performed by fundamental frequency ac current, restricting the speed of the dynamic response. Thus the DES must limit its dynamic so that the MMC can keep the joint system stable. It also must know the operating limitations of the MMC hardware, to avoid producing an imbalance that would cause an overcurrent in the converter clusters.

With DPC, in principle there is no need for operational communication between the DES and the MMC, as each one manages itself. Control-wise, they do not even need to know that they are connected together. Though some degree of communication is needed; for shutdowns, failures, and to ensure that the DES stays within the operating limits of the MMC if they change. This makes DPC a great alternative for hybrid MMC-DES, as having multiple decoupled and task-specific control schemes can be more convenient than a large scheme with a hybrid mixture of control objectives. This is coherent with the fact that most of the found DPC references are indeed proposed for hybrid MMC-DES.

2.2.2. Coupled power control (CPC)

Under CPC (Fig. 2.5), the DES acts as the controlled voltage source, subjecting its power operation to what the MMC establishes. This poses a departure from the standard MMC control, because the MMC now controls its internal power instead of energy. In consequence, the MMC now effectively controls the DES power as an extension of its power. The management of the different control objectives of the MMC-DES is hereby centralized, making CPC more practical for applications that need a cooperative operation of the MMC and the DES, rather than independent ones.

An effective CPC scheme must take some aspects into consideration as well. The MMC must be cautious not to enforce an operating point where the DES can no longer guarantee the SM voltage stability. Also, the MMC must perform secondary control tasks inherent from the DES. For example, in a MMC-BESS now the MMC must carry out the

batteries SoC balance, and in a MMC-PV the MMC has to handle the MPPT. The dynamic response times are no longer a worry for the MMC-DES integration, as the SMs voltage control will naturally be faster than the power control.

A more centralized control scheme means also a more integrated information flow. With CPC, there must be continuous operational communication between the MMC and the DES. This is critical to allow the stability of the joint system and to adequately utilize the DES energy, because the DES operational limits and requirements vary constantly over time, subject for example to a BMS or a MPPT. This differs greatly from the DPC, where the limitations of the MMC are fixed, so the DES does not need to be constantly informed. As CPC requires a higher information and control integration than DPC, it is more practical for non-hybrid DES. Most of the found references of CPC are indeed non-hybrid.

In CPC, the MMC must also carry out the DES energy control (power set point, SoC, MPPT, etc). In the case of the DPC, this was straight forward because every SM could individually control its own energy by modifying the dc current that flows only through itself, and the MMC simply treated this as a disturbance. However in a CPC scheme, the energy control requires that the MMC has a control loop to modify the dc and ac circulating currents that flow through all SMs, based on control inputs of each individual SM. There are two main ways to integrate this into the control scheme in the literature, but sometimes elements of each one are mixed.

2.2.2.1. Series energy control (SEC)

When operating under a CPC scheme, the circulating currents are already managed by the power control, so the energy control can be incorporated as an outer control loop that generates power references for each SM. Afterwards, the MMC can just follow its former control sequence, performing its inter-cluster and port power control generate current references, current control to generate cluster voltage references, and its intra-cluster power control to distribute the cluster powers among their SMs. This way, the DES energy

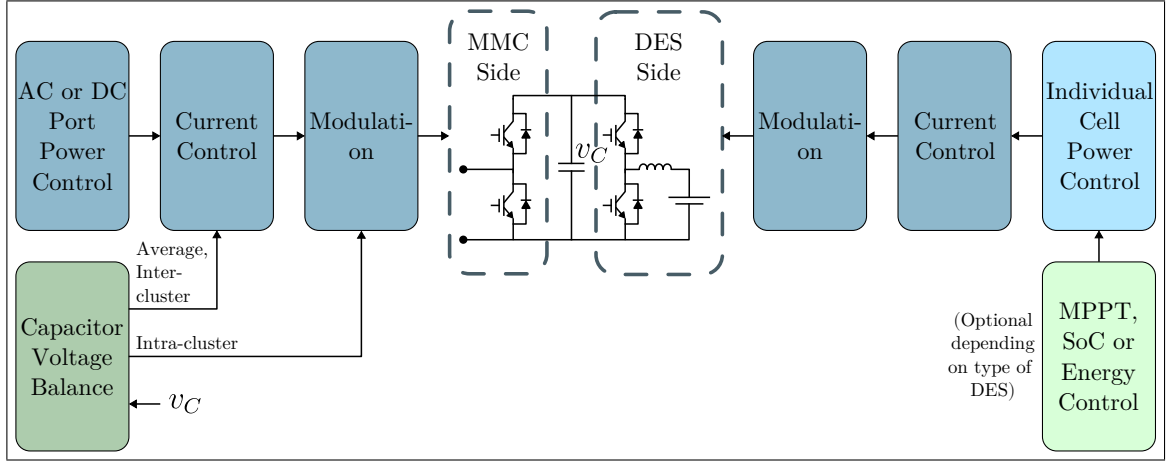


Figure 2.4. Decoupled power control overall scheme.

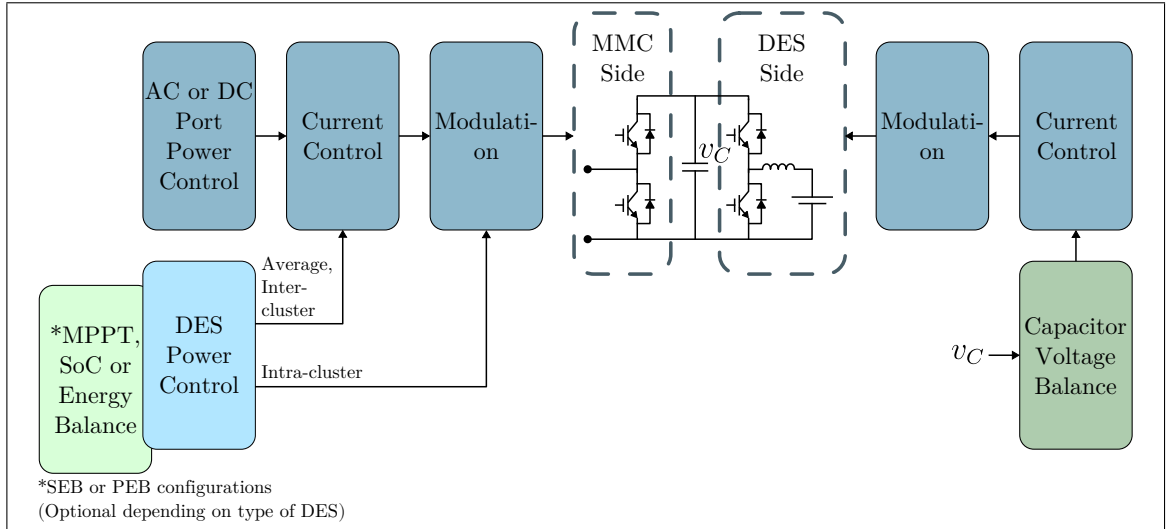


Figure 2.5. Coupled power control with missing energy control.

control is ultimately in series with the rest of the CPC (Vasiladiotis & Rufer, 2015) (Chen et al., 2017). This is called CPC-SEC (Fig. 2.6).

2.2.2.2. Parallel energy control (PEC)

The energy of the different DES devices can also be directly controlled by the circulating currents and modulation, bypassing the CPC power control stage. If done so, the DES power control only needs to handle the aggregated power of the SMs. The output of

the energy control stage has to merge with the control sequence of the CPC by modifying the control variables of the original scheme. This alteration can be done by adding a component to the control variables or multiplying them by a factor. This way, the DES energy control stage is essentially in parallel to the power control stage (Zhang et al., 2019) (Mao et al., 2019) (Quraan et al., 2016). This is called CPC-PEC (Fig. 2.7).

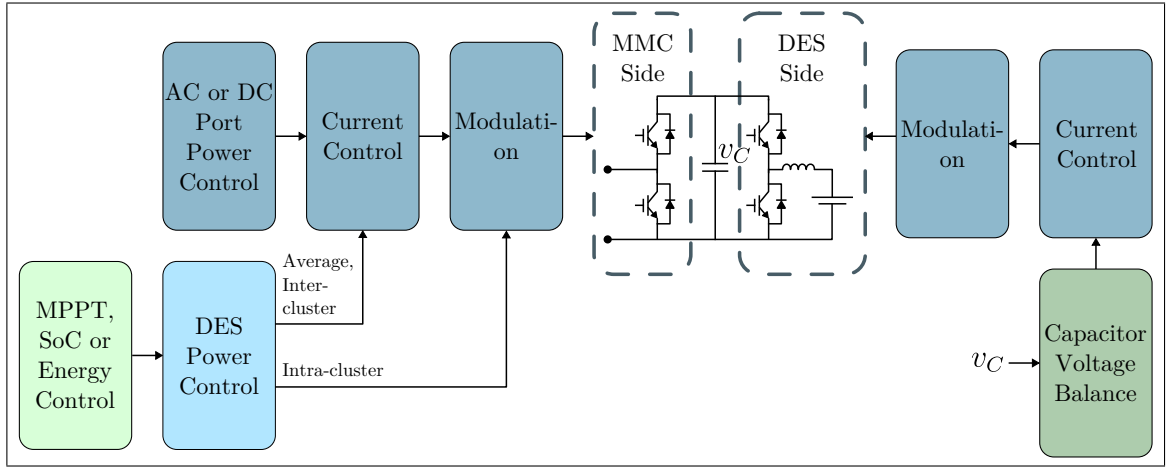


Figure 2.6. Coupled power control with series energy control.

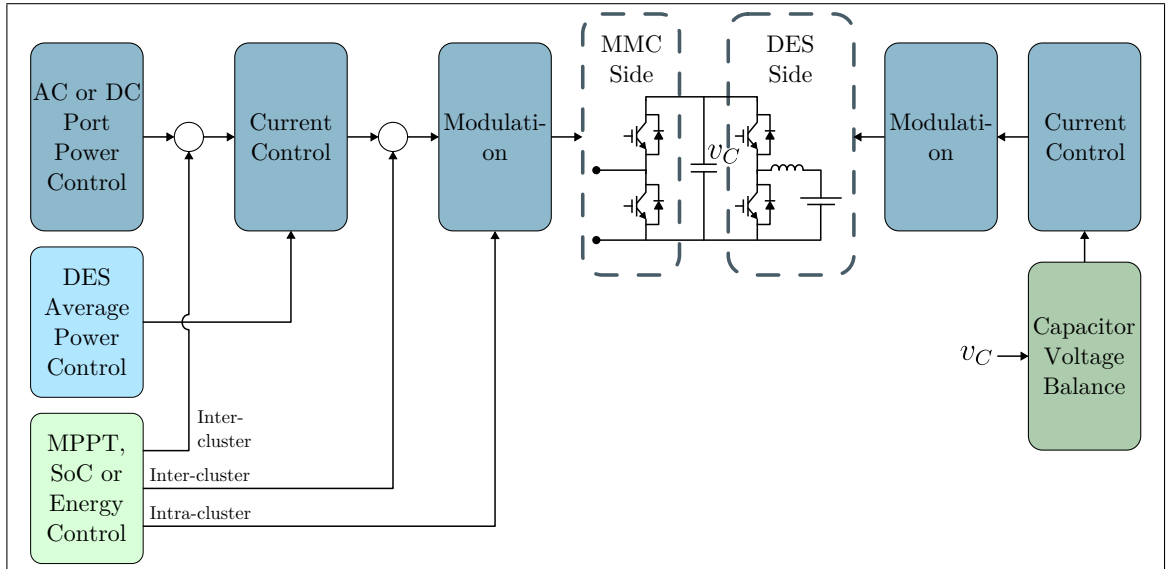


Figure 2.7. Coupled power control with parallel energy control.

Table 2.1. MMC-DES control schemes comparison.

Control	Advantages	Disadvantages
DPC	• Allows multiple independent DES power control systems • Convenient for hybrid DESs • Convenient for DESs that act as current source	• More control integration concerns (Distributed SM power must respect aggregate MMC limitations and dynamic response capabilities)
CPC	• Less control integration concerns • Convenient for DESs that act as voltage source	• Requires a centralized power control

2.3. Control scheme choice

To the best of the author knowledge, there are no comparisons in the literature between any of the former control scheme types. Thus, the advantages and disadvantages of each one are yet to be clarified. The lack of such study clouds the selection of a scheme. Perhaps this is the reason for so much variability in the schemes present in the literature. However, as stated before, some schemes can be intuitively more convenient for specific use cases. Table 2.1 presents some presumable advantages and disadvantages, but more research is needed to fathom their differences. Regarding CPC energy control, PEC may achieve a faster dynamic response because it bypasses the power control loop. Still, SEC may be easier to tune and design, as its arrangement is more modular because it generates references for only the outermost loop of the CPC.

While the scope of this work intends to encompass any MMC-DES, the experimental verification of this thesis is done on only one type of configuration. The converter used for the tests is a MMC with dc voltage sources connected in parallel to the SM capacitors. As the voltage sources can control the capacitor voltages by default, a CPC looks immediately convenient. But in fact, as the sources are directly connected to the capacitors without a controllable dc-dc conversion stage, the CPC scheme is actually enforced.

The selected control scheme is a CPC-SEC but with arbitrary DES power references instead of an energy control loop. An energy control stage is not included in the scheme because it is unnecessary to validate the hypothesis and would only complicate the analysis. The physical limitations of the MMC hardware relate to its power, currents, and voltages. The energy of the DES does not impose any direct strain on the MMC. Actually, it is the MMC capacities that limit the energy consumption and balance rates of the DES (plus the DES own capacities). Additionally, an operating DES energy control generates varying power and currents, which makes it difficult to analyze the steady state of the MMC variables.

2.4. Conclusion

This chapter presents the phenomenon of power imbalances that is needed for the adequate operation of a MMC-DES. It describes the mechanisms that produce them, which that are used to control them. These are circulating currents for inter-cluster power, and sorting or modulation adjustments for intra-cluster power. The chapter then explores the literature to showcase how these mechanisms are integrated into the control schemes of the converter.

Multiple MMC-DES control schemes have been proposed in the literature, along with numerous MMC-DES proposals. Each research group seems to develop its own custom scheme from the ground up, distinguishing themselves from the rest of the literature. This practice has resulted in a great variety of schemes, some very similar and some quite unlike.

This chapter showcases the schemes present in the literature and establishes a general categorization for them. While some do not exactly fit into one category, they all relate highly to one of the categories. This categorization provides order among a sparse literature. This helps to generate comparisons between multiple studies, and to better approach the design process of a MMC-DES control algorithm.

The categorization is used to choose how to approach the control of the experiments for this thesis, showing its usefulness. Nevertheless, it is established that more research is needed to reliably select the best control scheme for each application.

3. MMC-DES MODELING

This thesis focuses on modeling the MMC-side of the MMC-DES, as it primarily concerns MMC design and analysis. As discussed in Chapter 2, from the perspective of the MMC-side, the DES-side can be seen as either a dc voltage source (in the case of CPC) or a current disturbance (in the case of DPC) in the SMs. As a CPC-type control scheme was selected, the DES is modeled as dc voltage sources within the SMs, assuming stable DES-side CPC control. Furthermore, the DES-side modeling depends on the DES type and is beyond the scope of this thesis.

The MMC is a bidirectional multilevel ac/dc converter. As its name suggests, it consists of multiple modules that provide multiple voltage levels. These SMs are arranged in series with an inductor, forming a cluster or arm. Two clusters are connected in series, creating a phase or leg. The midpoint between the clusters is connected to an inductor that leads to the ac terminal, while the clusters tips form the dc terminals. This arrangement is known as a single-phase MMC or half-bridge MMC. When three of these single-phase arrangements are connected in parallel, they form a three-phase MMC, often referred to simply as a MMC. The MMC arrangement and naming is the same as the MMC-DES (Fig. 1.1) but without the DES modules.

3.1. Mathematical representation of the MMC

The MMC circuit is illustrated in Fig. 3.1. The subscript $\chi \in \{a, b, c\}$ represents the phases, while the subscript $y \in \{u, l\}$ represents the phase clusters. Each submodule, or $\text{SM}_{y\chi,n}$, $n \in \{1 \dots N\}$, contains a capacitor whose voltage $v_{Cy\chi,n}$ must be controlled to match the reference V_{SM} . Depending on the SM topology, the SM can connect its capacitor to the cluster circuit in various configurations. This work utilizes the half-bridge topology, which can connect the capacitor in only positive polarization or bypass it. These states can be represented by the binary term $S_{y\chi,n} \in \{0, 1\}$, resulting in the SM voltage $v_{y\chi,n} = S_{y\chi,n} \cdot v_{Cy\chi,n}$. The converter is considered to be connected to fixed dc and ac grids.

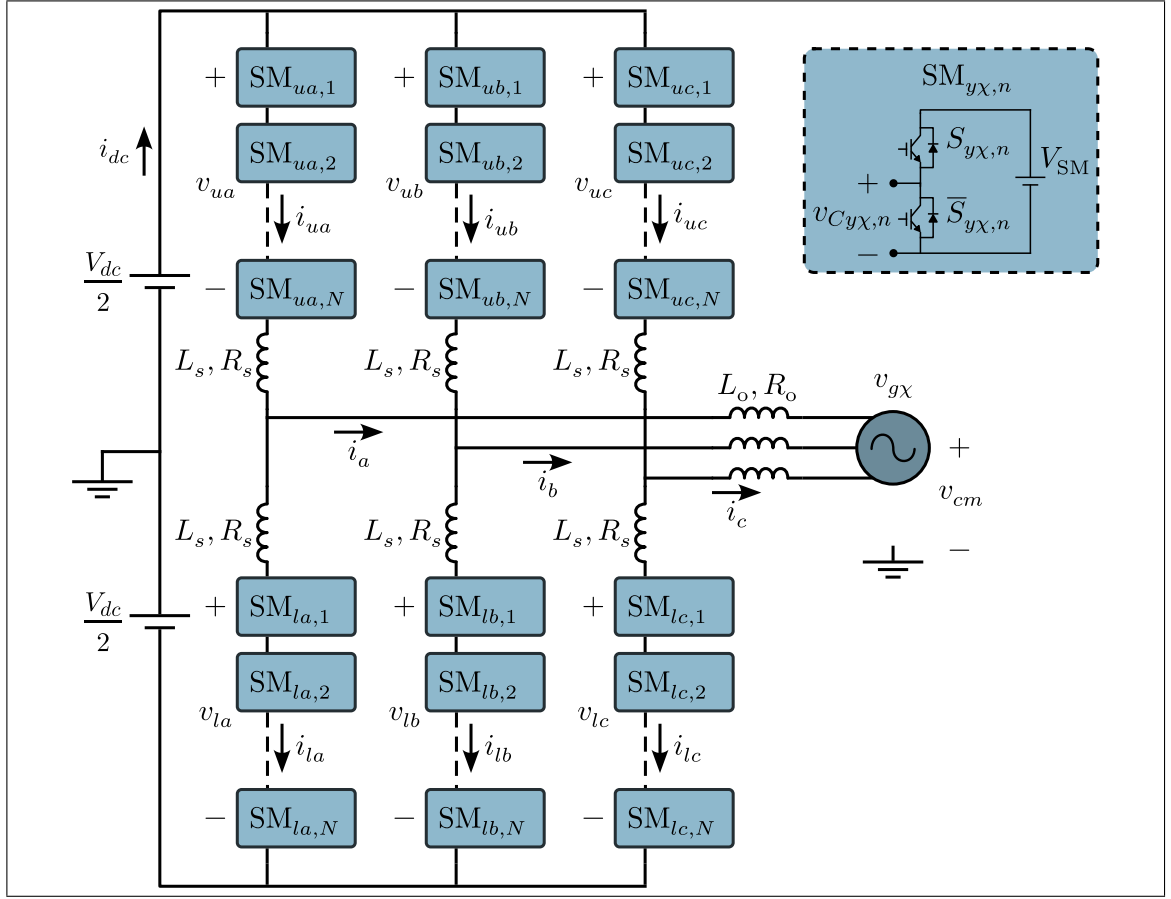


Figure 3.1. MMC model.

The voltage of all the connected SMs in one cluster adds up to form the cluster voltage $v_{y\chi}$. The converter model can be simplified by representing all the cluster SMs as a single voltage source of value $v_{y\chi}$. While the cluster voltage is a switched voltage, this analysis considers it to be equal to the modulated reference voltage, discarding the high frequency components that relate to the switching. This model depicted in Fig. 3.2 is sufficient to analyze and control inter-cluster power flow, and thus will be the one used in this work. Nevertheless, the former switched model is still necessary to maintain a stable control of the intra-cluster power flow of the converter, for keeping the SM voltages near its reference value.

There are three groups of currents that can be directly identified in the converter. The dc port current i_{dc} , the ac port currents i_{χ} , and the currents that flow through the clusters

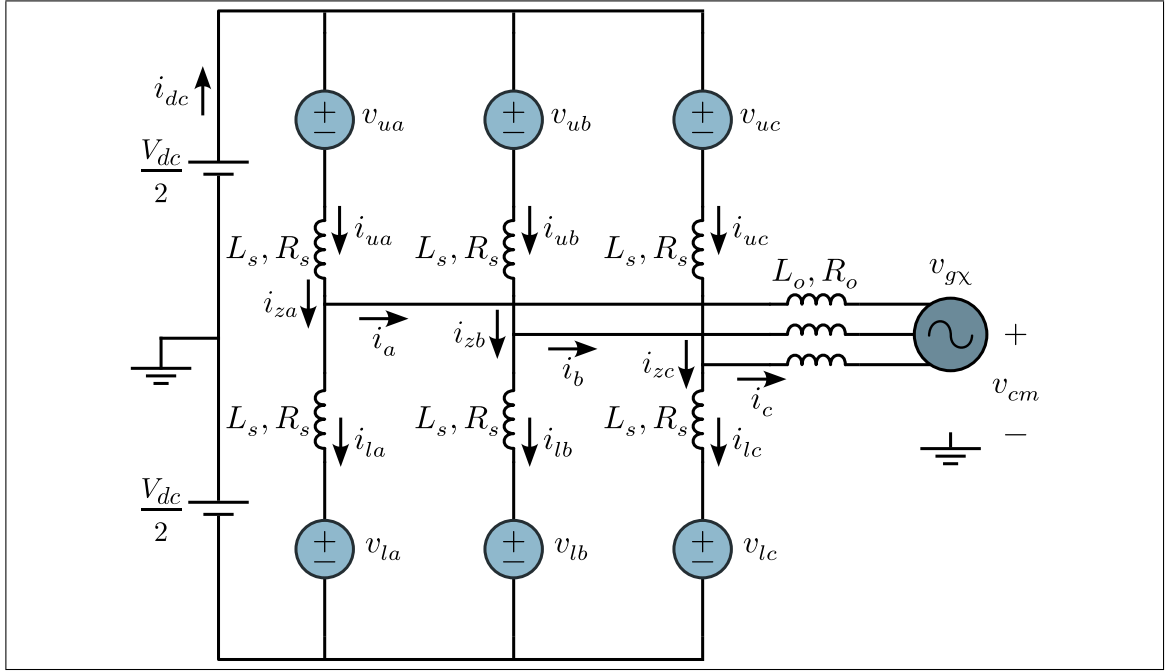


Figure 3.2. MMC simplified model.

$i_{y\chi}$. The cluster currents can be decomposed into two overlapping currents, the current $i_{z\chi}$ that circulates along the clusters of each phase without going to the ac port, and the currents that flow through the clusters into the ac port forming i_{χ} . The circulating current $i_{z\chi}$ is an important tool for the mathematical modeling and analysis of the converter, as will be seen in this chapter.

3.2. State equations

3.2.1. Voltage-Current model of the MMC

By performing the Kirchoffs Current Law (KCL) in the phase middle point, it can be observed that

$$i_{\chi} = i_{u\chi} - i_{l\chi} \quad (3.1)$$

The definition of the circulating current $i_{z\chi}$ implies that both clusters have the same amount of circulating current. Also, it is set that the upper and lower clusters of a phase

must contribute the same amount of current to the ac port. These two facts translate into the following relation:

$$i_{u\chi} = i_{z\chi} + \frac{i_\chi}{2}, \quad i_{l\chi} = i_{z\chi} - \frac{i_\chi}{2} \quad (3.2)$$

adding up both expressions shows that

$$i_{z\chi} = \frac{i_{u\chi} + i_{l\chi}}{2} \quad (3.3)$$

also it is evident that by definition that

$$i_{dc} = \sum_{\chi} i_{u\chi} = \sum_{\chi} i_{l\chi} = \frac{1}{2} \sum_{\chi} (i_{u\chi} + i_{l\chi}) = \sum_{\chi} i_{z\chi} \quad (3.4)$$

The Kirchoffs Voltage Law (KVL) can be performed between the ac and dc neutral points, along each cluster as shown in Fig. 3.2. This yields the relations

$$\frac{V_{dc}}{2} = v_{u\chi} + L_s \frac{di_{u\chi}}{dt} + R_s i_{u\chi} + L_o \frac{di_\chi}{dt} + R_o i_\chi + v_{g\chi} + v_{cm} \quad (3.5)$$

$$\frac{V_{dc}}{2} = v_{l\chi} + L_s \frac{di_{l\chi}}{dt} + R_s i_{l\chi} - L_o \frac{di_\chi}{dt} - R_o i_\chi - v_{g\chi} - v_{cm} \quad (3.6)$$

The equations have coupled state variables $i_{y\chi}$ and i_χ . A decoupled state equation for the i_χ current can be derived from the equations (3.1), (3.5) and (3.6), resulting in

$$\frac{di_\chi}{dt} = \frac{1}{L_s + 2L_o} [(v_{l\chi} - v_{u\chi}) - (R_s + 2R_o)i_\chi - 2v_{g\chi} - 2v_{cm}] \quad (3.7)$$

The same can be done for the $i_{z\chi}$ current through (3.3), (3.5) and (3.6)

$$\frac{di_{z\chi}}{dt} = \frac{1}{2L_s} (-(v_{l\chi} + v_{u\chi}) - 2R_s i_{z\chi} + V_{dc}) \quad (3.8)$$

As shown in (3.2) and (3.4), all the converter currents can be expressed as a combination of i_χ and $i_{z\chi}$. Thus (3.7) and (3.8) form a set of decoupled state equations that can jointly describe all of the converter currents.

3.2.2. $\Sigma\Delta$ model of the MMC

The state equations (3.7) and (3.8) show how the converter currents can be controlled by the cluster voltages. It can be seen that although their state variables are decoupled, their controlled variables are still coupled. This can be overcome by defining and applying a $\Sigma\Delta$ transform (Hagiwara & Akagi, 2009) (Antonopoulos, Angquist, & Nee, 2009) to the cluster voltages as followed

$$\begin{bmatrix} v_{\chi}^{\Sigma} \\ v_{\chi}^{\Delta} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} v_{u\chi} \\ v_{l\chi} \end{bmatrix} \quad (3.9)$$

Fig. 3.3 provides a conceptual visualization of the $\Sigma\Delta$ converter voltages.

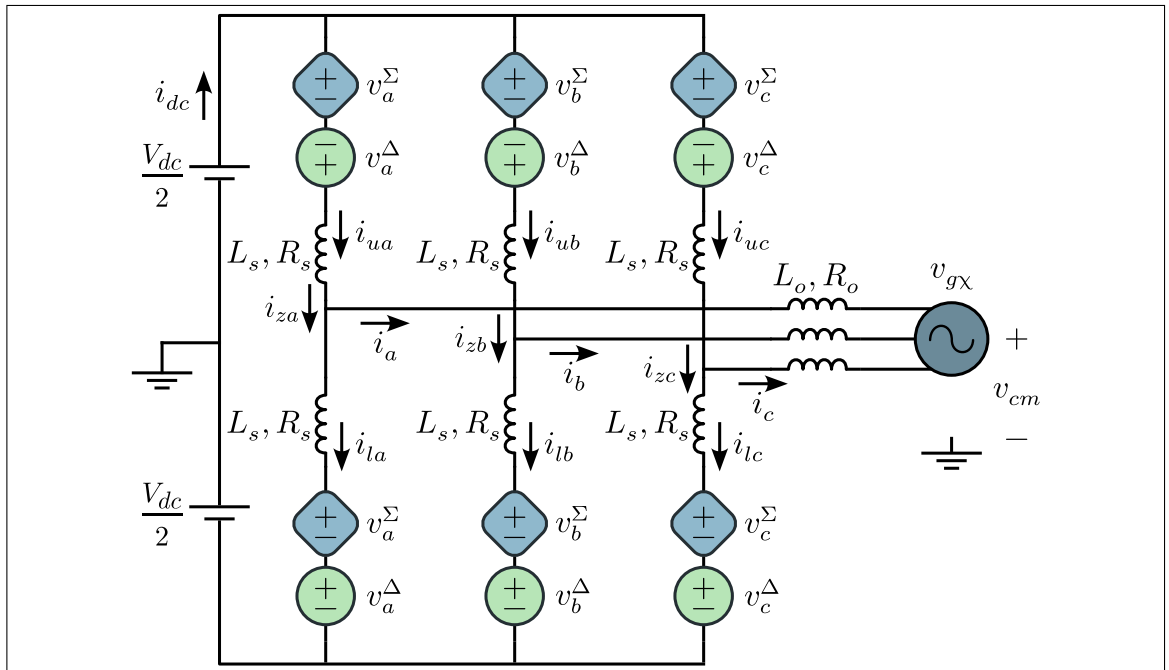


Figure 3.3. MMC $\Sigma\Delta$ model.

The Δ voltage can be decomposed into its sequence components $v_x^{\Delta+}$, $v_x^{\Delta-}$ and $v_x^{\Delta 0}$. By applying the $\Sigma\Delta$ transform to the common mode voltage expression (Du, Wu, &

Zargari, 2019),

$$v_{cm} = \frac{1}{6} \sum_{\chi} (v_{l\chi} - v_{u\chi}) \quad (3.10)$$

the following relation is found

$$\begin{aligned} v_{cm} &= \frac{1}{6} \sum_{\chi} v_{\chi}^{\Delta} = \frac{1}{3} \sum_{\chi} (v_{\chi}^{\Delta+} + v_{\chi}^{\Delta-} + v^{\Delta 0}) = \frac{1}{3} \sum_{\chi} v^{\Delta 0} \\ &= v^{\Delta 0} \end{aligned} \quad (3.11)$$

In addition, auxiliary terms are defined to write the state equations in a more compact manner.

$$R_{\epsilon} = \frac{R_s + 2R_o}{2}, \quad L_{\epsilon} = \frac{L_s + 2L_o}{2} \quad (3.12)$$

Replacing (3.9), (3.11) and (3.12) in the equations (3.7) and (3.8) yields the final state equations.

$$\frac{di_{\chi}}{dt} = \frac{1}{L_{\epsilon}} (v_{\chi}^{\Delta+-} - R_{\epsilon} i_{\chi} - v_{g\chi}) \quad (3.13)$$

$$\frac{di_{z\chi}}{dt} = \frac{1}{L_s} (-v_{\chi}^{\Sigma} - R_s i_{z\chi} + \frac{V_{dc}}{2}) \quad (3.14)$$

These equations allow a completely decoupled control of each converter current. It is noted that the common mode voltage does not affect the converter currents (neglecting the high frequency switching components), and thus it can be disregarded as far as currents are concerned. However, common mode voltage may be injected through v_{χ}^{Δ} (3.11) for other purposes.

3.3. Power equations

3.3.1. Power equations derivation

The MMC power analysis is typically summarized into the ac and dc ports power. These are

$$P_{ac} = -\frac{3I_g V_g}{2}, \quad P_{dc} = I_{dc} V_{dc} \quad (3.15)$$

The minus sign in the ac power arises from a convention discrepancy, but has no meaningful relevance. The ac port current is defined as positive when going outwards from the converter in Fig. 3.1, while the ac port power is defined as positive when going inwards in (3.32).

In presence of a DES, it is essential to also consider the cluster power in the analysis of the converter. The cluster power is the power that each cluster sends to, or receives from the DES. It can be obtained by multiplying the cluster current and voltage,

$$p_{u\chi} = v_{u\chi} i_{u\chi} = (v_{\chi}^{\Sigma} - v_{\chi}^{\Delta})(i_{z\chi} + \frac{i_{\chi}}{2}) \quad (3.16)$$

$$p_{l\chi} = v_{l\chi} i_{l\chi} = (v_{\chi}^{\Sigma} + v_{\chi}^{\Delta})(i_{z\chi} - \frac{i_{\chi}}{2}) \quad (3.17)$$

To further develop these expressions, the voltages and currents need to be better defined. Unlike i_{χ} , the steady state circulating current can contain multiple frequency components h for different purposes, therefore

$$i_{z\chi} = \sum_h i_{z\chi,h} \quad (3.18)$$

For the scope of this thesis, $i_{z\chi}$ is controlled to only contain a dc and fundamental ac frequency components, meaning that $h \in \{0, 1\}$ and $i_{z\chi} = i_{z\chi,0} + i_{z\chi,1}$. The variables are defined as

$$\begin{aligned} v_{g\chi} &= V_g \sin(\omega t + \theta_{\chi}) \\ i_{\chi} &= I_{\chi} \sin(\omega t + \theta_{\chi}) \\ i_{z\chi,0} &= I_{z\chi,0} \\ i_{z\chi,1} &= I_{z\chi,1} \sin(\omega t + \phi_{z\chi}) \end{aligned} \quad (3.19)$$

where θ_{χ} is the ac grid voltage phase angle and $\phi_{z\chi}$ is the angle of the circulating current respect to the phase angle. The converter is assumed to operate with unitary power factor, and thus it is assumed that the i_{χ} current is controlled to match the θ_{χ} ac grid voltage angle. The injection of common mode voltage is hereby not contemplated due to the

reasons reported at the end of section 2.1, yet Appendix B displays the full equations. Expressions for the converter voltages can be found by evaluating these newly defined terms in the state equations

$$v_{\chi}^{\Sigma} = -R_s(I_{z\chi,0} + I_{z\chi,1} \sin(\omega t + \phi_{z\chi})) - I_{z\chi,1} L_s \omega \cos(\omega t + \phi_{z\chi}) + \frac{V_{dc}}{2} \quad (3.20)$$

$$v_{\chi}^{\Delta} = I_{\chi} R_e \sin(\omega t + \theta_{\chi}) + I_{\chi} L_e \omega \cos(\omega t + \theta_{\chi}) + V_g \sin(\omega t + \theta_{\chi}) \quad (3.21)$$

Due to the principle of orthogonality of different frequencies h , the cluster powers can be disaggregated into each frequency contribution. Doing so makes easier to solve the power expression as there are less oscillating cross terms between frequencies. And separating the power components by frequency allows to better relate the dc and ac port currents with the different cluster power components.

$$P_{y\chi} = \sum_h P_{y\chi,h} \quad (3.22)$$

Now the power expressions (3.16) and (3.17) can be developed for the two present frequencies. The fundamental frequency power term contains a lot of oscillatory components that can be disregarded, as only the average power is of interest. Doing so results in

$$P_{u\chi,0} = -R_s I_{z\chi,0}^2 + \frac{1}{2} V_{dc} I_{z\chi,0} \quad (3.23)$$

$$P_{l\chi,0} = -R_s I_{z\chi,0}^2 + \frac{1}{2} V_{dc} I_{z\chi,0} \quad (3.24)$$

$$\begin{aligned} P_{u\chi,1} = & -\frac{1}{2} R_s (I_{z\chi,1}^2 + \frac{I_{\chi}^2}{4}) - \frac{1}{4} R_o I_{\chi}^2 - \frac{1}{4} V_g I_{\chi} \\ & + \frac{1}{2} I_{z\chi,1} [(R_s + R_o) I_{\chi} \cos(\phi_{z\chi} - \theta_{\chi}) + I_{\chi} L_o \omega \sin(\phi_{z\chi} - \theta_{\chi}) \\ & + V_g \cos(\phi_{z\chi} - \theta_{\chi})] \end{aligned} \quad (3.25)$$

$$\begin{aligned} P_{l\chi,1} = & -\frac{1}{2} R_s (I_{z\chi,1}^2 + \frac{I_{\chi}^2}{4}) - \frac{1}{4} R_o I_{\chi}^2 - \frac{1}{4} V_g I_{\chi} \\ & - \frac{1}{2} I_{z\chi,1} [(R_s + R_o) I_{\chi} \cos(\phi_{z\chi} - \theta_{\chi}) + I_{\chi} L_o \omega \sin(\phi_{z\chi} - \theta_{\chi}) \\ & + V_g \cos(\phi_{z\chi} - \theta_{\chi})] \end{aligned} \quad (3.26)$$

3.3.2. $\Sigma\Delta$ power transform

The equations (3.23)-(3.26) have many coupled terms that can complicate the individual control of upper and lower clusters, and cloud their analysis. The $\Sigma\Delta$ transform once more comes in handy to solve the coupling issue. The new power expressions are

$$P_{\chi,0}^{\Sigma} = \frac{1}{2}V_{dc}I_{z\chi,0} - R_s I_{z\chi,0}^2 \quad (3.27)$$

$$P_{\chi,0}^{\Delta} = 0 \quad (3.28)$$

$$P_{\chi,1}^{\Sigma} = -\frac{1}{4}V_g I_{\chi} - \frac{1}{2}R_s(I_{z\chi,1}^2 + \frac{I_{\chi}^2}{4}) - \frac{1}{4}R_o I_{\chi}^2 \quad (3.29)$$

$$P_{\chi,1}^{\Delta} = \frac{1}{2}I_{z\chi,1}[V_g \cos(\phi_{z\chi} - \theta_{\chi}) + (R_s + R_o)I_{\chi} \cos(\phi_{z\chi} - \theta_{\chi}) + \omega L_o I_{\chi} \sin(\phi_{z\chi} - \theta_{\chi})] \quad (3.30)$$

The original cluster powers can be easily observed through the $\Sigma\Delta$ powers, just by adding and subtracting

$$P_{u\chi} = P_{\chi}^{\Sigma} - P_{\chi}^{\Delta}, \quad P_{l\chi} = P_{\chi}^{\Sigma} + P_{\chi}^{\Delta} \quad (3.31)$$

The $\Sigma\Delta$ powers also carry valuable conceptual meaning. The Σ power is the average power that the clusters of a single phase contribute or consume. The Δ power is how much the power contribution of each phase cluster deviate from the mean value. Alternatively, Δ power can be seen as the power that one cluster transfers to the other phase cluster. Moreover, $\Sigma\Delta$ powers can be easily related to the inter-cluster balancing categories described in the literature and presented in section 2.1. Σ power can be used to manage horizontal imbalances, and Δ power can be used to manage vertical imbalances.

The equations (3.27) and (3.28) show that the dc circulating currents only appear in the Σ power, affecting the power of each phase but not how the power is distributed inside the phase. The equations (3.29) and (3.30) show that neglecting the losses, only the ac grid current affects the Σ power. Meanwhile, the ac circulating currents is the only current that significantly affects how the power is distributed among the clusters of a single phase.

3.4. Power relation and modes of operation

Usually, the sign convention of the MMC powers is tied to the converter operation. For instance, if the converter is operated as an inverter, the dc power is considered positive when entering the converter, and the ac power is considered positive when leaving the converter. As this work aims to generalize the analysis, it follows a different approach. The power sign convention is redefined, being considered positive when entering the converter and negative when leaving the converter. Note the minus sign in (3.15), due to having opposed ac port current and power sign conventions. Assuming a stable capacitor voltage control, this results in the following relation:

$$P_{dc} + P_{ac} + P_{des} = 0 \quad (3.32)$$

The cluster power definition of (3.16) and (3.17) yields a positive value whenever net power enters the cluster and consequently the DES. To follow the convention of (3.32), it must be noted that

$$P_{des,y\chi} = -P_{y\chi} \quad (3.33)$$

The integration of DES power into the MMC power flow is modeled by defining two categories, dc compensation and ac compensation. When the DES power is nonzero, only one port compensates the DES power contribution in order to maintain the relation (3.32). DC compensation is when the DES power flow is corresponded only with changes in the dc port power, maintaining an independent power control in the ac port. Conversely, ac compensation is when the DES power flow is corresponded only with changes in the ac port power, while maintaining an independent power control in the dc port. This work aims to analyze the requirements and limitations of the MMC-DES operation, so the forthcoming analysis will always consider the non-compensated port to be at nominal power operation.

In addition to the compensation category, the converter operation also has power directionality categories, acting as an inverter or rectifier. The combination of these categories

Table 3.1. Feasibility of power operation under nominal power at one the MMC ports. Feasible: ✓, unfeasible: ✗.

Power direction	Port power sign			Compensation	
	DC	AC	DES	DC	AC
Inverter	+	−	+	✓	✗
Inverter	+	−	−	✗	✓
Rectifier	−	+	+	✗	✓
Rectifier	−	+	−	✓	✗
Source	−	−	+	✓	✓
Sink	+	+	−	✓	✓

results in multiple operating modes. It is possible that the MMC acts by delivering or absorbing power on the ac and dc ports simultaneously (dc and ac with same signs), utilizing the DES as a sink or source, but these directionality categories were discarded because there is no mention of them in the literature and including them would considerably enlarge the analysis.

The nominal power value of the converter is set equal for both main ports, intending for them to operate nominally when the DES power is zero. It is set that the main ports cannot exceed its nominal power value, rendering some operating modes unfeasible. For example, if the ac port is set at its nominal value acting as an inverter, a negative DES power value would make the dc port contribute more than its nominal value, which is considered unfeasible. Table 3.1 identifies the unfeasible operating points.

The DES power or the compensated port power could also be set to zero, albeit considering all those particular cases as distinct operating modes would result in a counter-productive amount of operating modes. To incorporate these particular cases, the feasible operating points with zero power in a port are considered as just border cases of the above. Table 3.2 presents the feasible operating modes and Fig. 3.4 provides a diagram that illustrates them. Table 3.1 displays the sink and source cases to showcase all possible MMC-DES power flow combinations, but they are excluded from the studied operating modes for the reasons previously reported.

Table 3.2. MMC-DES operating modes.

			DC	AC	DES
I	DC compensation	Inverter	$+^0$	$-$	$+^0$
II	DC compensation	Rectifier	$-^0$	$+$	$-^0$
III	AC compensation	Inverter	$+$	$-^0$	$-^0$
IV	AC compensation	Rectifier	$-$	$+^0$	$+^0$

Note that the non-compensated port follows arbitrary references and is not necessarily fixed, it is just not affected by DES power. Therefore, transitions between different operating modes are contemplated in the modelling. However, this work focuses on the most straining operating points, and thus the non-compensated port will be always at nominal power in the forthcoming analysis and not intermediate points.

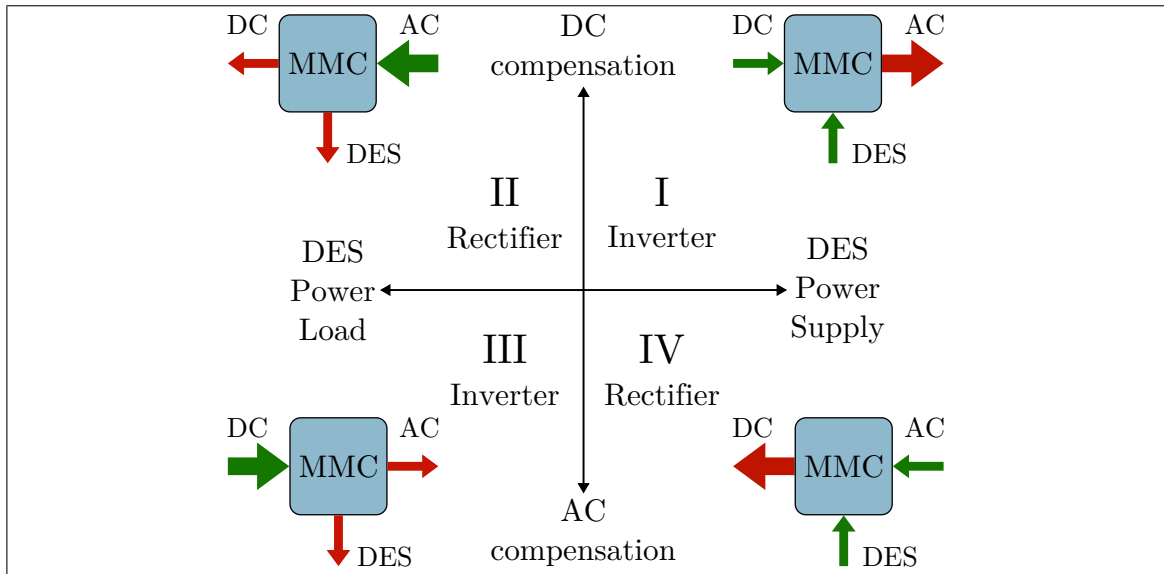


Figure 3.4. Operating modes diagram.

3.5. Conclusion

This chapter presents the classical mathematical modeling of the MMC and the simplified version. The state equations of the simplified model of the converter are derived through the Kirchhoff laws. The $\Sigma\Delta$ transform is introduced in order to decouple the state equations of the converter, and obtain the final form of the equations.

The power equations of the converter are derived, identifying that the DES power can be incorporated into the analysis as the negated value of the cluster power. The $\Sigma\Delta$ transform is utilized. This generates more manageable power expressions with deeper conceptual meaning, relating Σ power to horizontal power and Δ power to vertical power.

Finally, the DES interaction with the MMC regarding power is modeled as two possible categories, dc compensation and ac compensation. This new categories combined with the classical inverter/rectifier categories, yield 4 feasible power operating modes for a MMC-DES, summarized in Table 3.2.

4. MMC-DES CONTROL

The control scheme is of CPC-SEC type. This means that the DES power is managed on the MMC-side of the control. The DES-side control only ensures that the SMs stay at a stable voltage. As stated before, the design of the DES-side control depends on the DES at issue and will be left out of the scope of this thesis. This is not a complication, because by selecting a CPC scheme type, the inter-cluster power management of the converter can be analyzed through only the MMC-side.

Chapter 2 pointed out that each MMC-DES research develops its own control scheme with slight variations for their particular context, and this research is no exception. The hereby presented converter control is built from the equations presented in Chapter 3, making design choices that facilitate the evaluation of the hypothesis, some of which were commented on in Section 2.3. Relevant references for other variations of MMC-DES control include (Münch et al., 2010) (Soong & Lehn, 2014b) (Chen et al., 2017) (Vasiladiotis & Rufer, 2015) (Schroeder & Jaeger, 2019). The designed control does not include optional secondary MMC control objectives, like SM voltage ripple reduction or circulating current suppression. It encompasses only the essential control objectives for stable converter operation, aiming to study the MMC from the most possible general perspective while facilitating the hypothesis evaluation.

The control of the converter consists of three main layers. First, the power control loop that receives the power references and measurements of the MMC and the DES, and generates current references. Second, the current control loop receives current references and measurements of the MMC, and generates cluster voltage references. Third, the modulation stage receives the cluster voltage references and generates the trigger signals that drive the converter transistors. These stages are illustrated by Fig. 2.6 in the general CPC-SEC scheme, and are presented in further detail in Fig. 4.1.

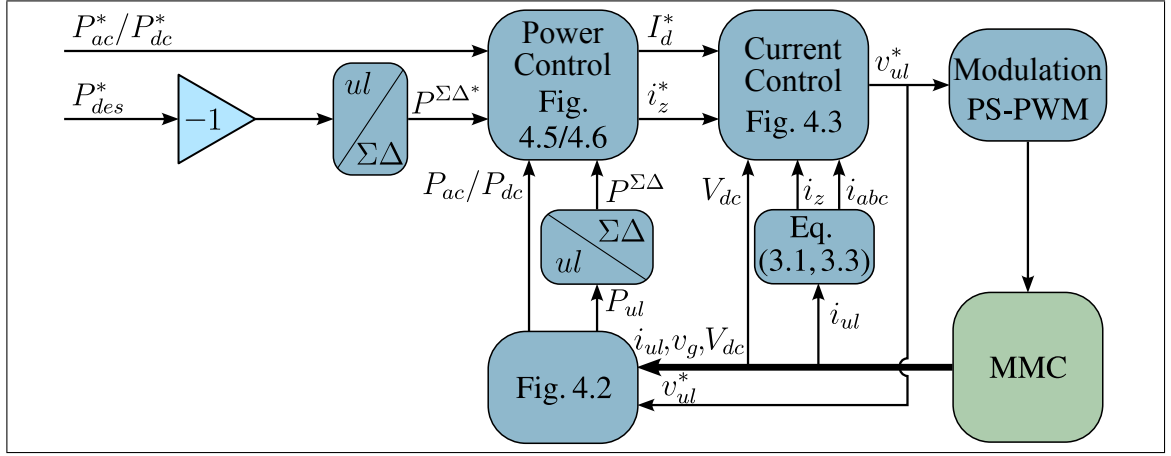


Figure 4.1. Implemented MMC-side control of the MMC-DES.

4.1. Modulation

The implemented modulation is a simple Phase Shift Pulse Width Modulation (PS-PWM). The modulation stage is where intra-cluster power is managed, by implementing algorithms that can treat each cluster SM in an individual manner depending on the operating requirements. However intra-cluster power is out of the scope of this thesis, as explained in the section 1.3, so there is no enhancement to the classical PS-PWM. The cluster voltage reference is just divided by the number of SMs, and the same voltage waveform is fed to all the SMs of a cluster.

4.2. Measurements and filters

The control feedback loops require measurements or estimations of the controlled variables. Measurements are more reliable than estimations, but also more costly. It is convenient to utilize sensors for the control loops but keeping them to a low amount. The minimum amount of current measurements that allow the reconstruction of all the converter currents is the six cluster currents, which are measured. All SM capacitor voltages are measured to observe if the DES-side control is indeed keeping them at the reference voltage. The cluster voltage was then reconstructed using modulation information. The ac

grid voltage was also measured to obtain the grid phase angle and for power calculation. The dc grid voltage was measured for accuracy in the power control, but it could not be measured if the dc grid voltage is known and considered stable and reliable enough.

The circulating and ac port currents were calculated from the cluster current measurements using equations (3.1) and (3.3), and fed directly to the current control. In the case of the power control, the cluster power was calculated multiplying the cluster current measurements and the cluster voltage reference. Assuming an accurate modulation, the voltage reference would be equal to the real cluster voltages minus high frequency components derived from the modulation. Those high frequency components do not make a relevant contribution to net power and would disturb the power control, so they can be neglected. The ports powers were calculated by multiplying the derived currents and the measured voltages.

The power calculation obtained by the previously described multiplication contains oscillatory power components that must be filtered to obtain the average dc power values. The oscillatory components are of two main frequencies, the fundamental frequency and the second harmonic, but there is also smaller presence of high frequency components mainly due to current ripple. To remove these components, the instant power $p_{y\chi}$ is passed through a low-pass filter and two notch filters. Filtering the power adds a lag to the result, which needs to be considered when designing the time constant of the controllers. The resulting power calculation pipeline is shown in Fig. 4.2.

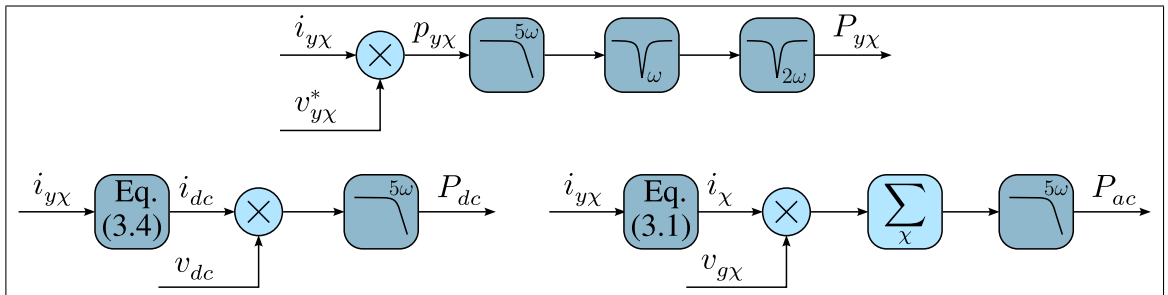


Figure 4.2. Power calculation.

4.3. Current control

The controlled currents are the ac port and circulating currents. The equations (3.13) and (3.14) show that they can be controlled independently. Thus, they could be jointly controlled by a single control algorithm, or be controlled by different algorithms. The equations (3.2) and (3.4) show that the dc port and cluster currents are linearly dependent on the ac port and circulating currents. This means that they do not need to be directly controlled.

4.3.1. AC port current

This thesis only considers the transfer of active power in the converter ports, meaning that the ac port currents are considered to be always balanced with a 120° phase shift between them. In consequence, a proportional integral (PI) control in the Clarke-Park (dq) transform space is perfectly suited and is the selected scheme. The output of this controller is the v_x^Δ voltage. The dq transform requires the desired current angle $\theta \approx \omega t$, which is obtained through a Phase Locked Loop (PLL) algorithm from voltage measurements of the ac grid.

The dq current measurements can be obtained through the Clarke-Park transformation

$$\begin{bmatrix} I_d \\ I_q \\ I_0 \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos(\theta) & \cos\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) \\ -\sin(\theta) & -\sin\left(\theta - \frac{2\pi}{3}\right) & -\sin\left(\theta + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \quad (4.1)$$

The transform is not required for the reference currents because the power control is designed to output the magnitude of the ac port current, which is equivalent to the d current I_d . The q current I_q is set to zero because only active power is transferred.

The dq voltages outputted by the controller are passed through the inverse Clarke-Park transform, to obtain the Δ voltages. Assuming a perfect balanced and symmetric system ($V_0 = 0$), the inverse transform is

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 1 \\ \cos\left(\theta - \frac{2\pi}{3}\right) & -\sin\left(\theta - \frac{2\pi}{3}\right) & 1 \\ \cos\left(\theta + \frac{2\pi}{3}\right) & -\sin\left(\theta + \frac{2\pi}{3}\right) & 1 \end{bmatrix} \cdot \begin{bmatrix} V_d \\ V_q \\ V_0 \end{bmatrix} \quad (4.2)$$

4.3.2. Circulating currents

The circulating currents vary across phases depending on the desired vertical imbalances. Therefore, their phasors are not necessarily equal in magnitude nor do they have a fixed 120° phase shift. To control them, it seems more convenient to use an algorithm that does not take those assumptions and can follow any waveform. Some alternatives are a Linear Quadratic Regulator (LQR) or a Model Predictive Control (MPC) algorithm. The first one is selected for its simplicity and the high availability of toolboxes to design it.

A LQR is a type of state-feedback control, where the plant state is fed back to the system through a gain K , effectively modifying the response dynamics of the system. The LQR is an automated way to determine the gain K , based on weight factors of the controlled variable and the manipulated variable. To design the controller, the circulating current plant is written in its state space matrix form:

$$\dot{\vec{x}} = A\vec{x} + B\vec{u} + \vec{w} \quad (4.3)$$

$$\vec{y} = C\vec{x} \quad (4.4)$$

where,

$$A = \begin{bmatrix} \frac{-R_s}{L_s} & 0 & 0 \\ 0 & \frac{-R_s}{L_s} & 0 \\ 0 & 0 & \frac{-R_s}{L_s} \end{bmatrix}, \quad B = \begin{bmatrix} \frac{-1}{L_s} & 0 & 0 \\ 0 & \frac{-1}{L_s} & 0 \\ 0 & 0 & \frac{-1}{L_s} \end{bmatrix}, \quad \vec{w} = \begin{bmatrix} \frac{V_{dc}}{2L_s} \\ \frac{V_{dc}}{2L_s} \\ \frac{V_{dc}}{2L_s} \end{bmatrix} \quad (4.5)$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

After choosing appropriate weights, the discrete time LQR feedback constant K_d can be calculated solving the discrete Riccati equation or obtained through a software toolbox (The MathWorks Inc., 2021).

As the LQR only changes the dynamic of the system, it does not yet follow a reference. There are a couple alternatives to close the control loop. The reference can be passed through an error integrator and then fed to the LQR. The LQR can be constructed in its tracking error form. The reference can be used to calculate the steady state value of the manipulated variable, and the result can be fed to the system. This last approach can be susceptible to permanent error, but is nonetheless selected due to its simplicity. The steady state manipulated variable can be obtained as followed:

$$\dot{\vec{x}}_{ss} = \vec{0} \implies \vec{x}_{ss} = -A^{-1}(B\vec{u}_{ss} + \vec{w}) \quad (4.6)$$

$$\vec{y}_{ss} = C\vec{x}_{ss} = -CA^{-1}B\vec{u}_{ss} - CA^{-1}\vec{w} \quad (4.7)$$

$$\implies \vec{u}_{ss} = -(CA^{-1}B)^{-1}(\vec{y}_{ss} + CA^{-1}\vec{w}) \quad (4.8)$$

Two constants are defined to abbreviate the LQR scheme,

$$K_u = -(CA^{-1}B)^{-1} \quad (4.9)$$

$$K_w = \frac{1}{2L_s}CA^{-1} \quad (4.10)$$

Finally, the current control is depicted in Fig. 4.3.

4.4. Power control

Not all powers are directly controlled. Only two are controlled and the third forcefully changes its value bounded by the expression (3.32). The ports power dictate how much power enters or exits the converter as a whole, while the DES power dictates how much power enters from the DES through each cluster. Thus to ensure proper control over the DES constituents, the DES power must always be controlled.

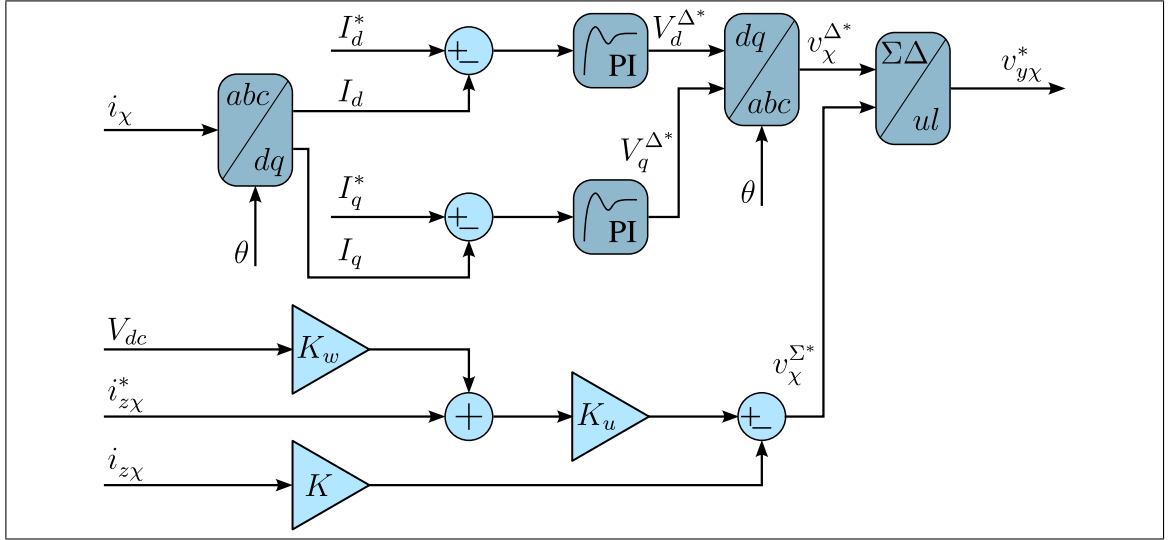


Figure 4.3. Converter current control.

The second controlled power depends on the type of compensation. The compensated port must adjust its value dynamically depending on the changes in the DES power. Meanwhile, the uncompensated or fixed port must maintain its value independently of the DES contribution. The approach that best fits the described behavior, is to control the fixed port and to leave the compensated port subject to the others.

4.4.1. $\Sigma\Delta$ power control

The cluster power control was designed using the $\Sigma\Delta$ power equations (3.27)-(3.30), so the measured cluster power is in first place transformed into their $\Sigma\Delta$ counterparts. As explained in the section 3.3.2, the control of Σ power enables the horizontal distribution of the DES power within the converter, and Δ power allows the vertical distribution of the DES power within the converter phases.

The term $P_{\chi,0}^\Sigma$ contains losses and power from the dc port. It can be noted that the power that flows into each phase χ can be regulated independently. The term $P_{\chi,1}^\Sigma$ contains losses and power from the ac port. But in this case, the power delivered from or to each

phase must be the same as the others to maintain balance in the ac port. This evidentiates that horizontal power control must be performed by the dc circulating currents.

The use of ac circulating current for vertical imbalances entails an issue. If an imbalance is required in one of the phases and not in the others, the ac current that will flow into the converter must come from the dc link to close the path for the current, as (3.4) will have ac component different 0. This is far from ideal as the dc grid will now contain an ac current component.

To solve this issue, the ac circulating currents that flow on all phases must add up to zero. So if power is required in one of the phases, compensatory currents must be injected into the other phases. To avoid causing undesired vertical imbalances on the other phases, the compensatory currents must have a phase angle such that they produce only reactive power and no active power. Disregarding losses, the equation (3.30) shows that the required angle is $\theta_\chi \pm 90^\circ$. The resultant currents are shown in Fig. 4.4.

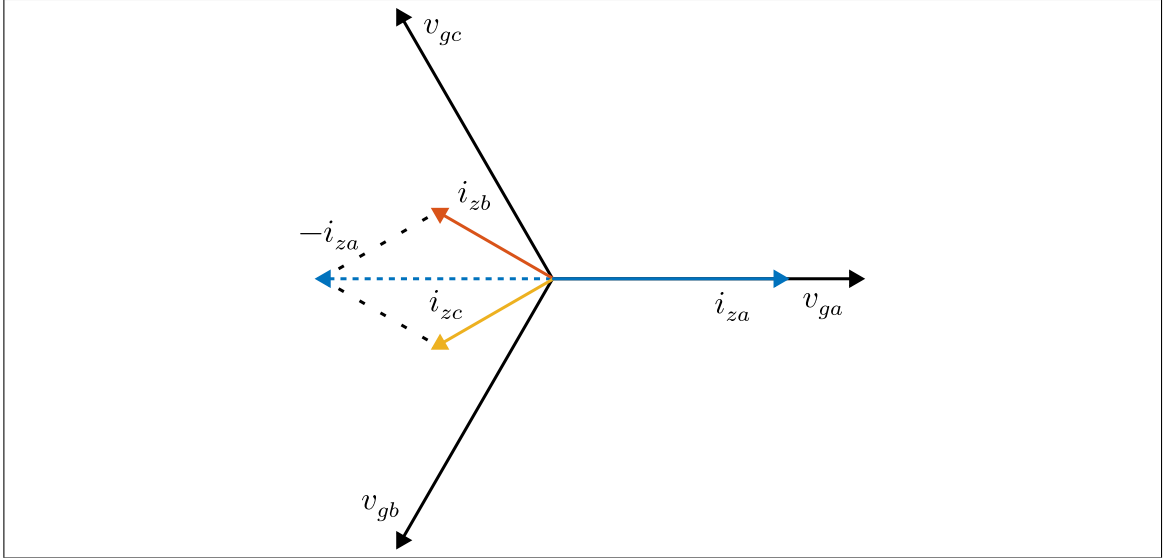


Figure 4.4. Compensatory currents. Vertical imbalance only in phase a .

This mechanism was first proposed by (Münch et al., 2010) and can be generalized for the ac components of all three circulating currents as followed,

$$i_{z\chi,1} = i_{z\chi}^p + i_{z\chi}^c \quad (4.11)$$

$$i_{z\chi}^p = I_{z\chi}^p \cos(\omega t + \theta_\chi) \quad (4.12)$$

$$i_{z\chi}^c = I_{z\chi}^c \cos(\omega t + \theta_\chi + 90^\circ) \quad (4.13)$$

$$I_{z\chi}^c = \begin{cases} \frac{1}{\sqrt{3}}(I_{zb}^p - I_{zc}^p), & \chi = a \\ \frac{1}{\sqrt{3}}(I_{zc}^p - I_{za}^p), & \chi = b \\ \frac{1}{\sqrt{3}}(I_{za}^p - I_{zb}^p), & \chi = c \end{cases} \quad (4.14)$$

where $i_{z\chi}^p$ is the primary current, which produces Δ power, and $i_{z\chi}^c$ is the compensatory current.

The $\Sigma\Delta$ powers are controlled by PI controllers. The output of the controllers are the dc and ac components of the circulating currents. The compensatory currents are added afterwards to obtain the reference circulating current waveform.

4.4.2. DC compensation

The bounds described in the section 3.4 imply that the dc compensation operates in two main ways. If the converter operates in inverter direction, when the DES supplies energy to the converter, the dc port must drain less energy from the dc grid. Conversely if the converter operates in rectifier direction, when the DES consumes energy from the converter, that energy is subtracted from the energy that is supplied to the dc grid.

The dc compensation control is designed to receive references for the ac port power and the DES power. The ac port control generates output ac current references. The DES control generates ac circulating current references and dc circulating current references that add up to the dc port current as shown in (3.4). Thus, the DES power control indirectly controls the dc port power.

The ac port power is controlled by a PI controller. The output of the controller is the ac port current reference. If the ac power reference changes, Σ power also changes, hence disturbing the DES power control. To counter this disturbance, the Σ power loop output receives a feed forward component from the output of the ac port power. The resulting control scheme is shown in Fig. 4.5.

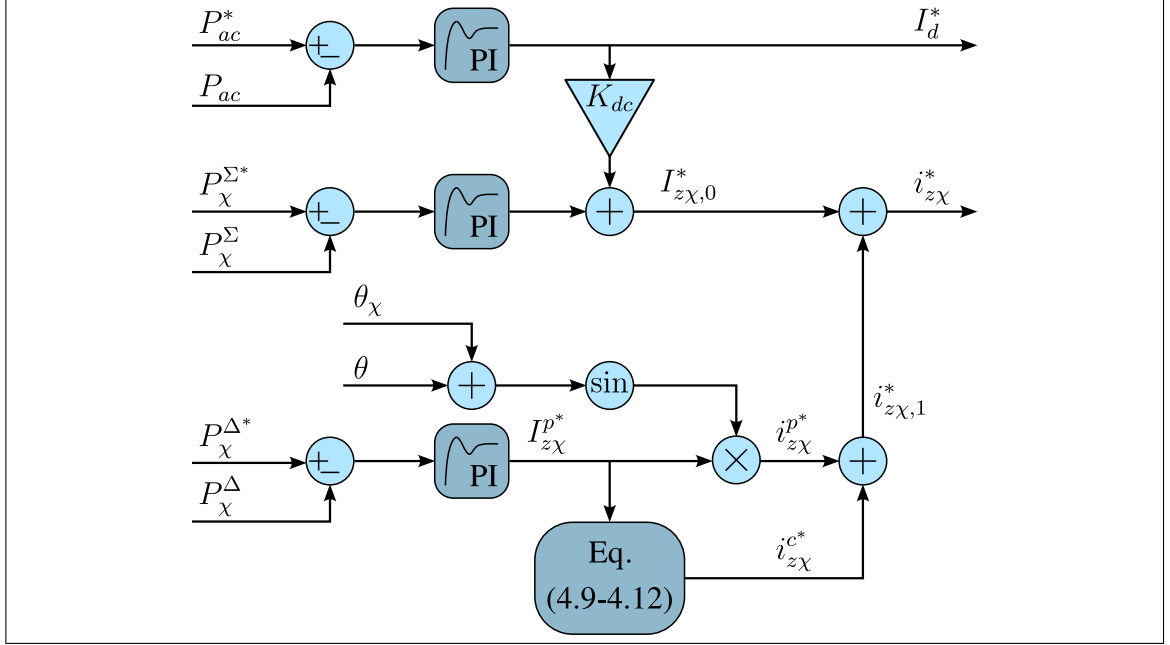


Figure 4.5. DC compensation power control. $K_{dc} = \frac{V_g}{2V_{dc}}$.

4.4.3. AC compensation

To the best of the author knowledge, there is no ac compensation CPC algorithm present in the literature, and only a single ac compensation DPC algorithm has been proposed (Schroeder & Jaeger, 2019). Therefore the forthcoming control algorithm is another contribution of this thesis.

The bounds described in the section 3.4 imply that the ac compensation operates in two main ways. If the converter operates in inverter direction, when the DES absorbs energy from the converter, that energy is subtracted from the energy that is supplied to the

ac grid. Conversely if the converter operates in rectifier direction, when the DES supplies energy to the converter, the ac port must drain less energy from the ac grid.

The ac compensation control is designed to receive references for the dc port power and the DES power. The dc port control generates the reference for the sum of all dc circulating currents. The DES control generates ac circulating current references and the ac output current reference. Thus, the DES power control indirectly controls the ac port power.

In the case of dc compensation, the dc circulating currents could be controlled independently from each other. However under ac compensation, they must comply the restriction of adding up to a fixed dc current value. Additionally, the ac port currents must have the same magnitude across all phases. To independently control Σ power across phases while complying with both restrictions, the Σ power is divided in two components. The average Σ power P_{avg}^{Σ} and the variational Σ power $P_{\delta\chi}^{\Sigma}$.

$$P_{avg}^{\Sigma} = \frac{1}{3} \sum_{\chi} P_{\chi}^{\Sigma} \quad (4.15)$$

$$P_{\delta\chi}^{\Sigma} = P_{\chi}^{\Sigma} - P_{avg}^{\Sigma} \quad (4.16)$$

The average Σ power is controlled by the ac port current, and the variational Σ powers are controlled by the dc circulating currents. The $P_{\delta\chi}^{\Sigma}$ control does not conflict with the dc port power control because by definition

$$\sum_{\chi} P_{\delta\chi}^{\Sigma} = 0 \quad (4.17)$$

so the $P_{\delta\chi}^{\Sigma}$ power consumed by one phase is supplied by the others and not by the dc port.

The dc port power is controlled by a PI controller. The output of the controller is the dc port current reference. If the dc power reference changes, Σ power also changes, disturbing the DES power control. This problem is the same as in the dc compensating

case. The same solution is implemented. The resulting control scheme is shown in Fig. 4.6.

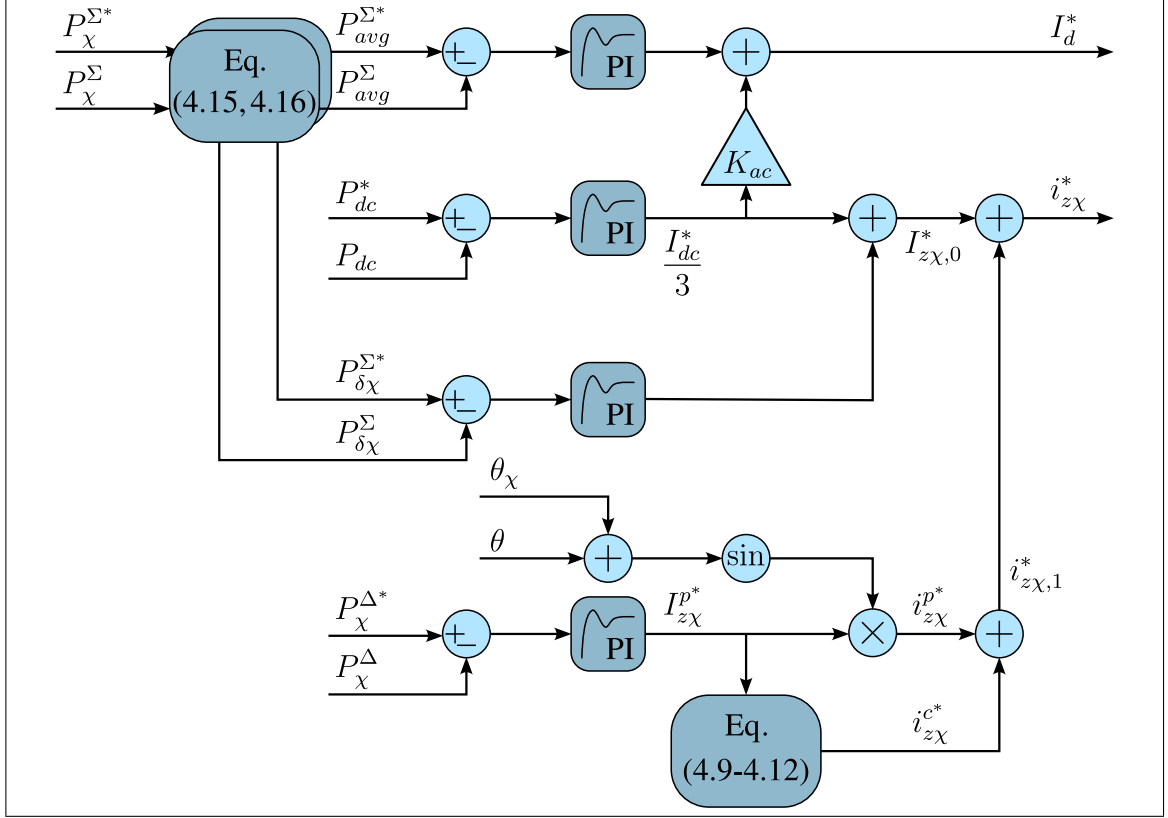


Figure 4.6. AC compensation power control. $K_{ac} = \frac{2V_{dc}}{V_g}$.

4.5. Conclusion

This chapter presented the proposed MMC-side CPC-SEC scheme for the a MMC-DES. It described the different control loops and what algorithms were chosen. The current control was separated into two controllers, a PI for the ac port currents and a LQR for the circulating currents. All powers were controlled by PI controllers.

The power control of the converter was subdivided into two control schemes, one for dc compensation and one for ac compensation. They are different due to differences

in their operating restrictions. The dc compensation has full freedom to control the dc circulating currents, while the ac compensation does not. The ac compensation has some freedom over the dc circulating currents and the ac port currents, but must obey partial restrictions regarding both. While the dc compensation controller is fairly standard, there is no ac compensation CPC scheme present in the literature, which makes the proposed scheme an additional contribution of this thesis.

The chapter also described the measurements that were chosen and their treatment to construct the feedback signals for the control loops. The controlled currents were not directly measured, but were obtained from the measurements through equations (3.1) and (3.3). The power is also obtained through measurements and equations, but in addition it needs to be filtered. This filter adds a lag to the resultant power signals, and must be considered in the controller tuning.

5. IMBALANCE MODELING

Relevant new symbology

λ : Imbalance factor

ζ : Ovrating factor

κ : Power directionality (inverter/rectifier)

Γ : Function (5.18) for $i_{z\chi,1}$ magnitude

The cluster power imbalance of a MMC-DES is directly related to the $\Sigma\Delta$ cluster power. As stated before, Δ power relates to vertical imbalances, and Σ power to horizontal imbalance and the DES power contribution to the system. To represent the imbalances, new imbalance factors λ_χ^Σ and λ_χ^Δ are defined. They are the $\Sigma\Delta$ DES powers normalized by the power that they would achieve if the DES were to supply all the nominal power of one of the converter grid ports P^{nom} .

$$\lambda_\chi^\Sigma = \frac{P_{des,\chi}^\Sigma}{P^{nom}/6} \implies P_{des,\chi}^\Sigma = \frac{\lambda_\chi^\Sigma P^{nom}}{6} \quad (5.1)$$

$$\lambda_\chi^\Delta = \frac{P_{des,\chi}^\Delta}{P^{nom}/6} \implies P_{des,\chi}^\Delta = \frac{\lambda_\chi^\Delta P^{nom}}{6} \quad (5.2)$$

The aim is to find expressions that can link the changes in the converter current and voltage as a function of these imbalance factors. This has to be done for both dc and ac compensation modes individually, as both modes manage the converter currents differently to achieve the same imbalanced DES states.

The rating analysis of the converter requires the definition of overrating factors. They are the maximum value that one of the converter variables of interest achieves in a determined unbalanced state, divided by the maximum value of that variable in its nominal balanced state:

$$\zeta_{i\chi}(\lambda) = \frac{\max_{y,t}\{|i_{y\chi}|\}}{I_{y\chi}^{nom}} \quad (5.3)$$

$$\zeta_{v\chi}(\lambda) = \frac{\max_{y,t}\{|v_{y\chi} - \frac{V_{dc}}{2}|\}}{\max_{y,t}\{|v_{y\chi}^{nom} - \frac{V_{dc}}{2}|\}} \quad (5.4)$$

where $I_{y\chi}^{nom}$ is the maximum maximum value of the cluster currents during nominal converter operation. It can be derived either from the power equations (3.15) and relations (3.2, 3.4), or by setting the imbalance factors to zero in the expressions (5.23-5.24),

$$\begin{aligned} I_{y\chi}^{nom} &= \max_{y,t}\{|i_{y\chi}^{nom}|\} \\ &= \max_{y,t}\{|i_{z\chi,0}^{nom}|\} + \max_{y,t}\{|i_{y\chi,1}^{nom}|\} \\ &= \frac{P^{nom}}{3} \left(\frac{1}{V_{dc}} + \frac{1}{V_g} \right) \end{aligned} \quad (5.5)$$

The voltage overrating expressions have a $\frac{V_{dc}}{2}$ subtraction to effectively center them in zero, as the cluster voltage $v_{y\chi}$ can reach values between 0 and V_{dc} and the steady state reference voltage must not exceed neither of those margins. The overrating factors are calculated for both compensation schemes, yielding $\zeta_{i\chi}^{dc}$, $\zeta_{i\chi}^{ac}$, $\zeta_{v\chi}^{dc}$, and $\zeta_{v\chi}^{ac}$.

The direction of the power between the converter grid ports is represented by a constant κ as

$$\kappa = \begin{cases} 1 & : \text{inverter} \\ -1 & : \text{rectifier} \end{cases} \quad (5.6)$$

A dc link overrating factor is also defined to enable cleaner analytic expressions,

$$\zeta_{dc} = \frac{V_{dc}}{2V_g} \quad (5.7)$$

The following trigonometric identity will be of practical use to simplify the forthcoming expressions.

$$a \sin(\omega t + \alpha) + b \sin(\omega t + \beta) = \sqrt{a^2 + b^2 + 2ab \cos(\alpha - \beta)} \cdot \sin \left(\omega t + \tan^{-1} \left(\frac{a \sin(\alpha) + b \sin(\beta)}{a \cos(\alpha) + b \cos(\beta)} \right) \right) \quad (5.8)$$

5.1. DC compensation

In dc compensation mode, the ac port power is always fixed at its nominal value. This means that the ac port power will always be

$$P_{ac} = -\kappa P^{nom} \quad (5.9)$$

Note that the ac port current is balanced by definition, meaning that

$$I_a = I_b = I_c = I_g \quad (5.10)$$

thus the relations (3.15) and (5.9) imply that in dc compensation:

$$\begin{aligned} -\kappa P^{nom} &= \frac{-3V_g I_g}{2} \\ \implies I_g &= \frac{2\kappa P^{nom}}{3V_g} \end{aligned} \quad (5.11)$$

5.1.1. Current overrating

All the overrating expressions are derived from applying the $\Sigma\Delta$ transform to the relation (3.33). Doing so results in

$$P_{\chi,0}^{\Sigma} + P_{\chi,1}^{\Sigma} = -P_{des,\chi}^{\Sigma} \quad (5.12)$$

$$P_{\chi,1}^{\Delta} = -P_{des,\chi}^{\Delta} \quad (5.13)$$

The forthcoming procedure discards the losses terms. This is of course not ideal, but including them results in huge and absolutely impractical expressions.

5.1.1.1. Horizontal imbalance

The impact of horizontal imbalances on the converter currents can be derived from the equations (5.10, 5.11, 5.12).

$$\begin{aligned}\frac{V_{dc}I_{z\chi,0}}{2} - \frac{V_g I_g}{4} &= -\frac{\lambda_\chi^\Sigma P^{nom}}{6} \\ V_{dc}I_{z\chi,0} &= \frac{\kappa P^{nom}}{3} - \frac{\lambda_\chi^\Sigma P^{nom}}{3} \\ I_{z\chi,0} &= \frac{P^{nom}}{6\zeta_{dc}V_g}(\kappa - \lambda_\chi^\Sigma)\end{aligned}\tag{5.14}$$

5.1.1.2. Vertical imbalance

The impact of vertical imbalances on the converter currents can be derived from the equation (5.13),

$$\begin{aligned}\frac{\lambda_\chi^\Delta P^{nom}}{6} &= -\frac{I_{z\chi,1}^p V_g}{2} \\ I_{z\chi,1}^p &= -\frac{\lambda_\chi^\Delta P^{nom}}{3V_g}\end{aligned}\tag{5.15}$$

This term is only the primary component of the ac circulating currents. Note that the primary current is by definition in phase with the phase voltage. The total ac circulating current of one phase depends also on the compensatory currents, as shown in Fig. 5.1. These currents were defined back in Fig. 4.4 and equations (4.11-4.14).

The total ac circulating current of phase a is

$$\begin{aligned}I_{za,1} &= \sqrt{I_{za}^p{}^2 + \left(\frac{I_{zb}^p}{\sqrt{3}} - \frac{I_{zc}^p}{\sqrt{3}}\right)^2} \\ I_{za,1} &= \frac{P^{nom}}{3V_g} \sqrt{\lambda_a^\Delta{}^2 + \frac{1}{3}(\lambda_c^\Delta - \lambda_b^\Delta)^2}\end{aligned}\tag{5.16}$$

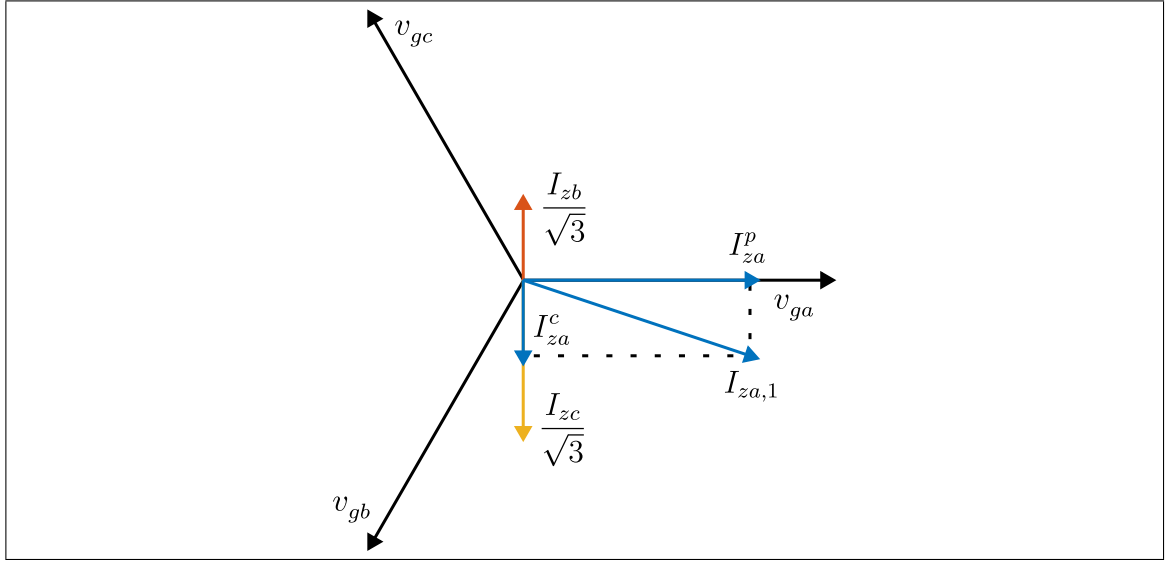


Figure 5.1. Magnitude of ac circulating current component.

This can be generalized for all phases, obtaining

$$I_{z\chi,1} = \frac{P^{nom}}{3V_g} \Gamma_\chi(\vec{\lambda}^\Delta) \quad (5.17)$$

where

$$\Gamma_\chi(\vec{\lambda}^\Delta) = \sqrt{\lambda_\chi^{\Delta^2} + \frac{1}{3} \sum_{j \neq \chi} \lambda_j^{\Delta^2} - \frac{2}{3} \prod_{j \neq \chi} \lambda_j^\Delta} \quad (5.18)$$

It is also evident in Fig. 5.1 that

$$\cos(\phi_{z\chi}) = \frac{I_{z\chi,1}^*}{I_{z\chi,1}} = \frac{-\lambda_\chi^\Delta}{\Gamma_\chi} \quad (5.19)$$

thus the phase of the current with respect to the voltage is

$$\phi_{z\chi}(\lambda^\Delta) = \cos^{-1} \left(\frac{-\lambda_\chi^\Delta}{\Gamma_\chi} \right) \quad (5.20)$$

Note: Before continuing, it is important to remark the effects of discarding the losses.

The expression (5.14) describes a linear relation between the dc circulating current and the horizontal imbalance, but this relation is actually slightly quadratic. The

Δ simplification is far more inconvenient, because the compensatory currents injection accounts for non negligible losses with a magnitude related to $\omega L_o I_\chi^c$. In consequence, it is expected that the ac circulating current will be miscalculated in a noticeable amount, but the significance of that error its not clear yet.

The ac cluster current can be obtained from the ac circulating current and the ac port current as in (3.2), by applying the identity (5.8).

$$\begin{aligned}
i_{y\chi,1} &= i_{z\chi,1} \pm \frac{i_\chi}{2} \\
&= I_{z\chi,1} \sin(\omega t + \theta_\chi + \phi_{z\chi}) \pm \frac{I_g}{2} \sin(\omega t + \theta_\chi) \\
&= \sqrt{I_{z\chi,1}^2 + \frac{I_g^2}{4} \pm I_{z\chi,1} I_g \cos(\phi_{z\chi})} \cdot \sin \left(\omega t + \theta_\chi + \tan^{-1} \left(\frac{I_{z\chi,1} \sin(\phi_{z\chi})}{I_{z\chi,1} \cos(\phi_{z\chi}) + \frac{I_g}{2}} \right) \right) \\
&= \sqrt{I_{z\chi,1}^2 + \frac{I_g^2}{4} \mp I_{z\chi,1} I_g \frac{\lambda_\chi^\Delta}{\Gamma_\chi}} \cdot \sin \left(\omega t + \theta_\chi + \tan^{-1} \left(\frac{I_{z\chi,1} \sin(\phi_{z\chi})}{I_{z\chi,1} \cos(\phi_{z\chi}) + \frac{I_g}{2}} \right) \right)
\end{aligned} \tag{5.21}$$

5.1.1.3. Overrating factor

The maximum cluster current can be disaggregated into two components,

$$\max_{y,t} \{|i_{y\chi}|\} = \max_{y,t} \{|i_{z\chi,0}|\} + \max_{y,t} \{i_{y\chi,1}\} \tag{5.22}$$

So to determine the converter current overrating, first is necessary to find the maximum absolute values of the dc and ac components of the cluster currents

$$\begin{aligned}
\max_{y,t} \{|i_{z\chi,0}|\} &= |I_{z\chi,0}| \\
&= \frac{P^{nom}}{6\zeta_{dc} V_g} |\kappa - \lambda_\chi^\Sigma|
\end{aligned} \tag{5.23}$$

$$\begin{aligned}
\max_{y,t}\{i_{y\chi,1}\} &= \max_y \left\{ \sqrt{I_{z\chi,1}^2 + \frac{I_g^2}{4} \mp I_{z\chi,1} I_g \frac{\lambda_\chi^\Delta}{\Gamma_\chi}} \right\} \\
&= \max_y \left\{ \sqrt{\left(\frac{P^{nom}}{3V_g}\right)^2 \Gamma_\chi^2 + \left(\frac{P^{nom}}{3V_g}\right)^2 \mp 2 \left(\frac{P^{nom}}{3V_g}\right)^2 \Gamma_\chi \frac{\lambda_\chi^\Delta}{\Gamma_\chi}} \right\} \\
&= \frac{P^{nom}}{3V_g} \sqrt{\max_y \{\Gamma_\chi^2 + 1 \mp 2\lambda_\chi^\Delta\}} \\
&= \frac{P^{nom}}{3V_g} \sqrt{\Gamma_\chi^2 + 2|\lambda_\chi^\Delta| + 1}
\end{aligned} \tag{5.24}$$

The maximum cluster current can be obtained from the sum of the ac and dc current magnitudes, (5.3) thus results in

$$\begin{aligned}
\zeta_{i\chi}^{dc}(\lambda) &= \frac{\max_{y,t}\{|i_{y\chi}|\}}{I_{y\chi}^{nom}} \\
&= \frac{\frac{P^{nom}}{6\zeta_{dc}V_g} |\kappa - \lambda_\chi^\Sigma| + \frac{P^{nom}}{3V_g} \sqrt{\Gamma_\chi^2 + 2|\lambda_\chi^\Delta| + 1}}{\frac{P^{nom}}{3V_g} (1 + \frac{1}{2\zeta_{dc}})} \\
&= \frac{|\kappa - \lambda_\chi^\Sigma| + 2\zeta_{dc} \sqrt{\Gamma_\chi^2 + 2|\lambda_\chi^\Delta| + 1}}{1 + 2\zeta_{dc}}
\end{aligned} \tag{5.25}$$

5.1.2. Voltage overrating

To find an expression the cluster voltage, we first analyze each component, evaluating the newfound current component expressions.

5.1.2.1. Voltage components

Beginning with the DC component, from (3.14) it can be found that

$$\frac{d}{dt}i_{z\chi,0} = \frac{1}{2L_s}(-2v_{\chi,0}^{\Sigma} - 2R_s i_{z\chi,0} + V_{dc}) \quad (5.26)$$

$$\begin{aligned} \Rightarrow v_{\chi,0}^{\Sigma} &= -R_s I_{z\chi,0} + \frac{V_{dc}}{2} \\ &= -\frac{R_s P^{nom}}{6\zeta_{dc} V_g}(\kappa - \lambda_{\chi}^{\Sigma}) + \zeta_{dc} V_g \end{aligned} \quad (5.27)$$

Note that

$$v_{y\chi,0} = v_{\chi,0}^{\Sigma} \quad (5.28)$$

Similarly, the ac component of the equation (3.14) is

$$\frac{d}{dt}i_{z\chi,1} = \frac{1}{2L_s}(-2v_{\chi,1}^{\Sigma} - 2R_s i_{z\chi,1}) \quad (5.29)$$

from which it can be derived that the ac Σ component of the converter voltage is

$$v_{\chi,1}^{\Sigma} = -I_{z\chi,1}(\omega L_s \cos(\omega t + \phi_{z\chi}) + R_s \sin(\omega t + \phi_{z\chi})) \quad (5.30)$$

The trigonometric identity (5.8) is of use now, applying it and evaluating (5.17) results in

$$v_{\chi,1}^{\Sigma} = -\frac{P^{nom}}{3V_g} Z_s \Gamma_{\chi} \sin\left(\omega t + \phi_{z\chi} + \tan^{-1}\left(\frac{\omega L_s}{R_s}\right)\right) \quad (5.31)$$

where

$$Z_s = \sqrt{\omega^2 L_s^2 + R_s^2} \quad (5.32)$$

The found expression can be rewritten as

$$v_{\chi,1}^{\Sigma} = V_{\chi,1}^{\Sigma} \sin(\omega t + \psi_{\chi,1}^{\Sigma}) \quad (5.33)$$

where

$$\begin{aligned} V_{\chi,1}^{\Sigma}(\lambda^{\vec{\Delta}}) &= -\frac{P^{nom}}{3V_g} Z_s \Gamma_{\chi}(\lambda^{\vec{\Delta}}) \\ \psi_{\chi,1}^{\Sigma}(\lambda^{\vec{\Delta}}) &= \theta_{\chi} + \phi_{z\chi}(\lambda^{\Delta}) + \tan^{-1} \left(\frac{\omega L_s}{R_s} \right) \end{aligned} \quad (5.34)$$

The same procedure can be applied to $v_{\chi,1}^{\Delta}$ through (3.13), yielding

$$v_{\chi,1}^{\Delta} = V_1^{\Delta} \sin(\omega t + \psi_{\chi,1}^{\Delta}) \quad (5.35)$$

$$\begin{aligned} V_1^{\Delta} &= \sqrt{\left(\frac{2P^{nom} Z_s}{3V_g} \right)^2 + \frac{4\kappa P^{nom} R_{\epsilon}}{3} + V_g^2} \\ \psi_{\chi,1}^{\Delta} &= \theta_{\chi} + \tan^{-1} \left(\frac{I_g \omega L_{\epsilon}}{I_g R_{\epsilon} + V_g} \right) \end{aligned} \quad (5.36)$$

The ac cluster voltage relates to the $\Sigma\Delta$ components as

$$\begin{aligned} v_{y\chi,1} &= v_{\chi,1}^{\Sigma} \mp v_{\chi,1}^{\Delta} \\ &= V_{\chi,1}^{\Sigma} \sin(\omega t + \psi_{\chi,1}^{\Sigma}) \mp V_{\chi,1}^{\Delta} \sin(\omega t + \psi_{\chi,1}^{\Delta}) \end{aligned} \quad (5.37)$$

The identity (5.8) can be used again to find an expression for the ac component of the cluster voltage, resulting in

$$v_{y\chi,1} = V_{y\chi,1} \sin(\omega t + \psi_{y\chi,1}) \quad (5.38)$$

$$\begin{aligned} V_{y\chi,1} &= \sqrt{V_{\chi,1}^{\Sigma 2} + V_{\chi,1}^{\Delta 2} \mp 2V_{\chi,1}^{\Sigma} V_{\chi,1}^{\Delta} \cos(\psi_{\chi,1}^{\Sigma} - \psi_{\chi,1}^{\Delta})} \\ \psi_{y\chi,1} &= \tan^{-1} \left(\frac{V_{\chi,1}^{\Sigma} \sin(\psi_{\chi,1}^{\Sigma}) \mp V_{\chi,1}^{\Delta} \sin(\psi_{\chi,1}^{\Delta})}{V_{\chi,1}^{\Sigma} \cos(\psi_{\chi,1}^{\Sigma}) \mp V_{\chi,1}^{\Delta} \cos(\psi_{\chi,1}^{\Delta})} \right) \end{aligned} \quad (5.39)$$

5.1.2.2. Overrating factor

Just like the cluster currents, the maximum cluster voltage can be disaggregated into two components,

$$\max_{y,t} \left\{ \left| v_{y\chi} - \frac{V_{dc}}{2} \right| \right\} = \max_{y,t} \left\{ \left| v_{y\chi,0} - \frac{V_{dc}}{2} \right| \right\} + \max_{y,t} \{ v_{y\chi,1} \} \quad (5.40)$$

To find the voltage overrating factor, we first find the maximum value of the nominal voltage components. Note that

$$V_{\chi,1}^{\Sigma nom} = V_{\chi,1}^{\Sigma}(0) = 0 \quad (5.41)$$

then the nominal ac component is

$$\begin{aligned} \max_{y,t} \{v_{y\chi,1}^{nom}\} &= V_{y\chi,1}^{nom} \\ &= V_1^{\Delta} \end{aligned} \quad (5.42)$$

and the nominal dc shift from the mean cluster voltage value is

$$\begin{aligned} \max_{y,t} \left\{ \left| v_{y\chi,0}^{nom} - \frac{V_{dc}}{2} \right| \right\} &= | -R_s I_{z\chi,0}^{nom} | \\ &= \frac{R_s P^{nom}}{6\zeta_{dc} V_g} \end{aligned} \quad (5.43)$$

Similarly, the maximum dc shift from the mean cluster voltage value is

$$\begin{aligned} \max_{y,t} \left\{ \left| v_{y\chi,0} - \frac{V_{dc}}{2} \right| \right\} &= \left| -\frac{R_s P^{nom}}{6\zeta_{dc} V_g} (\kappa - \lambda_{\chi}^{\Sigma}) \right| \\ &= \frac{R_s P^{nom}}{6\zeta_{dc} V_g} |\kappa - \lambda_{\chi}^{\Sigma}| \end{aligned} \quad (5.44)$$

The maximum value of the ac component is

$$\max_{y,t} \{v_{y\chi,1}\} = \max_y \left\{ \sqrt{V_{\chi,1}^{\Sigma 2} + V_1^{\Delta 2} \mp 2V_{\chi,1}^{\Sigma} V_1^{\Delta} \cos(\psi_{\chi,1}^{\Sigma} - \psi_{\chi,1}^{\Delta})} \right\} \quad (5.45)$$

The cosine term depends on converter parameters and operating points, it is simplified assuming the worst case,

$$\begin{aligned} \max_{y,t} \{v_{y\chi,1}\} &\approx \sqrt{V_{\chi,1}^{\Sigma 2} + V_1^{\Delta 2} + 2|V_{\chi,1}^{\Sigma}| |V_1^{\Delta}|} \\ &= |V_{\chi,1}^{\Sigma}| + V_1^{\Delta} \quad , \quad V_1^{\Delta} \geq 0 \\ &= \frac{P^{nom}}{3V_g} Z_s |\Gamma_{\chi}| + V_1^{\Delta} \end{aligned} \quad (5.46)$$

Finally, the over-nominal voltage factor of each phase is

$$\begin{aligned}
\zeta_{v\chi}^{dc}(\lambda) &= \frac{\max_{y,t} \left\{ \left| v_{y\chi} - \frac{V_{dc}}{2} \right| \right\}}{\max_{y,t} \left\{ \left| v_{y\chi}^{nom} - \frac{V_{dc}}{2} \right| \right\}} \\
&= \frac{\frac{R_s P^{nom}}{6\zeta_{dc} V_g} |\kappa - \lambda_\chi^\Sigma| + \frac{P^{nom}}{3V_g} Z_s |\Gamma_\chi| + V_1^\Delta}{\frac{R_s P^{nom}}{6\zeta_{dc} V_g} + V_1^\Delta} \\
&= \frac{\frac{P^{nom}}{3V_g} (Z_s |\Gamma_\chi| + \frac{R_s}{2\zeta_{dc}} |\kappa - \lambda_\chi^\Sigma|) + V_1^\Delta}{\frac{R_s P^{nom}}{6\zeta_{dc} V_g} + V_1^\Delta} \tag{5.47}
\end{aligned}$$

5.2. AC compensation

In ac compensation mode, the dc port power is always fixed at its nominal value. This means that the dc port power will always be

$$P_{dc} = \kappa P^{nom} \tag{5.48}$$

The expressions (3.15), (5.48) and (3.4) imply that in dc compensation:

$$\begin{aligned}
\kappa P^{nom} &= I_{dc} V_{dc} \\
\Rightarrow \sum_{\chi} I_{z\chi,0} &= \frac{\kappa P^{nom}}{V_{dc}} \tag{5.49}
\end{aligned}$$

In (5.49) ac compensation mode establishes a condition for the sum of the dc circulating currents, whereas dc compensation established a condition for the currents themselves.

Therefore, in this case is of practical use to define new auxiliary imbalance factors.

$$\lambda_{avg}^{\Sigma} = \frac{1}{3} \sum_{\chi} \lambda_{\chi}^{\Sigma} \quad (5.50)$$

$$\lambda_{\delta\chi}^{\Sigma} = \lambda_{\chi}^{\Sigma} - \lambda_{avg}^{\Sigma} \quad (5.51)$$

5.2.1. Current overrating

5.2.1.1. Horizontal imbalance

From the equation (3.33) is evident that

$$\sum_{\chi} (P_{\chi,0}^{\Sigma} + P_{\chi,1}^{\Sigma}) = \sum_{\chi} -P_{des,\chi}^{\Sigma} \quad (5.52)$$

Thus

$$\frac{V_{dc}}{2} \sum_{\chi} I_{z\chi,0} - \frac{V_g}{4} \sum_{\chi} I_{\chi} = -\frac{P^{nom}}{6} \sum_{\chi} \lambda_{\chi}^{\Sigma} \quad (5.53)$$

Evaluating (5.49), (5.10) and (5.50) yields,

$$\begin{aligned} \kappa P^{nom} - \frac{3V_g}{2} I_g &= -\lambda_{avg}^{\Sigma} P^{nom} \\ \Rightarrow I_g &= \frac{2P^{nom}}{3V_g} (\kappa + \lambda_{avg}^{\Sigma}) \end{aligned} \quad (5.54)$$

This expression relation partially describes the impact of horizontal imbalances on the converter in ac compensation mode. To find the missing part, the newfound ac port current

expression (5.54) and (5.10) are evaluated in the relation (5.12),

$$\begin{aligned}
\frac{V_{dc}I_{z\chi,0}}{2} - \frac{1}{4}V_gI_g &= -\frac{\lambda_\chi^\Sigma P^{nom}}{6} \\
V_{dc}I_{z\chi,0} - \frac{P^{nom}}{3}(\kappa + \lambda_{avg}^\Sigma) &= -\frac{\lambda_\chi^\Sigma P^{nom}}{3} \\
V_{dc}I_{z\chi,0} &= \frac{P^{nom}}{3}(\kappa - (\lambda_\chi^\Sigma - \lambda_{avg}^\Sigma)) \\
I_{z\chi,0} &= \frac{P^{nom}}{6\zeta_{dc}V_g}(\kappa - \lambda_{\delta\chi}^\Sigma)
\end{aligned} \tag{5.55}$$

5.2.1.2. Vertical imbalance

While the implications of horizontal imbalances varies between dc and ac compensation, vertical imbalances cause the same currents in both modes. The additional currents relate to the vertical imbalances as described in (5.13). AC compensating mode does not establish new conditions on the ac circulating current, thus the ac cluster current component is also (5.17).

5.2.1.3. OVERRATING FACTOR

Now to determine the converter current overrating, again first is necessary to find the maximum absolute values of the dc and ac components of the cluster currents

$$\begin{aligned}
\max_{y,t}\{|i_{z\chi,0}|\} &= |I_{z\chi,0}| \\
&= \frac{P^{nom}}{6\zeta_{dc}V_g} |\kappa - \lambda_{\delta\chi}^\Sigma|
\end{aligned} \tag{5.56}$$

$$\begin{aligned}
\max_{y,t}\{i_{y\chi,1}\} &= \max_y \left\{ \sqrt{I_{z\chi,1}^2 + \frac{I_g^2}{4} \mp I_{z\chi,1} I_g \frac{\lambda_\chi^\Delta}{\Gamma_\chi}} \right\} \\
&= \max_y \left\{ \sqrt{\left(\frac{P^{nom}}{3V_g}\right)^2 \Gamma_\chi^2 + \left(\frac{P^{nom}}{3V_g}\right)^2 (\kappa + \lambda_{avg}^\Sigma)^2 \mp 2 \left(\frac{P^{nom}}{3V_g}\right)^2 (\kappa + \lambda_{avg}^\Sigma) \Gamma_\chi \frac{\lambda_\chi^\Delta}{\Gamma_\chi}} \right\} \\
&= \frac{P^{nom}}{3V_g} \sqrt{\max_y \{ \Gamma_\chi^2 + (\kappa + \lambda_{avg}^\Sigma)^2 \mp 2(\kappa + \lambda_{avg}^\Sigma) \lambda_\chi^\Delta \}} \\
&= \frac{P^{nom}}{3V_g} \sqrt{\Gamma_\chi^2 + (\kappa + \lambda_{avg}^\Sigma)^2 + 2 |(\kappa + \lambda_{avg}^\Sigma) \lambda_\chi^\Delta|} \quad (5.57)
\end{aligned}$$

Following the same procedure as in dc compensation, the current overrating factor for a determined operating point is

$$\begin{aligned}
\zeta_{i\chi}^{ac}(\lambda) &= \frac{\max_{y,t}\{|i_{y\chi}|\}}{\max_{y,t}\{|i_{y\chi}^{nom}|\}} \\
&= \frac{\frac{P^{nom}}{6\zeta_{dc}V_g} |\kappa - \lambda_{\delta\chi}^\Sigma| + \frac{P^{nom}}{3V_g} \sqrt{\Gamma_\chi^2 + (\kappa + \lambda_{avg}^\Sigma)^2 + 2 |(\kappa + \lambda_{avg}^\Sigma) \lambda_\chi^\Delta|}}{\frac{P^{nom}}{3V_g} (1 + \frac{1}{2\zeta_{dc}})} \\
&= \frac{|\kappa - \lambda_{\delta\chi}^\Sigma| + 2\zeta_{dc} \sqrt{\Gamma_\chi^2 + (\kappa + \lambda_{avg}^\Sigma)^2 + 2 |(\kappa + \lambda_{avg}^\Sigma) \lambda_\chi^\Delta|}}{1 + 2\zeta_{dc}} \quad (5.58)
\end{aligned}$$

5.2.2. Voltage overrating

5.2.2.1. Voltage components

Replicating what was done for the dc compensation case, the dc voltage component of the ac compensated converter is now

$$v_{y\chi,0} = -\frac{R_s P^{nom}}{6\zeta_{dc} V_g} (\kappa - \lambda_{\delta\chi}^\Sigma) + \zeta_{dc} V_g \quad (5.59)$$

The ac Σ component of the converter voltage is the same as in the dc compensation case (5.33)-(5.34). However the Δ component is now

$$v_{\chi,1}^\Delta = V_1^\Delta \sin(\omega t + \psi_{\chi,1}^\Delta) \quad (5.60)$$

$$V_1^\Delta(\vec{\lambda}^\Sigma) = \sqrt{\left(\frac{2P^{nom}Z_s}{3V_g}\right)^2 (\kappa + \lambda_{avg}^\Sigma)^2 + \frac{4P^{nom}R_\epsilon}{3}(\kappa + \lambda_{avg}^\Sigma) + V_g^2} \quad (5.61)$$

$$\psi_{\chi,1}^\Delta(\vec{\lambda}^\Sigma) = \theta_\chi + \tan^{-1}\left(\frac{I_g\omega L_\epsilon}{I_g R_\epsilon + V_g}\right)$$

and the total ac voltage has again the form (5.38)-(5.39).

5.2.2.2. Overrating factor

The maximum value of the nominal voltage components are the same as in dc compensation (5.42)-(5.43), but the general expressions differ. The maximum value of the ac component is similar but not the same as V_1^Δ is now different and a function of $\vec{\lambda}^\Sigma$.

$$\begin{aligned} \max_{y,t}\{v_{y\chi,1}\} &\approx \sqrt{V_{\chi,1}^\Sigma{}^2 + V_1^\Delta{}^2 + 2|V_{\chi,1}^\Sigma||V_1^\Delta|} \\ &= |V_{\chi,1}^\Sigma| + V_1^\Delta \quad , \quad V_1^\Delta \geq 0 \\ &= \frac{P^{nom}}{3V_g} Z_s |\Gamma_\chi| + V_1^\Delta(\vec{\lambda}^\Sigma) \end{aligned} \quad (5.62)$$

The over-nominal voltage factor of each phase is now

$$\begin{aligned}
\zeta_{v\chi}^{ac}(\lambda) &= \frac{\max_{y,t} \left\{ \left| v_{y\chi,0} - \frac{V_{dc}}{2} \right| \right\} + \max_{y,t} \{v_{y\chi,1}\}}{\max_{y,t} \left\{ \left| v_{y\chi,0}^{nom} - \frac{V_{dc}}{2} \right| \right\} + \max_{y,t} \{v_{y\chi,1}^{nom}\}} \\
&= \frac{\frac{R_s P^{nom}}{6\zeta_{dc} V_g} |\kappa - \lambda_{\delta\chi}^{\Sigma}| + \frac{P^{nom}}{3V_g} Z_s |\Gamma_{\chi}| + V_1^{\Delta}(\lambda^{\vec{\Sigma}})}{\frac{R_s P^{nom}}{6\zeta_{dc} V_g} + V_1^{\Delta}(\vec{0})} \\
&= \frac{\frac{P^{nom}}{3V_g} (Z_s |\Gamma_{\chi}| + \frac{R_s}{2\zeta_{dc}} |\kappa - \lambda_{\delta\chi}^{\Sigma}|) + V_1^{\Delta}(\lambda^{\vec{\Sigma}})}{\frac{R_s P^{nom}}{6\zeta_{dc} V_g} + V_1^{\Delta}(\vec{0})} \tag{5.63}
\end{aligned}$$

5.3. Conclusion

This chapter introduces a metric and symbolic representation for the magnitude of power imbalances in a MMC-DES, which are the imbalance factors λ_{χ}^{Σ} and λ_{χ}^{Δ} . Then it derives equations that express the converter currents and voltages as a function of the newly defined imbalance factors and converter parameters. These equations are then used to find analytic expressions that showcase the over-current and over-voltage that each phase of the converter must withstand in any point of imbalance, meaning for any set of $\lambda^{\vec{\Sigma}}$ and $\lambda^{\vec{\Delta}}$ values. This is done for both dc and ac compensation modes.

It is quite interesting to point out that both current overrating factors (5.25, 5.58) do not depend on any of the converter physical parameters. They only depend on the characteristics of the imbalance ($\lambda^{\vec{\Sigma}\Delta}$), the operating mode (κ and compensation), and the proportion between the grid voltages (ζ_{dc}). This denotes the generality of the imbalance effects on MMCs, and the generality of the found expressions. Nevertheless, the expressions would for sure incorporate converter parameters if the losses had been taken into consideration. While the imbalance phenomenon, in principle, behaves in the same manner in all MMCs

(regarding currents), the losses do not. On the other hand, the voltage overrating does depend on the converter parameters.

Another interesting finding arises by contrasting the found current overrating expressions (5.25, 5.58) and the feasible modes of operation in Table 3.1. Under the assumptions of the predefined operating modes, the λ_{χ}^{Σ} terms and κ have always the same sign under dc compensation, and opposed signs under ac compensation. This means that at least any balanced participation of the DES on the power transfer decreases the cluster currents. The expressions show that horizontal imbalances can increase or decrease the converter currents, but any vertical imbalance will always produce an increase on the converter currents. Closer inspection reveals that, while pure horizontal imbalances can produce current increases, those increases cannot exceed converter nominal rating. Therefore, the overrating expressions demonstrate that the converter nominal current rating can only be exceeded in presence of vertical imbalances.

6. ASSESSMENT OF MMC-DES IMBALANCES

The imbalance and overrating factors are convenient tools to determine the currents and voltages that a MMC-DES needs to withstand in a particular state of imbalance. On top of that, they serve as the grounds for a more profound analysis. An MMC-DES is most likely expected to operate within a range of imbalances, rather than in a fixed imbalance. It is relevant to know in detail the maximum overrating that a converter may need to withstand over a range of different imbalances, the imbalances that a converter can reach for a given overrating, and what is the progressive tradeoff between overrating requirements and different kinds of imbalances. This chapter utilizes the found overrating expressions to provide answers to these matters.

6.1. DES representation

In practical terms for MMC rating, a DES can be represented by the imbalances it may impose on the joint system. Different DES may impose different imbalances, as explained in Chapter 2. A type of DES can be prone to experience higher imbalances than others, for example a hybrid DES is expected to have higher imbalances than a SST DES. Nonetheless, a particular type of DES can also have various degrees of imbalances depending on other factors. For example a BESS may experience very different imbalances depending on whether its batteries are brand-new, old, or second-life. A PV DES can have different expected imbalances depending on the weather characteristics of its geographic location.

The term *coherent* DES will be hereafter used to refer to DESs in which all the clusters have either non-negative power or non-positive power, and never clusters that provide and consume power simultaneously.

The imbalance ranges are represented by a Λ space. Conceptually, this space is the region formed by all the $\lambda^{\vec{\Sigma}\Delta}$ values that a converter can experience during its operation. It is defined by boundaries for the values of each λ term, and constrains that encompass

the relations between $\vec{\lambda}^\Sigma$ and $\vec{\lambda}^\Delta$. Both kinds of restrictions are important. A MMC-PV can expect to never produce more than the nominal PV capacity, and never less than zero, meaning $0 \leq \lambda_x^\Sigma \leq 1$. It also may expect in an extreme case to have one phase cluster fully operational and the other fully shaded, meaning $-0.5 \leq \lambda_x^\Delta \leq 0.5$. These are the boundaries for $\vec{\lambda}^{\Sigma\Delta}$. But considering only them, allows the imbalance state $\lambda_x^\Sigma = 1, \lambda_x^\Delta = 0.5$, which would mean that one converter cluster can reach 1.5 times its nominal power and does not reflect reality. Thus a constrain on $\vec{\lambda}^{\Sigma\Delta}$ is needed to establish that $0 \leq \lambda_x^\Sigma + \lambda_x^\Delta \leq 1$ and $0 \leq \lambda_x^\Sigma - \lambda_x^\Delta \leq 1$. This logical procedure can be used to determine the restrictions that define the Λ space of any DES. More detailed information about a particular DES implementation can be used to establish further restrictions, such as non-linear constraints over the vertical and horizontal imbalances.

Table 6.1 defines set of general Λ spaces that can represent a variety of DES types and implementations. Λ_1 is a space where the DES provides power, it can supply the nominal power of one port, and multiple clusters can be shut down while the others remain nominal. Λ_2 is the same but with a DES that consumes power. The space Λ_1 can represent a PV DES with high variance, as in the previous example. It also can represent the discharging mode of a BESS with high cell variance, while Λ_2 represents the charging state or a FC DES. In general terms, Λ_1 and Λ_2 represent two fully rated coherent DES. Λ_3 represents a partially-rated DES (20% P^{nom} rating) that supplies power with very low expected imbalances. Λ_4 represents a DES in which some clusters can provide energy while others consume energy, i.e., incoherent. This last space may not be necessary nor desirable in a properly managed non-hybrid DES, but can represent some hybrid DES.

6.2. Global overrating factors

Once having defined the Λ space at issue, the overrating factors can be fed into an optimization problem to find the global converter overrating needed. The optimization problem must find the maximum overrating that will be needed in any cluster across all the Λ space. After approaching the problems with different optimization algorithms, the

Table 6.1. Sample Λ spaces.

Space	Boundaries	Constrains	Application Example
Λ_1	$0 \leq \lambda_x^\Sigma \leq 1$ $-0.5 \leq \lambda_x^\Delta \leq 0.5$	$0 \leq \lambda_x^\Sigma + \lambda_x^\Delta \leq 1$ $0 \leq \lambda_x^\Sigma - \lambda_x^\Delta \leq 1$	PV, FC or BESS under unbalanced discharging
Λ_2	$-1 \leq \lambda_x^\Sigma \leq 0$ $-0.5 \leq \lambda_x^\Delta \leq 0.5$	$-1 \leq \lambda_x^\Sigma + \lambda_x^\Delta \leq 0$ $-1 \leq \lambda_x^\Sigma - \lambda_x^\Delta \leq 0$	BESS under unbalanced charging
Λ_3	$0 \leq \lambda_x^\Sigma \leq 0.2$ $-0.05 \leq \lambda_x^\Delta \leq 0.05$	$0.1 \leq \lambda_x^\Sigma + \lambda_x^\Delta \leq 0.2$ $0.1 \leq \lambda_x^\Sigma - \lambda_x^\Delta \leq 0.2$	Partially-Rated BESS discharging
Λ_4	$-1 \leq \lambda_x^\Sigma \leq 1$ $-1 \leq \lambda_x^\Delta \leq 1$	$-1 \leq \lambda_x^\Sigma + \lambda_x^\Delta \leq 1$ $-1 \leq \lambda_x^\Sigma - \lambda_x^\Delta \leq 1$	Hybrid FC+BESS or Hybrid PV+EV

Table 6.2. Operating modes feasibility in Λ spaces.

Space	Feasible modes
Λ_1	I, IV
Λ_2	II, III
Λ_3	I, IV
Λ_4	I, II, III, IV

Sequential Least Squares Programming (SLSQP) algorithm proved to be capable of finding the optimal solutions. Appendix C provides a vectorized implementation of the model and optimization problem in Python.

Not all converter modes can operate in all Λ s, two of them admit supplying DESs and the other two admit consuming DESs. Refer to Table 3.2 to see the operating modes. The optimization is applied on the feasible operating modes of each Λ , which are shown in Table 6.2. The space Λ_4 can be applied on all operating modes by intersecting it with the mode restrictions, the shown optimization results are from the mode that yielded the highest overrating.

6.2.1. Current global overrating under dc compensation

The optimization problem (6.1) yields the global converter current overrating that is needed to operate across a Λ space under dc compensation. To implement the problem on a solver, the condition $\lambda \in \Lambda$ can be replaced by the respective constraints that define Λ .

$$\begin{aligned} \zeta_i^{dc} = & \max_{\chi, \lambda} \left\{ \frac{|\kappa - \lambda_\chi^\Sigma| + 2\zeta_{dc}\sqrt{\Gamma_\chi^2 + 2|\lambda_\chi^\Delta| + 1}}{1 + 2\zeta_{dc}} \right\} \\ \text{s.t. } & \chi \in \{a, b, c\} \\ & \lambda \in \Lambda \end{aligned} \quad (6.1)$$

Table 6.3 presents the solutions of the optimization problem under the predefined Λ spaces. The problem leads to six $\lambda^{\Sigma\Delta}$ solutions depending on the initial point given to the algorithm. However, all solutions yield the same optimal value. They also have a shared particularity. All the solutions lay in $\lambda^{\Sigma\Delta}$ points with the maximum vertical imbalance in one direction on two phases and the maximum vertical imbalance in the opposite direction on the remaining phase. Therefore the six solutions are all the possible combinations of having three clusters at nominal power, three clusters at minimal power, and at least one upper and one lower belonging to each group. All solutions have an equivalent power distribution but allocated in different clusters. The Σ power adopts the same value in all phases, because the constraints imply that the λ_χ^Σ coordinates must be placed at their mid-points to allow the maximum λ_χ^Δ values. The Λ_4 space yields the same result whether in inverter or rectifier operation.

The results are quite significant. The optimization over Λ_1 and Λ_2 shows that an MMC-DES with a fully rated coherent DES achieves its most demanding imbalance when the three phases have a cluster on nominal operation and another at minimal power, in a distribution with at least one upper cluster and one lower cluster at nominal power. Moreover, the results show that the maximum imbalances require approximately a 30% current overrating. The optimization over Λ_3 verifies something that could be foreseen in the

Table 6.3. Current global overrating under dc compensation.

Space	Solution (worst op. point)	Orrating (ζ_i^{dc})
Λ_1	$\vec{\lambda}^\Sigma = [0.5, 0.5, 0.5]^T$ $\vec{\lambda}^\Delta = [0.5, -0.5, -0.5]^T$	1.296
Λ_2	$\vec{\lambda}^\Sigma = [-0.5, -0.5, -0.5]^T$ $\vec{\lambda}^\Delta = [0.5, -0.5, -0.5]^T$	1.296
Λ_3	$\vec{\lambda}^\Sigma = [0.15, 0.15, 0.15]^T$ $\vec{\lambda}^\Delta = [0.05, -0.05, -0.05]^T$	0.995
Λ_4	$\vec{\lambda}^\Sigma = [0, 0, 0]^T$ $\vec{\lambda}^\Delta = [1, -1, -1]^T$	1.942

overrating equations, which is that if the vertical imbalances can be kept under certain values, a MMC-DES can require no current overrating. The optimization over Λ_4 shows that the most demanding theoretical imbalance is when three clusters are providing their full power to the other three clusters while nominal power flows between the main MMC ports, which requires a 94% current overrating.

6.2.2. Current global overrating under ac compensation

The optimization problem (6.2) yields the global converter current overrating for the ac compensation operating mode.

$$\zeta_i^{ac} = \max_{\chi, \lambda} \left\{ \frac{|\kappa - \lambda_{\delta\chi}^\Sigma| + 2\zeta_{dc} \sqrt{\Gamma_\chi^2 + (\kappa + \lambda_{avg}^\Sigma)^2 + 2|(\kappa + \lambda_{avg}^\Sigma)\lambda_\chi^\Delta|}}{1 + 2\zeta_{dc}} \right\} \quad (6.2)$$

s.t. $\chi \in \{a, b, c\}$

$\lambda \in \Lambda$

The results are presented in Table 6.4. The solutions for Λ_1 , Λ_2 and Λ_3 are now different. Now the highest global overrating occurs when one phase is contributing half its nominal power with the maximum vertical imbalance, while the other phases contribute

Table 6.4. Current global overrating under ac compensation.

Space	Solution (worst op. point)	OVERRATING (ζ_i^{ac})
Λ_1	$\vec{\lambda}^\Sigma = [0, 0.5, 0]^T$ $\vec{\lambda}^\Delta = [0, -0.5, 0]^T$	1.333
Λ_2	$\vec{\lambda}^\Sigma = [0, -0.5, 0]^T$ $\vec{\lambda}^\Delta = [0, -0.5, 0]^T$	1.333
Λ_3	$\vec{\lambda}^\Sigma = [0.1, 0.1, 0.15]^T$ $\vec{\lambda}^\Delta = [0, 0, 0.05]^T$	0.961
Λ_4	$\vec{\lambda}^\Sigma = [0, 0, 0]^T$ $\vec{\lambda}^\Delta = [1, -1, -1]^T$	1.942

the minimum power. The ac compensation mode achieves its maximum over-currents under both vertical and horizontal imbalances, whereas the dc compensation mode does so under only vertical imbalances. Λ_1 and Λ_2 require now a higher over-current, near 33% instead of 30%, but Λ_3 requires 3% less. The problem yields the same solution for the Λ_4 space, requiring near 94% over-current.

It is interesting to note why the solutions are different in dc and ac compensation. DES Σ power contributions decrease the converter current, whereas vertical imbalances increase it. The optimization naturally tries to add vertical imbalances and reduce Σ power. However to add vertical imbalances, the Σ power must be higher than zero. So the maximization of the overrating must choose between adding a vertical imbalance and reducing the DES power. In dc compensation the tradeoff always was inclined to add more vertical imbalance, but that is not the case in ac compensation.

6.2.3. Voltage global overrating under dc compensation

The optimization problem (6.3) yields the global converter voltage overrating that is needed to operate across a Λ space under dc compensation. As mentioned previously, the condition $\lambda \in \Lambda$ can be replaced by the respective constraints that define Λ to implement

Table 6.5. Voltage global overrating under dc compensation.

Space	Solution (worst op. point)	Orrating (ζ_v^{dc})
Λ_1	$\vec{\lambda}^\Sigma = [0.5, 0.5, 0.5]^T$ $\vec{\lambda}^\Delta = [0.5, -0.5, -0.5]^T$	1.039
Λ_2	$\vec{\lambda}^\Sigma = [-0.5, -0.5, -0.5]^T$ $\vec{\lambda}^\Delta = [0.5, -0.5, -0.5]^T$	1.042
Λ_3	$\vec{\lambda}^\Sigma = [0.15, 0.15, 0.15]^T$ $\vec{\lambda}^\Delta = [0.05, -0.05, -0.05]^T$	1.003
Λ_4	$\vec{\lambda}^\Sigma = [0, 0, 0]^T$ $\vec{\lambda}^\Delta = [1, -1, -1]^T$	1.092

the problem on a solver.

$$\zeta_v^{dc} = \max_{\chi, \lambda} \left\{ \frac{\frac{P^{nom}}{3V_g} \left(Z_s |\Gamma_\chi| + \frac{R_s}{2\zeta_{dc}} |\kappa - \lambda_\chi^\Sigma| \right) + V_1^\Delta}{\frac{R_s P^{nom}}{6\zeta_{dc} V_g} + V_1^\Delta} \right\} \quad (6.3)$$

s.t. $\chi \in \{a, b, c\}$
 $\lambda \in \Lambda$

Table 6.5 presents the solutions of the optimization problem under the predefined Λ spaces. The problem leads to the same solutions as in the current overrating case. They exhibit the same behavior of being in $\vec{\lambda}^{\Sigma\Delta}$ points with the maximum vertical imbalance in one direction on two phases and the maximum vertical imbalance in the opposite direction on the remaining phase. The required overrating is relatively small in all cases, below 5% except for Λ_4 . These results differ with the currents, as the first indicate that an imbalance always requires a voltage overrating, whereas some imbalances do not require a current overrating. Nevertheless, the required overvoltages are always small and should not pose a big complication.

Table 6.6. Voltage global overrating under ac compensation.

Space	Solution (worst op. point)	Orrating (ζ_v^{ac})
Λ_1	$\vec{\lambda}^\Sigma = [0.5, 0.5, 0.5]^T$ $\vec{\lambda}^\Delta = [0.5, -0.5, -0.5]^T$	1.056
Λ_2	$\vec{\lambda}^\Sigma = [-0.5, -0.5, -0.5]^T$ $\vec{\lambda}^\Delta = [0.5, -0.5, -0.5]^T$	1.024
Λ_3	$\vec{\lambda}^\Sigma = [0.15, 0.15, 0.15]^T$ $\vec{\lambda}^\Delta = [0.05, -0.05, -0.05]^T$	1.007
Λ_4	$\vec{\lambda}^\Sigma = [0, 0, 0]^T$ $\vec{\lambda}^\Delta = [1, -1, -1]^T$	1.092

6.2.4. Voltage global overrating under ac compensation

The optimization problem (6.4) yields the global converter current overrating for the ac compensation operating mode.

$$\zeta_v^{dc} = \max_{\chi, \lambda} \left\{ \frac{\frac{P^{nom}}{3V_g}(Z_s|\Gamma_\chi| + \frac{R_s}{2\zeta_{dc}}|\kappa - \lambda_{\delta\chi}^\Sigma|) + V_1^\Delta(\vec{\lambda}^\Sigma)}{\frac{R_s P^{nom}}{6\zeta_{dc}V_g} + V_1^\Delta(\vec{0})} \right\} \quad (6.4)$$

s.t. $\chi \in \{a, b, c\}$

$\lambda \in \Lambda$

The results are presented in Table 6.6. The problem solutions add the most vertical imbalance in all cases, like in the dc compensation case. The results again show that performing power imbalances in a MMC-DES requires quite low voltage overrating in all cases.

6.3. Imbalance Regions

The analysis of the previous section revolved around the question: How much overrating needs the converter to operate under a determined imbalance region? It is also relevant to analyze the problem from the opposing perspective: What imbalances can a converter reach while operating below a determined overrating? The answer to this question leads to the determination of feasible imbalance regions, instead of a single value. The study of such regions regarding voltage overrating is not very relevant, as the previous section showed that voltage overrating is not a significant concern. Therefore, this section will focus on current overrating.

The imbalance regions can be conveniently visualized and studied through contour graphs, but there is a major issue. The problem has a high dimensionality, 6 λ coordinates to be precise. This complicates the visualization of the problem. Nevertheless, the analysis can take advantage of symmetries in the overrating factors. The symmetries allows the generation of low dimensionality plots with valuable insights about the shape and extent of the feasible imbalance regions. The previous section showed that the global overrating highly related with the presence of two vertical imbalances in one direction and one in the opposing direction, without being affected by which phases had the imbalances. The overrating factor equations show that the same happens with horizontal imbalances. Thus, the graphs do not need to address all the possible imbalance combinations to show information representative of all imbalances.

There are also four main operating modes of the MMC-DES (3.2), but it is not necessary to address all of them to analyze the imbalance regions. In terms of the problem modeling, the only difference between modes I and II is that they have opposing κ and DES power signs. In mode I, κ is 1 and the DES spans non-negative values. Whereas in mode II, κ is -1 and the DES spans non-positive values. Looking, at the overrating expression (5.25), the overrating is not affected if both signs change. Therefore the imbalance regions are the same in modes I and II, but with opposing axis signs. The same occurs

with modes II and III in the overrating expression (5.58). Considering this, the graphs will only encompass modes I and IV, for dc and ac compensation respectively.

6.3.1. Horizontal imbalances

To visualize the effects of only the horizontal imbalances, the Δ power is set to zero in all phases. Each plot shows the effects of Σ variations along the phases a and b , while the c phase is held constant.

The equation (5.25) indicates that regarding horizontal imbalances in operating mode I, the maximum current will be simply determined by the phase with the least DES power contribution. This occurs because Σ power only appears in the modulus $|\kappa - \lambda_\chi^\Sigma|$, and the modulus increases when λ_χ^Σ decreases (remembering that κ and λ_χ^Σ have the same signs in the dc compensated modes). Fig 6.1a shows that the global overrating does not change with most Σ power variation in phases a and b , because phase c has the maximum current, independently of them. Similarly, the global overrating in Fig 6.1b is determined solely by the phase with the least Σ power.

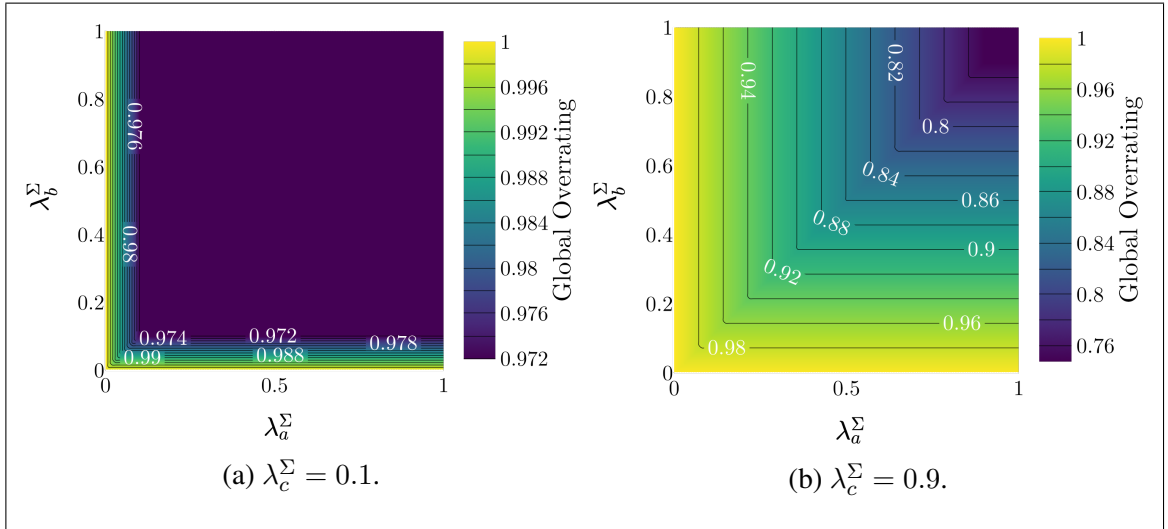


Figure 6.1. Effect of horizontal imbalances on current overrating under dc compensation.

The situation changes with ac compensation in mode IV as (5.58) contains the λ_{avg}^Σ term that produces an effect from the Σ power of one phase to the overrating of the others. When one phase contributes low power, the global overrating is now noticeably influenced by the others, and it can be brought to a much lower value than in the dc compensation case (Fig. 6.2a). The ac compensated modes also achieve lower global overrating values in general, as it is more related to the total Σ contribution, and not so much on the lowest contributing phase (Fig. 6.2b). The illustrations indicate that the converter current rating cannot be exceeded through only horizontal imbalances, whether in dc or ac compensation mode.

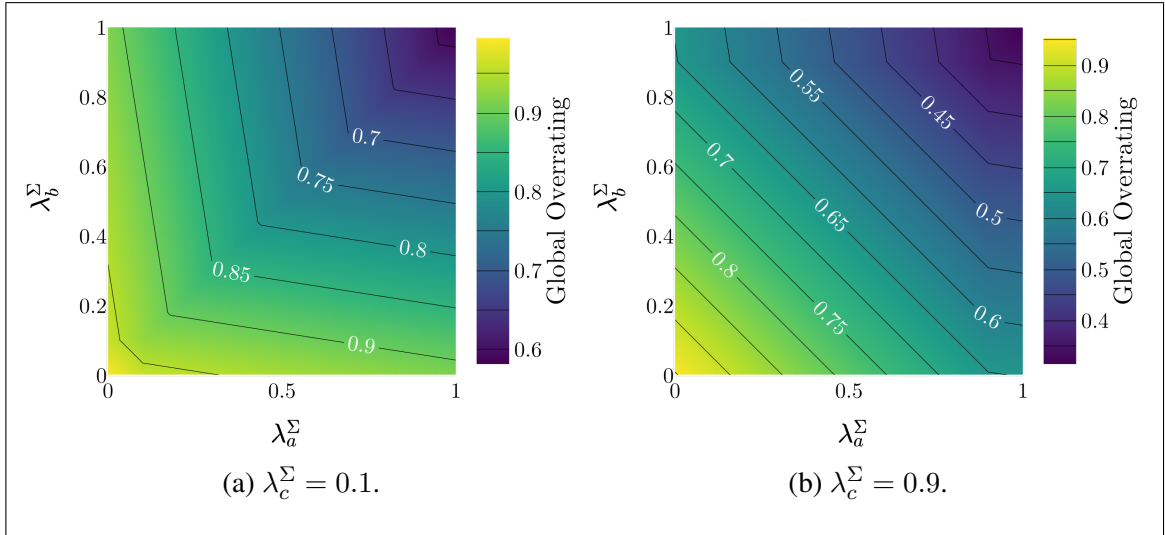


Figure 6.2. Effect of horizontal imbalances on current overrating under ac compensation.

6.3.2. Vertical imbalances

The vertical imbalances in general demand much higher overratings than horizontal imbalances under dc compensation (Fig. 6.3). The global overrating responds mainly to the phase with the highest imbalance magnitude (Fig. 6.3a), and when one phase has a high vertical imbalance, the others do not have much impact and can only worsen the global

current overrating (Fig. 6.3b). An important observation is that the feasible imbalance region with nominal rating is much narrower than under horizontal imbalances (Fig. 6.3a).

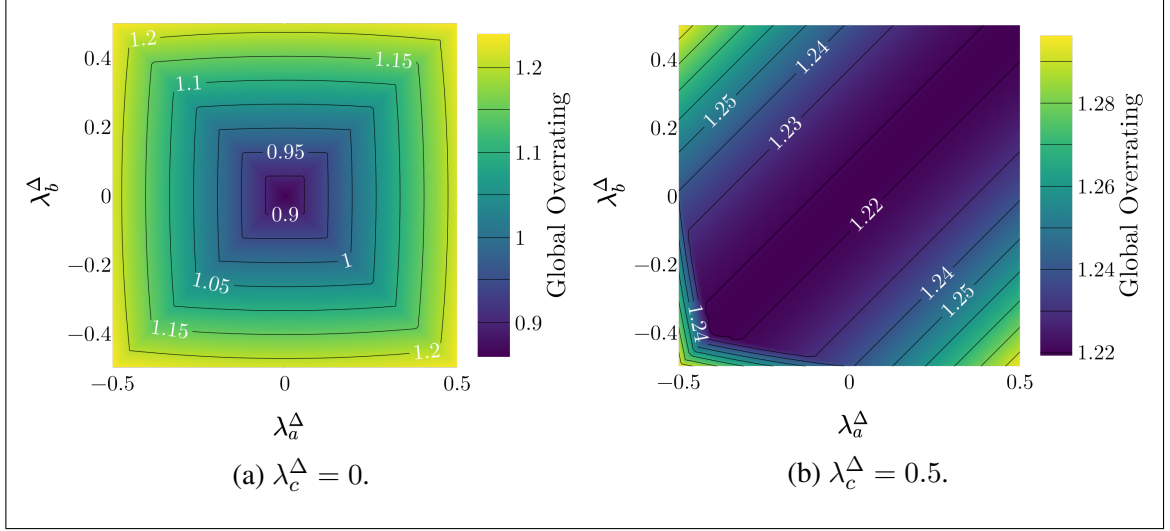


Figure 6.3. Effect of vertical imbalances on current overrating under dc compensation.

Vertical imbalances in ac compensation mode have the same behavior, but demand much lower global overratings (Fig. 6.4). They can also exceed the nominal current rating, but under much heavier imbalances. Figs. 6.3b, 6.4b corroborate an observation made in section 6.2, which is that the highest overratings tend to occur when two phases have imbalances in one direction, and the remaining phase in the opposite direction.

6.3.3. Cluster imbalances

The use of the $\Sigma\Delta$ transform enables a convenient mathematical modeling of the power imbalance problem. The decoupling of the imbalances into two components (horizontal and vertical) also allows to visualize the effects of imbalances in high detail. However, the broad picture must not be overlooked by focusing only on the details. Ultimately, what matters is the effect of power variations in clusters, which are a mixture of Σ and Δ

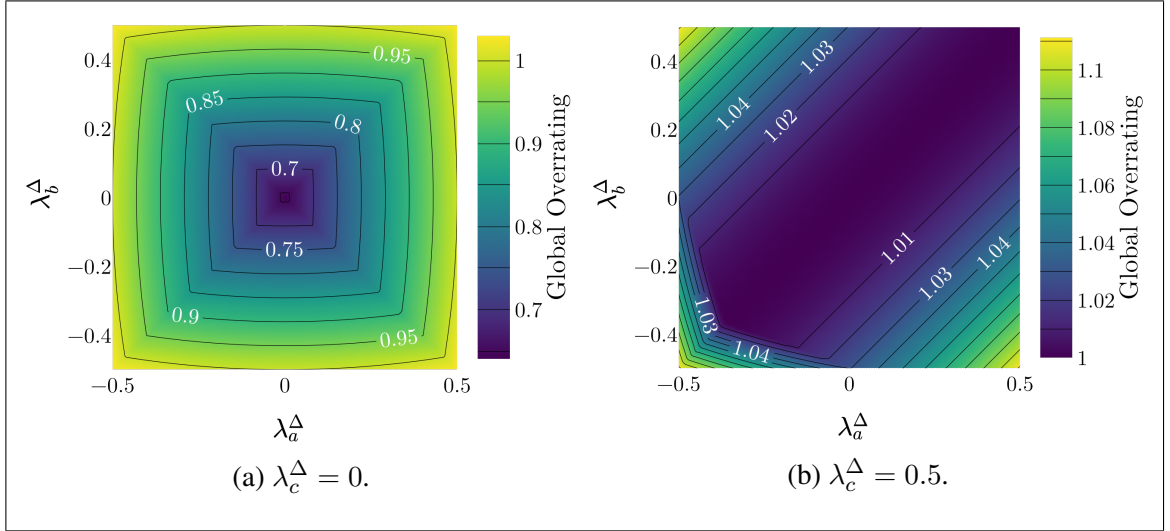


Figure 6.4. Effect of vertical imbalances on current overrating under ac compensation.

power variations. A new imbalance factor $\lambda_{y\chi}$ is defined to represent the power contribution of the clusters.

$$\lambda_{y\chi} = \begin{cases} \lambda_{\chi}^{\Sigma} - \lambda_{\chi}^{\Delta}, & y = u \\ \lambda_{\chi}^{\Sigma} + \lambda_{\chi}^{\Delta}, & y = l \end{cases} \quad (6.5)$$

The general effects of $\lambda_{y\chi}$ variations are a bit more cumbersome to visualize, because there are now six variables to monitor simultaneously instead of 3. But looking at a few cases can provide a good idea of the behavior of the imbalances. In particular, the two evaluated cases observe the global overrating when, a) the DES is turned off except for two clusters, and b) the DES is at nominal power except for two clusters. The two variable clusters belong to different phases, and one is upper and the other is lower. Choosing them in this manner generates results that combine horizontal and opposing vertical imbalances.

A cluster cannot transfer power alone without exceeding the nominal converter current under dc compensation (Fig. 6.5a). A cluster also cannot transfer zero power while the others transfer nominal power, but it can decrease its power up to nearly half the nominal value (Fig. 6.5b). This indicates that increasing the DES power enables higher imbalances, but along a constrained region.

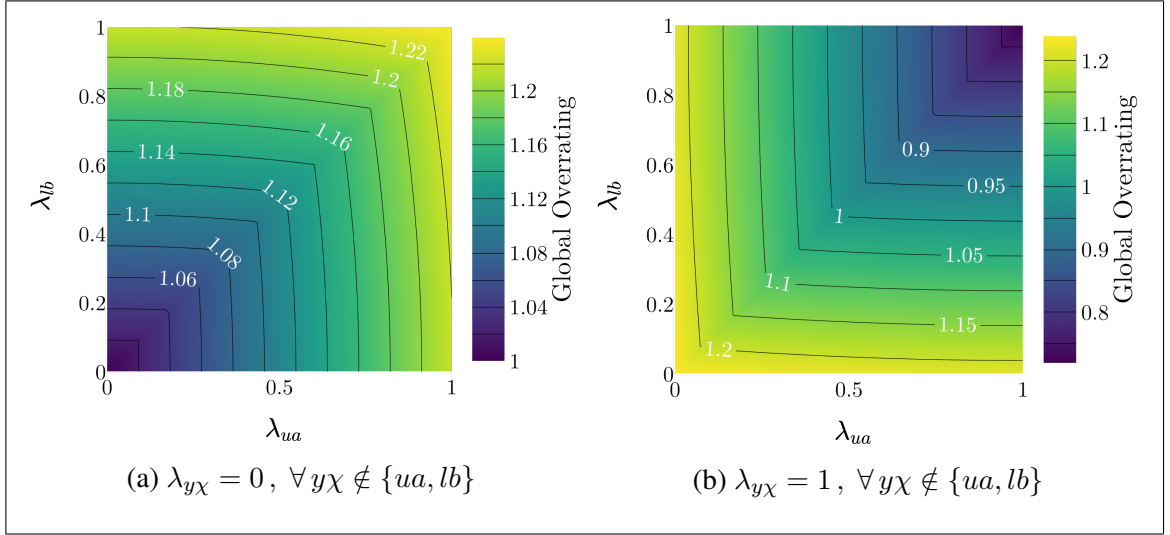


Figure 6.5. Effect of cluster imbalances on current overrating under dc compensation.

A cluster again cannot be turned on alone without exceeding the nominal converter current under ac compensation (Fig. 6.6a), but now it can be turned off while the others contribute nominal power (Fig. 6.6b). The gradients of Fig. 6.6 once more indicate that increasing the DES power enables higher imbalances, but the region of imbalances is much less constrained under ac compensation.

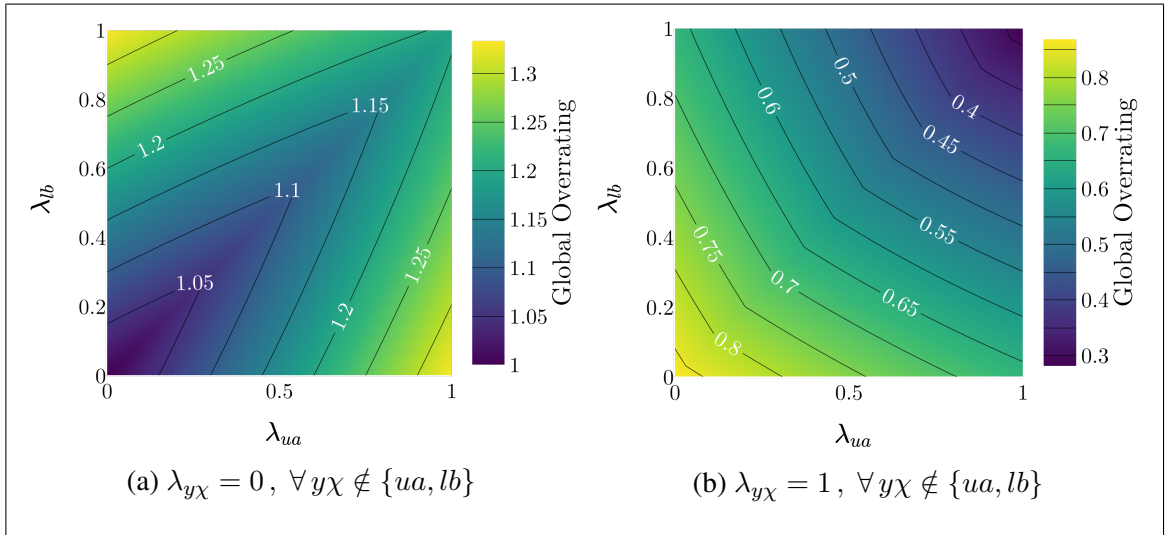


Figure 6.6. Effect of cluster imbalances on current overrating under ac compensation.

The plotted range can be extended to encompass areas where the DES operates incoherently. This is depicted in Fig. 6.7, where the areas left and under the dashed line describe an incoherent DES operation. The non-plotted clusters are set to a power level that ensures that the converter is within the feasible operating modes across the plotted power range.

The converter is not capable of entering the incoherent area in the evaluated scenario under dc compensation, without exceeding the nominal rating (6.7a). Actually, (5.25) indicates that a dc compensated MMC is never capable of breaking coherence without exceeding the nominal rating, under the predefined operating modes. This is because any λ_{χ}^{Σ} value that changes its sign, will cause an overcurrent in that same phase, regardless of the overall DES power operation.

On the other hand, the ac compensated MMC is capable of operating incoherently (6.7b). However, it can only do so in areas very close to the coherent region, and most of the incoherent area seems to be out of reach.

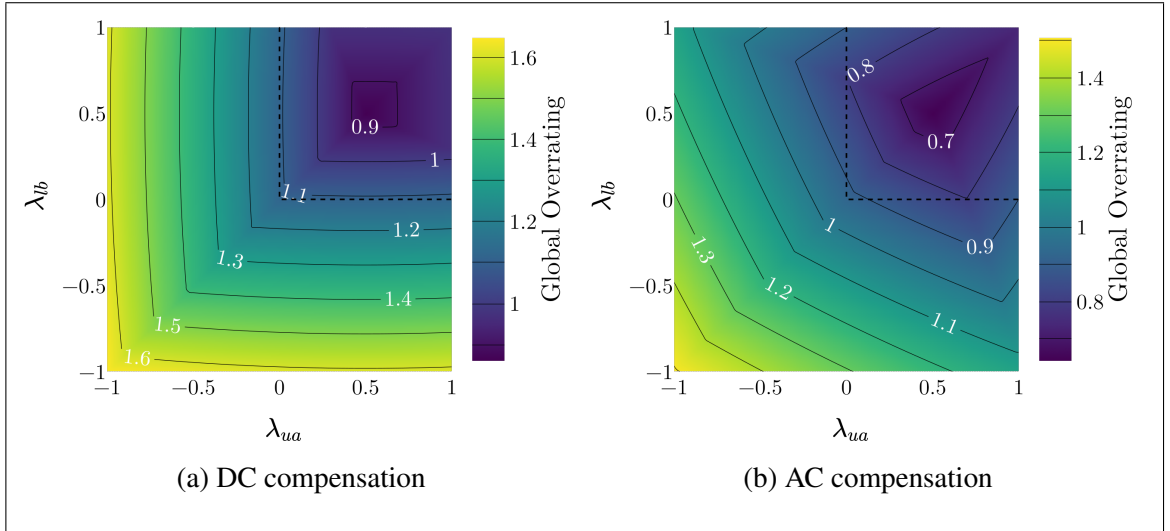


Figure 6.7. Effect of cluster imbalances on current overrating under incoherent operation. $\lambda_{y\chi} = 0.5$, $\forall y\chi \notin \{ua, lb\}$.

6.4. Imbalance metrics

This chapter has thus far analyzed the capacities and behavior of power imbalances within an MMC. This provides a good understanding of the MMC, but it is not clear yet how to compare the MMC with other converters regarding imbalances. There is a need for a generic metric that can be applied to any converter susceptible to imbalances, and that can indicate valuable information about how converters tolerate them.

(Yu et al., 2017) identified this issue and proposed a metric called Power Balance Factor (PBF). However that metric only takes into consideration the over-voltages that imbalances produce in converters, even for converters that inject currents to manage imbalances. That does not result in a fair comparison, as it assumes that a converter can bear any current magnitude. A new metric is hereby proposed, the Power Imbalance Factor (PIF). It follows the same logic of the former, but considering both voltage and current limitations.

The PIF is the division between the volumes of the Power Imbalance Region (PIR) and the Power Imbalance Space (PIS). The PIS is defined as the multidimensional space formed by the $\vec{\lambda}$ factors that describe the imbalance of the converter, and it is bounded by the individual power limitations of the converter subdivisions that are represented by each coordinate of $\vec{\lambda}$. The PIS is the generalization of the previously presented Λ spaces. The broadest case of a mode I or IV coherent MMC-DES is a six-dimensional space enclosed between the boundaries $0 \leq \lambda_{y\chi} \leq 1$, equivalent to Λ_1 . The PIR is the multidimensional region within the PIS that is bounded by restrictions over the converter current and/or voltage. The consideration of a current or voltage restriction depends solely on their relevance. For example, current restrictions are not relevant for a CHB in Star configuration, because its currents are always below nominal under imbalances (Yu et al., 2017). Section 6.2 showed that over-voltages in MMC imbalances should be noted, but are not much of an issue. Therefore, the PIF calculation for the MMC hereby only considers current limitations.

6.4.1. Power Imbalance Region

The PIR of the MMC cannot be fully visualized, as it is a 6-dimensional region. The PIR of a converter can be visualized if its imbalances are represented in a 3-dimensional vector or lower, such as the half-bridge MMC (2 dimensions) or the CHB (3 dimensions). The best approximation for the MMC is to observe a 3-dimensional slice of the region. Slicing the region and lowering three dimensions conveys a great loss of information, but it can still provide a valuable sense of the feasibility of different imbalances in the MMC.

The slices were done by maintaining fixed the imbalances λ_{la} , λ_{ub} , λ_{uc} . The resulting shapes illustrate the cube-like nature of dc compensation imbalance limitations (Fig. 6.8a), in contrast to the ones in ac compensation which are more prism-like (Fig. 6.8b). The actual shape of the ac compensation PIR is hard to describe and recognize based on Fig. 6.8b, because much of it is bordered by the PIS instead of the overrating restriction. Fig. 6.9 displays the PIR with a lower DES power, enabling to better observe the shape that the overrating restriction produces in the ac compensation PIR. The shape resembles a triangular base prism but with a cross-section that narrows when closer to the bases, ending in a point instead of a triangular base. The intersection of the overrating restriction with the PIS results in a pyramid-like shape.

Ultimately, the PIRs from Fig. 6.8 indicate that a group of clusters in ac compensation can comfortably deviate from the others if there is at least a bit of DES power ($> \sim 0.2P^{nom}$). But the same system in dc compensation tolerates much less variability between its clusters power. Or seen from another perspective, in most cases it is much more costly to achieve the same imbalances under dc compensation than in ac compensation. Plotting more PIRs shows that the ac compensation region quickly grows with DES power, to encompass most of the PIS. Meanwhile, the dc compensation region tends to grow much slowly in relation to DES power.

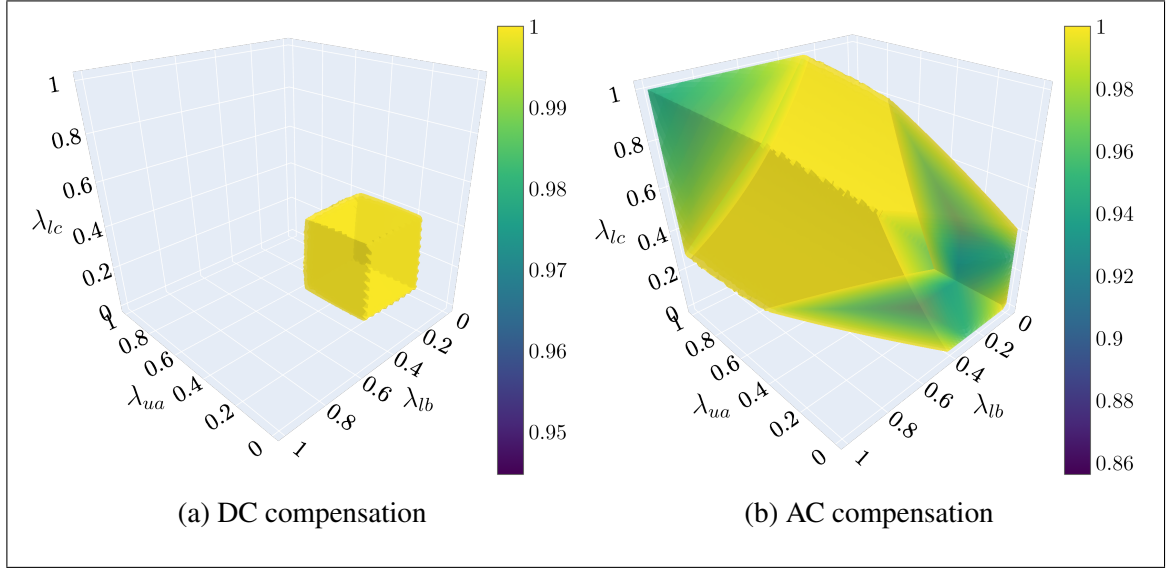


Figure 6.8. PIR 3D slices. $\lambda_{y\chi} = 0.3$, $\forall y\chi \notin \{ua, lb, lc\}$.

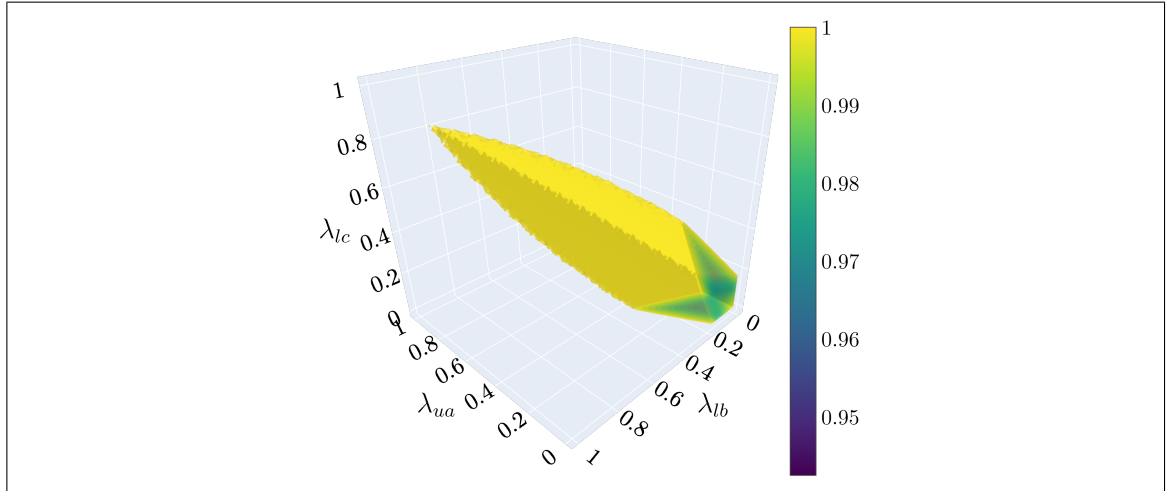


Figure 6.9. PIR shape of ac compensation. $\lambda_{y\chi} = 0.08$, $\forall y\chi \notin \{ua, lb, lc\}$.

6.5. Power Imbalance Factor

The PIF is an indicator of the capacity of power converters to achieve internal imbalances. In the context of this thesis, it is applied for inter-cluster imbalances. However it can also be applied for other imbalances, such as the intra-cluster imbalances of an MMC, by modeling of the imbalance in a similar manner.

Table 6.7. MMC PIFs. * for dc compensated modes, ** for ac compensated modes

Mode	PIF
I*	0.1674
II*	0.1674
II**	0.8837
IV**	0.8837

The PIF of an MMC can be calculated as

$$\text{PIF} = \frac{\int \cdots \int_{\Lambda} Z(\vec{\lambda}) d\Lambda}{\int \cdots \int_{\Lambda} 1 d\Lambda} \quad (6.6)$$

where,

$$Z(\vec{\lambda}) = \begin{cases} 1, & \max_{\chi} \left\{ \zeta_{i\chi}(\vec{\lambda}) \right\} \leq \zeta^{max} \\ 0, & \max_{\chi} \left\{ \zeta_{i\chi}(\vec{\lambda}) \right\} > \zeta^{max} \end{cases} \quad (6.7)$$

Λ is the PIS (Λ_1 or Λ_2 for default), and ζ^{max} is the design overrating boundary (1 for default).

The equation (6.6) does not have an analytic solution. Furthermore, the high variable count and the discontinuities of the Z function make the expression extremely resource demanding for software integration algorithms, and a solution was not reached through those tools. The integral was approximated using Monte Carlo integration. However, this approximation technique posed another challenge. Having a matrix of sufficient points for an accurate estimation is highly memory demanding, again due to the dimensionality of the problem. Performing the integration algorithm iteratively is extremely slow for the same reason. This was solved by complementing the Monte Carlo integration with the stratified sampling technique (James, 1980), iterating in a manner that leverages the available computer memory without collapsing it. Appendix C provides a vectorized implementation of the overrating model and PIF estimation in Python.

The resulting PIF values (Table 6.7) are consistent with the visualizations of the PIRs and the previous contour graphs. They verify that the ac compensation mode is much more capable of achieving imbalances than the dc compensation mode, reaching 88.37% and 16.74% of the imbalance points respectively. The difference is quite significant. This result indicates that the operation mode should be a major factor to consider in the design of an MMC-DES. The PIFs are also congruent with the expected symmetry between modes, yielding the same results for modes I and II, and for modes III and IV.

The PIF can also be calculated for an incoherent DES. The default PIS to do so is Λ_4 , but adding the restriction $\sum_x \lambda_x^\Sigma \geq 0$ or $\sum_x \lambda_x^\Sigma \leq 0$. This restriction ensures that the evaluated PIS remains within one of the defined operating modes. The obtained values are 0.52% and 8.91% for dc and ac compensation respectively. Such a low value was expected for dc compensation, for the reasons reported at the end of Subsection 6.3.3. However the results confirm that ac compensation is not suited for incoherent operation, although it can perform it in small amounts. Subtracting the coherent sub-region of Λ_4 shows that ac compensation can only operate in 6.34% of the purely incoherent region of Λ_4 , and corroborates that dc compensation cannot operate incoherently.

6.6. Conclusion

This chapter analyses the effect of power imbalances in an MMC under different scopes. To do this, first it presents a way to represent a DES through its power and imbalance capacities, which results in the definition of an imbalance space.

Analyzing the imbalances through an optimization algorithm shows the most straining imbalances in ac compensation mode require more current overrating (33%) than in dc compensation mode (30%), and that voltage overrating is not much of an issue. Also, the most straining imbalances lay on different $\lambda^{\Sigma\Delta}$ points depending on the type of compensation. The worst case for dc compensation is having the maximum amount of vertical imbalance on all phases with the DES at half nominal power, but with at least one phase

on each vertical imbalance direction. The worst case for ac compensation is having the maximum amount of vertical imbalance on one phase, and having the others with minimal DES power.

The results of the optimization problem lead to think that imbalances under ac compensation are worse because they require more overrating in the worst case. However, plotting the imbalances reveals that ac compensation can reach much more imbalances than dc compensation, and it generally requires less overrating. They also respond differently to imbalances, dc compensation demands overrating mainly in relation to the most unbalanced cluster, whereas ac compensation is also related to said cluster but it is mostly affected by the net power of the DES.

The chapter presents a new metric called PIF to quantify the compatibility of a converter with imbalances. The PIF translates the yet complex results into an easily understandable figure, and facilitates comparisons between different converters and modes of operation. It reveals that the differences between ac and dc compensation are much larger than what could be presumed with previous analysis, yielding values of 88.37% and 16.75% respectively.

Ultimately, the analysis indicates that using the power of a DES to compensate the power requirements of the dc MMC port (or vice versa), is generally a more current demanding task than using the DES power to compensate power requirements of the ac MMC port. And that an MMC can be comfortably imbalanced in most scenarios if controlled under ac compensation, even though the optimization indicated otherwise. This means that it is generally far more convenient to incorporate a DES in applications that require or allow flexibility in the ac port of the MMC instead of the dc port.

7. THEORY VALIDATION

The equations presented in Chapter 5 are theoretical and lack the loss components of the converter operation. Therefore, it is essential to verify that they indeed represent the phenomena that occur in an MMC with inter-cluster imbalances, and that the absence of the losses in the analysis does not compromise the accuracy of the expressions in a significant degree.

It is also important to validate the designed controllers. The implemented control scheme is of CPC-SEC type, but with manual DES power references instead of an energy control loop (Section 2.3). As explained in Chapter 4, the scheme does not include any secondary control objectives, like SM voltage ripple reduction. The dc compensation control is fairly standard in the literature. However, the efficacy of the proposed ac compensation CPC-SEC scheme is yet to be uncovered. Nevertheless, this thesis is not focused on MMC-DES control, so there will be no performance analysis of the converter control.

The overrating model and control are partially validated through laboratory experiments. The available experimental setup can operate in only two of the four identified modes because the experimental prototype only has unidirectional power sources operating as DES. The remaining two modes are validated through simulations with the same parameters as the experiments. Doing so also verifies that the simulations are representative of real hardware, as the simulated results of the experimental converter should be similar to the laboratory results. Afterwards, simulations of an industrial MMC are conducted to evaluate the accuracy of the overrating model in industrial conditions. All experiments and simulations use the same imbalance sequence to standardize the tests and have unbiased comparisons between modes. Table 7.1 presents the test sequence. It draws the converter to: nominal operation without DES contribution, DES contributing half nominal power without imbalances, with horizontal imbalances, with vertical imbalances, with both imbalances.

Table 7.1. Test imbalance sequence.

Section	$\lambda^{\vec{\Sigma}}$	$\lambda^{\vec{\Delta}}$
1	$[0, 0, 0]^T$	$[0, 0, 0]^T$
2	$[0.5, 0.5, 0.5]^T$	$[0, 0, 0]^T$
3	$[0.3, 0.5, 0.7]^T$	$[0, 0, 0]^T$
4	$[0.5, 0.5, 0.5]^T$	$[0.1, 0.3, -0.2]^T$
5	$[0.3, 0.5, 0.7]^T$	$[0.1, 0.3, -0.2]^T$

Table 7.2. Experiment Parameters.

Description	Parameter	Value
Nominal Power	P^{nom}	600W
DC grid Voltage	V_{dc}	192V
AC grid Voltage	V_g	75V
Cluster Inductance	L_s	5mH
Output Inductance	L_o	1mH
Cluster Resistance	R_s	0.96Ω
Output Resistance	R_o	0.31Ω
SM DC Source Voltage	V_{SM}	48V
SMs per cluster	N	4
SM Capacitance	C_{SM}	$987\mu F$
SM Switching Frequency	f_{sw}	2.5kHz

7.1. Experiments

The experimental MMC-DES hardware is comprised by an MMC with dc power sources attached to the SM capacitors. Circuit-wise, the converter setup resembles a MMC-SST. Nevertheless, taking into consideration the fixed voltage of the dc sources, the DES is more akin to a BESS or FCs. The experimental converter is actually designed to emulate a discharging MMC-BESS or a MMC-FC, as the MMC-BESS is the most widely researched type of MMC-DES. The setup is depicted in Fig. 7.1, and Table 7.2 displays the converter parameters.

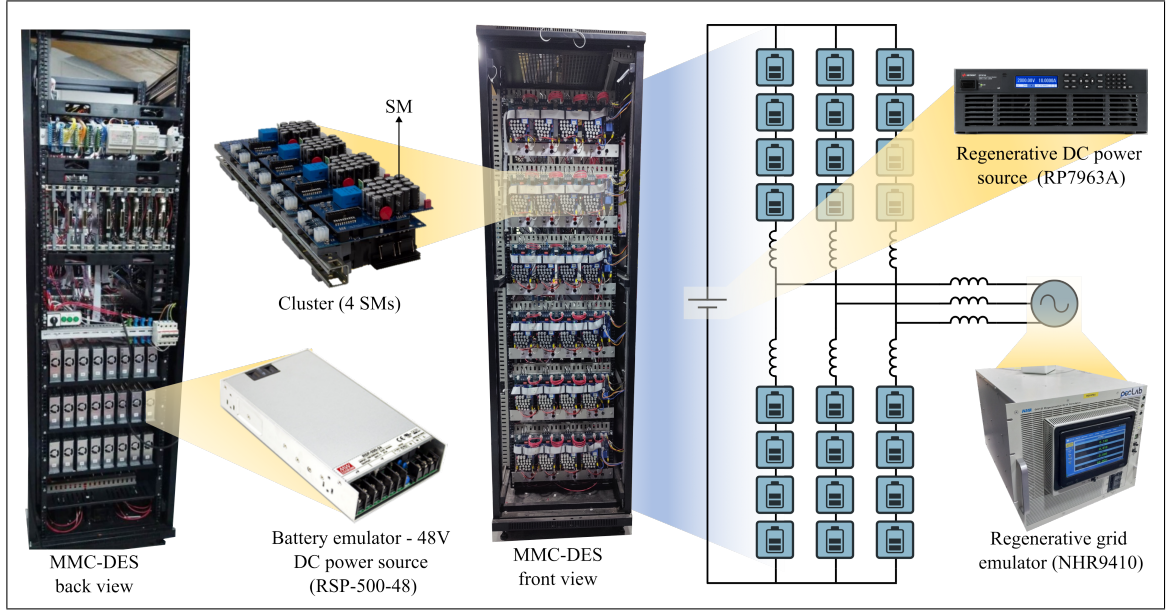


Figure 7.1. Experimental setup.

7.1.1. Mode I

The converter control successfully manages the distribution of inter-cluster power without disturbing the MMC main ports power (Fig. 7.2). This was expected as the dc compensation control used in mode I is relatively common in the MMC-DES literature.

Fig. 7.2 also displays the cluster currents, modulating voltages, and their corresponding maximum global values determined by the overrating model. The maximum global current can be calculated as

$$\max\{|i_{y\chi}|\} = \zeta_i \cdot I_{y\chi}^{nom} \quad (7.1)$$

The experiments verify that the changes in converter voltage are not significant in relation to the nominal voltage. On the other hand, the cluster currents change in a more noticeable degree. The model follows the changes in magnitude correctly but not with perfect precision. The imprecisions can be tracked down to the first section of the test, where the model determines an inaccurate nominal current value, allegedly due to the lack

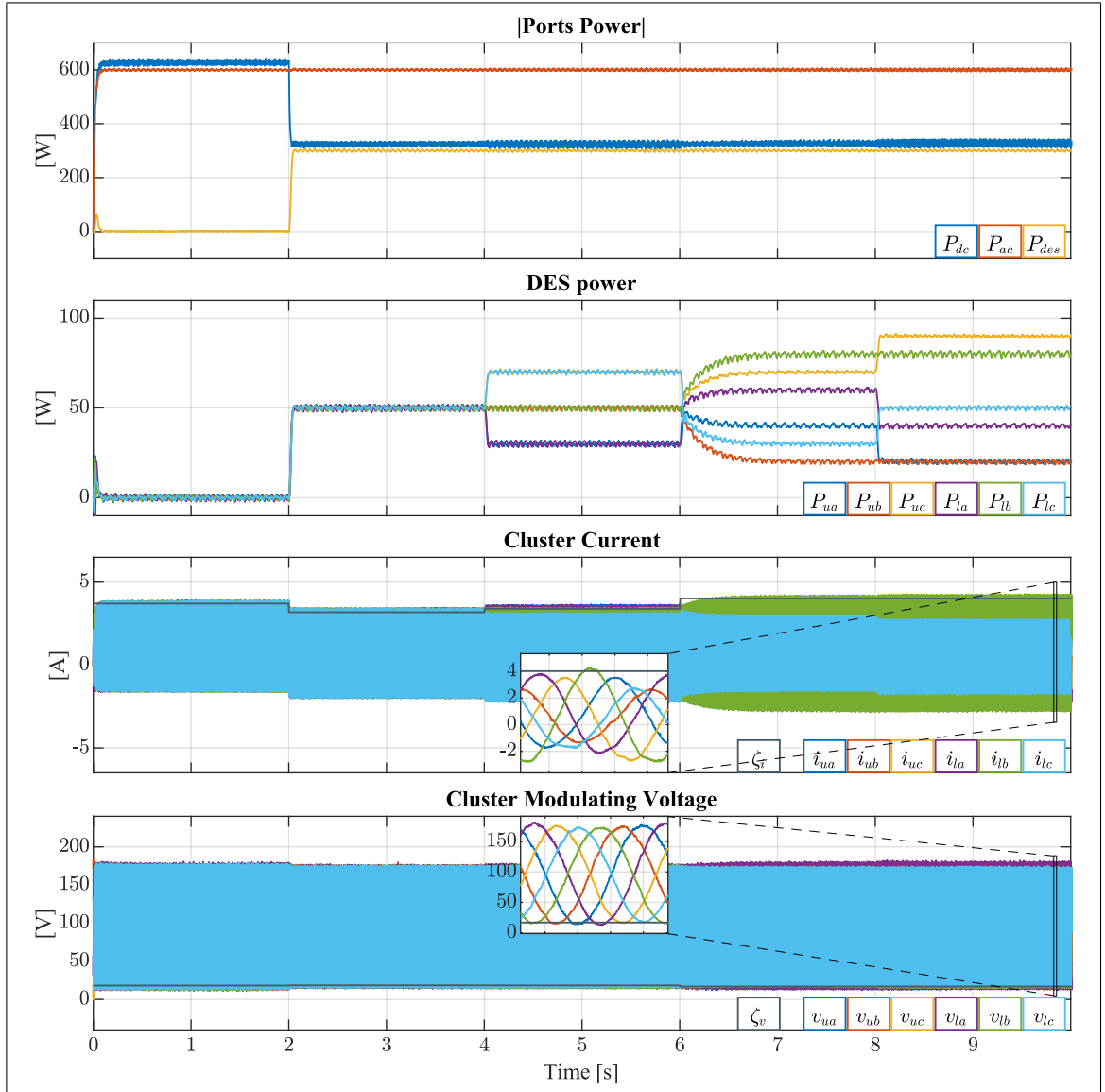


Figure 7.2. Mode I experiment.

of losses in the model. From there, it is not surprising that there is a slight current miscalculation in the subsequent test sections. In consequence, it is expected to observe smaller relative imprecisions in an industrial converter, as such converters have lower losses.

7.1.2. Mode IV

The proposed ac compensation CPC algorithm also manages the distribution of inter-cluster power without disturbing the MMC main ports power (Fig. 7.3). This validates the scheme depicted in Fig. 4.6 and the contribution of a new control algorithm for the MMC. There is, however, a minor new disturbance in the fixed MMC port when the DES net power changes. Perhaps the scheme or PI controllers could be improved to eliminate this disturbance or diminish it. Nevertheless, the dynamic performance of the control algorithms is not a key concern for this research.

As the model predicted, the current magnitude changes under ac compensation (Fig. 7.3) are different than the ones under dc compensation (Fig. 7.2) and generally lower. The model again follows the changes in the maximum currents but without perfect accuracy.

7.1.3. Model evaluation

The model fails to perfectly estimate the nominal current in both compensation modes. Therefore, the imbalanced current magnitude inaccuracies may be, to some extent, attributable to the error in the nominal current estimation, which derives from disregarding the converter losses. While an overrating model that considers losses would be of vast mathematical complexity, it is pretty simple to incorporate the losses in the calculation of the nominal currents, given a good estimate of the converter efficiency.

Table 7.3 presents the current overestimation error when the maximum current is calculated by applying the overrating to the ideal nominal currents and when the maximum current is corrected by applying the overrating to the actual nominal currents. It also shows the global overrating error. The corrected maximum current is just

$$\max\{|i_{y\chi}|\}^* = \zeta_i \cdot (\text{Actual } I_{y\chi}^{nom})$$

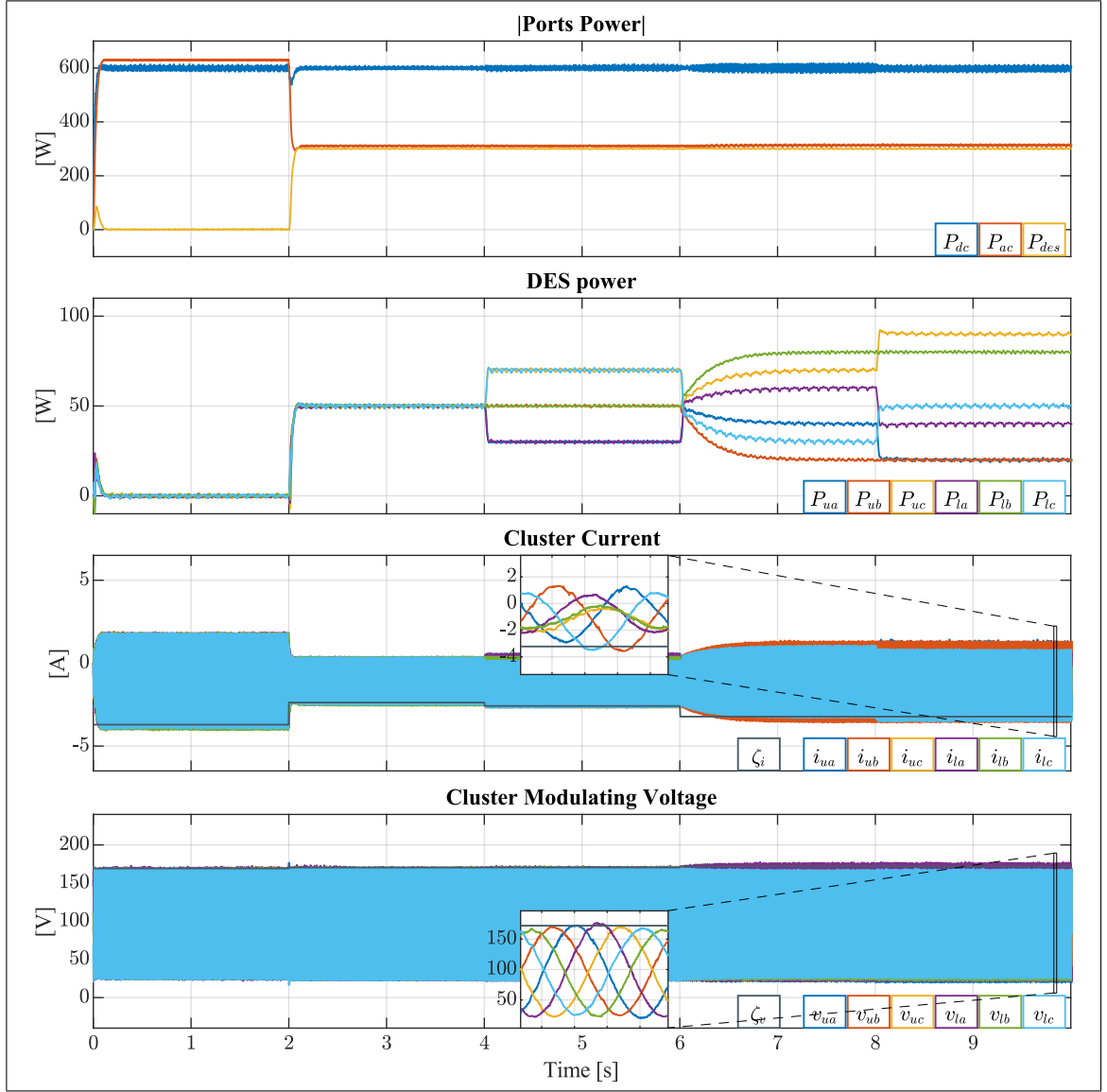


Figure 7.3. Mode IV experiment.

and the errors are calculated as

$$\max\{|i_{y\chi}|\}_{\text{Error}} = \frac{\max\{|i_{y\chi}|\} - (\text{Actual } \max\{|i_{y\chi}|\})}{\text{Actual } \max\{|i_{y\chi}|\}} \quad (7.2)$$

$$\max\{|i_{y\chi}|\}^*_{\text{Error}} = \frac{\max\{|i_{y\chi}|\}^* - (\text{Actual } \max\{|i_{y\chi}|\})}{\text{Actual } \max\{|i_{y\chi}|\}} \quad (7.3)$$

$$\zeta_{i \text{ Error}} = \zeta_i - (\text{Actual } \zeta_i) \quad (7.4)$$

Table 7.3. Overrating model evaluation along test sequence in experiments.

Section	Error	Mode I	Mode IV
1	$\max\{ i_{y\chi} \}$	-1.07%	-4.71%
	$\max\{ i_{y\chi} \}^*$	0%	0%
	ζ_i	0%	0%
2	$\max\{ i_{y\chi} \}$	-0.86%	-3.57%
	$\max\{ i_{y\chi} \}^*$	0.22%	1.19%
	ζ_i	0.19%	0.76%
3	$\max\{ i_{y\chi} \}$	-1.23%	-2.30%
	$\max\{ i_{y\chi} \}^*$	-0.16%	2.52%
	ζ_i	-0.15%	1.71%
4	$\max\{ i_{y\chi} \}$	0.47%	-5.91%
	$\max\{ i_{y\chi} \}^*$	1.56%	-1.27%
	ζ_i	1.66%	-1.12%
5	$\max\{ i_{y\chi} \}$	-0.22%	-5.82%
	$\max\{ i_{y\chi} \}^*$	0.86%	-1.16%
	ζ_i	0.92%	-1.02%

The experimental results are satisfactory. The maximum current estimation error is below 6% in all sections of the test sequence. The corrected current estimation tends to result in lower errors but can yield higher errors in some cases. Furthermore, the corrected current generally results in an overestimation of the actual current, while the standard current is usually an underestimation. The errors tend to be bigger in sections four and five of the sequence. Those are the sections with vertical imbalances, and therefore their error is not only influenced by the losses but also by the disregarded effect of the compensating currents in the vertical balance, which was mentioned in Chapter 4. The errors of the ac compensation global overrating ζ_i^{dc} are mostly smaller than the errors of ζ_i^{ac} . They correspond to modes I and IV respectively. They are both below 2% along the sequence.

7.2. Experimental setup simulation

The experimental setup simulation has two purposes. First, to evaluate the operating modes II and III, which cannot be tested experimentally due to the unidirectionality of the SM dc voltage sources. Second, it intends to validate the quality of the information provided by the simulation, as the results should be similar to the experimental ones. If so, it would indicate that the simulation of other converters provides trustworthy information and can lead to reliable conclusions. The simulation parameters are the same of Table 7.2.

7.2.1. Model evaluation of modes II & III

The currents of modes II and III behave similarly to the ones of modes I and IV respectively, but with negated values (Figs. 7.4, 7.5). This is expected, as they operate under the same control algorithms, and what changes is the power directionality. The model seems capable of estimating the maximum cluster currents with acceptable accuracy in these modes as well.

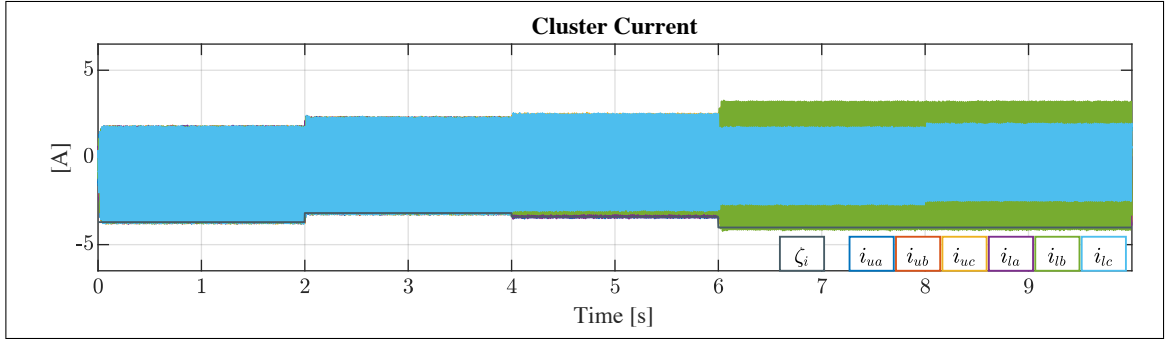


Figure 7.4. Mode II experimental setup simulation.

Table 7.4. presents the resulting error in current and global overrating estimations. The magnitudes are pretty close to the ones of modes I and IV, which indicated consistency between the simulation and the experiment. Nevertheless, there are some new interesting behaviors. The model underestimates the overrating and maximum currents in all sections under mode II, with or without the use of corrected currents. Mode III exhibits an

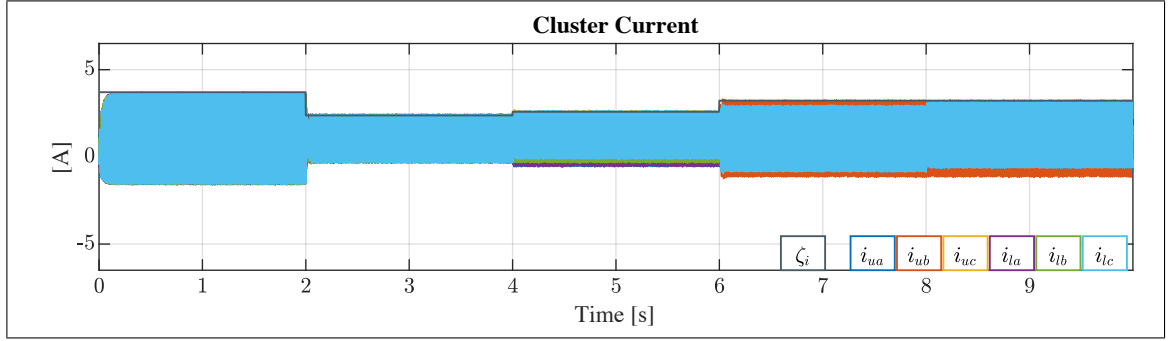


Figure 7.5. Mode III experimental setup simulation.

Table 7.4. Overrating model evaluation along test sequence in simulations of experimental setup.

Section	Error	Mode II	Mode III
1	$\max\{ i_{yX} \}$	-0.54%	4.44%
	$\max\{ i_{yX} \}^*$	0%	0%
	ζ_i	0%	0%
2	$\max\{ i_{yX} \}$	-0.84%	2.60%
	$\max\{ i_{yX} \}^*$	-0.29%	-1.77%
	ζ_i	-0.25%	-1.15%
3	$\max\{ i_{yX} \}$	-0.65%	2.34%
	$\max\{ i_{yX} \}^*$	-0.11%	-2.01%
	ζ_i	-0.10%	-1.43%
4	$\max\{ i_{yX} \}$	-2.54%	2.84%
	$\max\{ i_{yX} \}^*$	-2.01%	-1.53%
	ζ_i	-2.22%	-1.36%
5	$\max\{ i_{yX} \}$	-2.48%	2.85%
	$\max\{ i_{yX} \}^*$	-1.95%	-1.53%
	ζ_i	-2.15%	-1.35%

overestimation with the ideal currents and an underestimation with the corrected currents, contrary to the tendency of modes I and IV. Ultimately, these simulations allow to say that global overrating error is within a narrow margin in all cases, at least for the experimental setup.

Table 7.5. Industrial converter simulation parameters.

Description	Parameter	Value
Nominal Power	P^{nom}	1MW
DC grid Voltage	V_{dc}	2kV
AC grid Voltage	V_g	680V
Cluster Inductance	L_s	0.2mH
Output Inductance	L_o	0.1mH
Cluster Resistance	R_s	0.022 Ω
Output Resistance	R_o	0.01 Ω
SM DC Source Voltage	V_{SM}	333.3V
SMs per cluster	N	6
SM Capacitance	C_{SM}	1mF
SM Switching Frequency	f_{sw}	1.6kHz

7.3. Industrial converter simulation

The MMC is a topology used for high-power applications, much more than the 600W of the experiments. They usually have numerous SMs per cluster to achieve high voltages. A higher SM count translates into smaller L_s and L_o inductors, as there are more voltage levels to modulate the current. Commercial MMCs can comfortably reach 98% efficiency, whereas the experimental setup has near 95% efficiency. Both the reduction of the converter inductance and losses should increase the accuracy of the model, and it is important to have a notion of how the model would perform in an industrial-grade MMC. Table 7.5 displays the parameters of the simulated industrial converter.

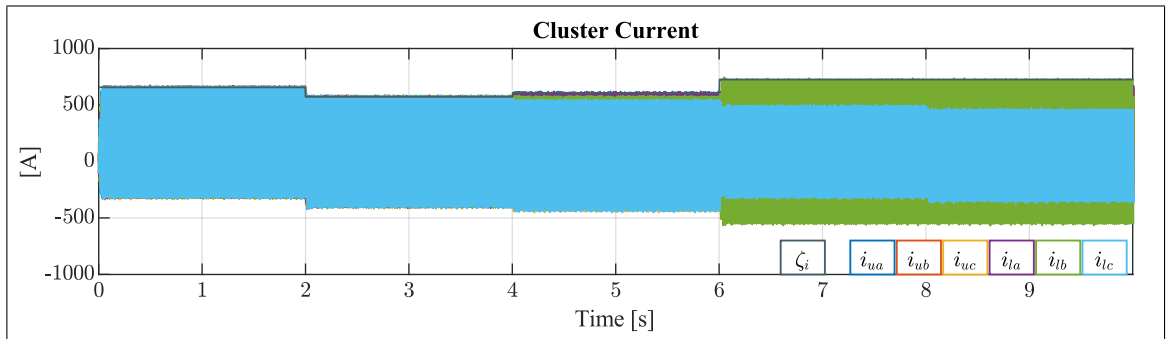


Figure 7.6. Mode I industrial converter simulation.

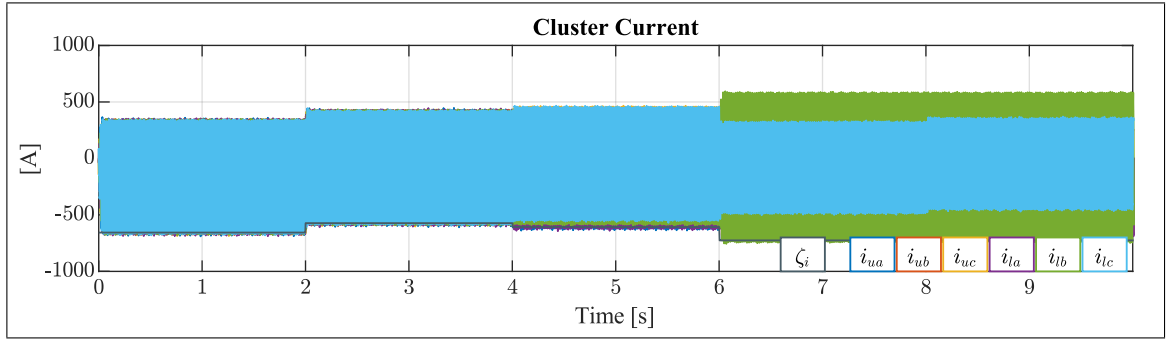


Figure 7.7. Mode II industrial converter simulation.

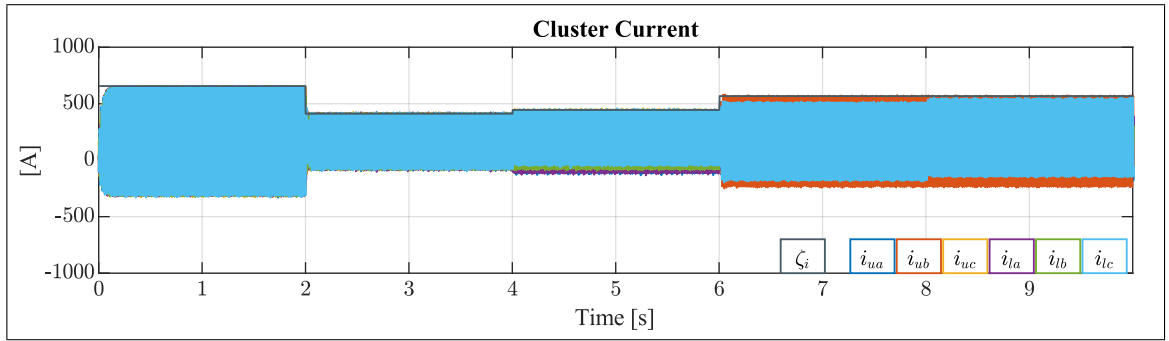


Figure 7.8. Mode III industrial converter simulation.

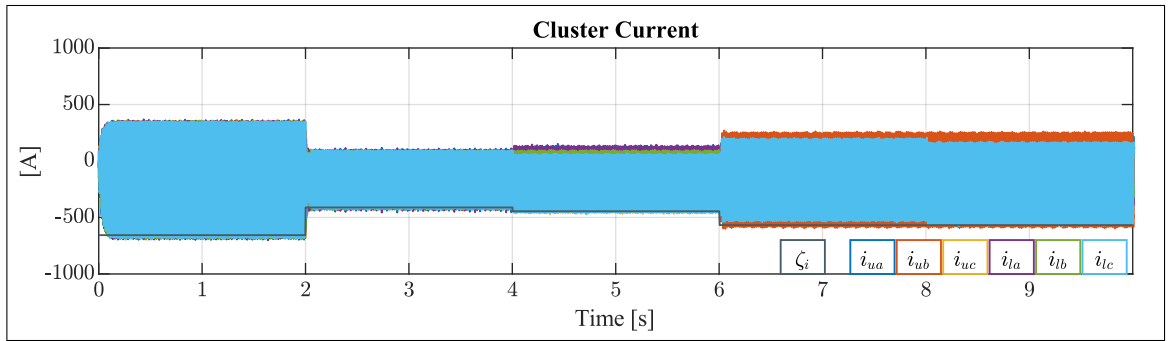


Figure 7.9. Mode IV industrial converter simulation.

The converter was successfully simulated in all modes (Figs. 7.6-7.9). Table 7.6 presents the errors of the model. The accuracy indeed increases with respect to the experimental setup, the estimated maximum current error is below 2% in all cases, and the worst global overrating error is just 1%. Some tendencies of the experimental setup errors are much more in this case. The overrating is always overestimated in modes I and IV, and

Table 7.6. Overrating model evaluation along test sequence in simulations of industrial converter.

Section	Error	Mode I	Mode II	Mode III	Mode IV
1	$\max\{ i_{yX} \}$	0.51%	-0.62%	2.44%	-2.70%
	$\max\{ i_{yX} \}^*$	0%	0%	0%	0%
	ζ_i	0%	0%	0%	0%
2	$\max\{ i_{yX} \}$	0.68%	-0.80%	1.31%	-1.42%
	$\max\{ i_{yX} \}^*$	0.17%	-0.18%	-1.11%	1.31%
	ζ_i	0.14%	-0.16%	-0.70%	0.81%
3	$\max\{ i_{yX} \}$	0.66%	-0.73%	1.20%	-1.28%
	$\max\{ i_{yX} \}^*$	0.14%	-0.11%	-1.21%	1.46%
	ζ_i	0.13%	-0.10%	-0.83%	0.97%
4	$\max\{ i_{yX} \}$	0.95%	-1.17%	1.29%	-1.65%
	$\max\{ i_{yX} \}^*$	0.44%	-0.55%	-1.13%	1.08%
	ζ_i	0.48%	-0.61%	-0.99%	0.92%
5	$\max\{ i_{yX} \}$	0.95%	-1.20%	1.25%	-1.62%
	$\max\{ i_{yX} \}^*$	0.44%	-0.58%	-1.16%	1.10%
	ζ_i	0.48%	-0.64%	-1.02%	0.94%

is always underestimated in modes II and III. The consistent bias in the overrating estimation indicates that the overrating is in fact not symmetrical between modes of the same compensation type, contrary to what the model and Table 6.7 indicated. Nevertheless, the difference is quite small, so the PIF values still provide valuable reference information, and it was expected that the theoretic values derived from the model would not be perfect. Regarding currents, the corrected model again produces better estimations in most cases but not all of them. Furthermore, the corrected currents and ideal currents have opposing underestimation and overestimation biases in the ac compensated modes, whereas they always coincide in the estimation error direction in the dc compensated modes. In those last modes (I and II), the corrected currents always yield a more accurate value. The dc compensated model performs slightly better than the ac, and its corrected current error is always below 0.6%, which is considerably good. It is also worth noting that mode I

is the best performing, and has outstanding accuracy in the presence of only horizontal imbalances.

7.4. Conclusions

This chapter aims to validate the control algorithms presented in Chapter 4, the overrating model formulated in Chapter 5, and the subsequent analysis derived in Chapter 6. It does so through both experiments and simulations that jointly evaluate the behavior of the MMC variables along multiple operating points and under different parameters.

The control algorithms achieve a successful control of the converter. This is particularly remarkable for the ac compensation CPC algorithm, which is first developed and proposed in this thesis. The algorithms manage the cluster power in a completely independent manner without disturbing the main MMC ports.

The results show that the cluster currents change notoriously along the different imbalance points, whereas the cluster voltage changes are minor. This corroborates that current overrating must be the main focus of attention regarding inter-cluster imbalances, which was first indicated in Chapter 6 (Tables 6.5,6.6). That chapter also pointed out the overrating symmetry between modes of the same compensation, but it was hereby disproven by the experiments and simulations. Nevertheless, the asymmetries are very small.

The overall accuracy of the model is considered satisfactory, having respective overrating and maximum current errors below 2% and 6% in the experimental results, and below 1% and 2% in the simulations of the industrial converter. The experiments and simulations visually show that the models correctly follow the changes in current magnitudes, but with small imprecisions. Those imprecisions can be partially attributed to the neglected losses in the nominal current calculation and the overrating model. In most cases, the model errors are reduced by calculating the maximum currents with the actual nominal current, and considerably reduced when applied to a MMC with parameters more akin to an industrial converter.

The model errors exhibit general biases and behaviors along the operating modes, which are quite consistent in the industrial-grade MMC simulations. Some of them are: 1) The model yields consistent overestimations or underestimations of the global overrating across all imbalances, depending on the operating mode and the consideration of losses in the nominal current calculation. 2) The dc compensated modes yield more accurate values across all imbalances. 3) The model has higher errors in presence of vertical imbalances than with horizontal ones. The converter design stage can consider these biases to establish different safety margins for the model results depending on the intended operation of the designed converter.

8. CONCLUSIONS

The MMC converter can be coupled with a DES, performing three simultaneous conversion stages. However, there are many challenges tied to the design and operation of an MMC-DES. Two of them are the increased control complexity of the joint system, and changes in the stationary set-points of the MMC currents and voltages caused by imbalances in the inter-cluster power distribution of the DES.

The literature encompasses a sparse variety of MMC-DES control algorithms. This research establishes a categorization for the existing algorithms, providing order and a reference frame for future algorithms and comparisons between them. The categorization allows to identify that DPC is more convenient for current-source DES and hybrid DES, but its control implementation raises more concerns. Meanwhile, CPC is more straightforward and convenient for voltage-source DES, but requires a centralized power controller. Complementary to the categorization, the converter power control can be managed under dc compensation or ac compensation, which link the DES power contribution to the dc and ac ports respectively (Fig. 3.4).

The imbalance factors (λ_{χ}^{Σ} and λ_{χ}^{Δ}) and the overrating factors ($\zeta_{i\chi}$ and $\zeta_{v\chi}$) corroborate that the required overrating of an unbalanced MMC can indeed be represented as an analytic function of its imbalance. The current overrating expressions do not depend on converter parameters, which remarks the generality of the imbalance phenomena in MMCs. They only depend on the characteristics of the imbalance ($\lambda^{\vec{\Sigma}\Delta}$), the operating mode (κ and compensation), and the proportion between grid voltages (ζ_{dc}). Nevertheless, the voltage overrating expressions depend on the converter parameters, and the current expressions would as well if losses had been taken into account. The overrating expressions show that horizontal imbalances can increase or decrease the converter currents, but their increases cannot exceed the converter nominal rating. Meanwhile vertical imbalances will always produce an increase on the converter currents. Therefore, the converter nominal current rating can only be exceeded in presence of vertical imbalances.

A DES can be represented by the full spectrum of potential imbalance states it might reach, leading to the definition of an *imbalance space*. Through the application of an optimization algorithm to various imbalance spaces, analysis shows that the most straining imbalances in the ac compensated modes require more current overrating (33%) than in the dc compensated modes (30%), and also shows that voltage overrating is not much of an issue. However, the imbalance contour plots indicate that ac compensation requires less overrating to reach most imbalances. The PIF analysis translates the complex multidimensional feasible imbalance volumes into an easily understandable figure. It reveals that with nominal current rating, dc compensation can only reach 16.75% of all possible imbalances, while ac compensation can reach 88.37%.

The analysis indicates that using the power of a DES to compensate the power requirements of the dc MMC port (or vice versa), is generally a more current demanding task than using the DES power to compensate power requirements of the ac MMC port. MMCs can actually be comfortably imbalanced in most scenarios if controlled under ac compensation. Therefore, it is far more convenient to incorporate variable DESs in applications that require or allow flexibility in the ac port of the MMC than in the dc port. As most papers encompass only dc compensation, the consideration of ac compensation can make some existing proposals more appealing.

The different analysis tools also show that the most straining imbalances lay on different $\lambda^{\vec{\Sigma}\Delta}$ points depending on the type of compensation, meaning that, if possible, the variability of a DES should be allocated differently depending on the planned type of compensation. Furthermore, applications that require both compensations intermittently would necessitate a differentiated treatment of the imbalances depending on the operating mode. The worst case for dc compensation is having the maximum amount of vertical imbalance on all phases with the DES at half nominal power, but with at least one phase on each vertical imbalance direction. The worst case for ac compensation is having the maximum amount of vertical imbalance on one phase, and having the others with minimal DES power.

Experiments and simulations successfully validate the proposed ac compensation CPC algorithm, the formulated imbalance model, and its subsequent analysis. Currents do vary considerably across imbalances, whereas voltage has only minor variations, verifying that currents are the main concern for unbalanced MMC-DESSs. Operation modes of the same compensation are almost symmetric regarding their current variations, but not quite. The differences can be attributed to the converter losses and diminish as the efficiency increases. The model predicts overrating with errors under 2% for the experimental converter and under 1% for industrial converters. The effects of excluding losses in the modeling is therefore marginal, as the model represents the converter operation with high accuracy. There are consistent biases or patterns in the found errors:

- The model may yield underestimated or overestimated predictions depending on the operating mode.
- The dc compensated modes yield more accurate predictions than the ac compensated ones.
- Vertical imbalances convey higher model errors than horizontal imbalances.

The converter design stage can consider these biases to establish different safety margins for the model results depending on the intended operation of the designed converter.

The conducted research validates the hypothesis, demonstrating that

H An analytical expression can be derived to accurately determine the current and voltage rating requirements of a MMC with inter-cluster power imbalances.

This statement is sustained on the basis of three fundamental findings that were proven in the presented research.

- The unbalanced operation of a MMC-DES can inherently create non-negligible increases in the converter current, while the voltage increases can be considered negligible.

- These increases in the converter operating variables can be analytically expressed as a function of the imbalance.
- Given an expected imbalance region of operation, the minimal required current hardware ratings of an MMC can be accurately derived.

Therefore, this thesis has successfully achieved its primary objective of developing equations for precise and reliable hardware design of MMC-DESs regarding current and voltage ratings, and generating a deeper understanding of MMC power imbalances.

The journey to better understand MMC-DES overratings yielded some interesting additional contributions. Like the decomposition of Σ power into its average and variational components, which was used as a tool to design the proposed CPC ac compensation scheme, but may be a useful symbolic representation of DES power as the average component portrays the net DES power and the variational component portrays the pure horizontal imbalances. Or the PIF, which was initially conceived as a tool to compare the imbalance capabilities of the different MMC-DES operating modes, but is also a powerful new tool to analyze and compare other converter topologies.

Future work could improve the developed expressions by incorporating the effects of non-unitary power factors into the analysis, or by finding a practical way to model the effects of the system losses in the current and voltage. The overrating model could also be extended for control algorithms that utilize common mode voltage, where the voltage overrating would gain relevance. This work also clears the way for rigorous comparisons between the MMC and CHB converters, whose unbalanced overrating models have already been devised. There is a need to clarify the circumstances under which each converter can offer advantages against the others, which could be partially evaluated through the metrics proposed in this work. As raised in Chapter 2, further research is needed to compare the existing control methods with rigorous analysis and reliably determine which are more convenient for different MMC-DES configurations and possible operating conditions.

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APPENDIX

A. LIST OF SYMBOLS AND ABBREVIATIONS

Variables and parameters

N	Number of SMs in a MMC cluster
i	Instant current [A]
I	Current magnitude [A]
v	Instant voltage [V]
V	Voltage magnitude [V]
S	SM switching state
L	Inductance [H]
R	Resistance [Ω]
Z	Impedance [Ω]
C	Capacitance [F]
f_{sw}	Switching frequency [Hz]
$\Sigma\Delta$	Sigma-Delta transform
t	Time [s]
ω	Frequency of ac grid voltage [rad/s]
θ	Angle [rad]
ϕ	Phase with respect to angle [rad]
P	Power [W]
A, B, C	State space model matrices
$\vec{x}$	State space system state vector
$\vec{u}$	State space system input vector
$\vec{w}$	State space system disturbance vector
$\vec{y}$	State space system output vector
K	Constant
λ	Imbalance factor
ζ	OVERRATING factor
κ	Power directionality between MMC main ports, $\kappa \in \{-1, 1\}$
Γ	Custom function for overrating model compactness
ψ	Custom angle for overrating model derivation compactness
Λ	Bounded space of imbalance factors
Z	Imbalance feasibility function

Special operations

- ⊙ Element-wise vector multiplication

Subscripts

a, b, c	Phases of a three-phase system
χ	Generic phase of a three-phase system, $\chi \in \{a, b, c\}$
u, l	Cluster vertical positions, upper and lower
y	Generic vertical position of a MMC cluster, $y \in \{u, l\}$
n	Generic numeration of a SM inside a cluster, $n \in \{1, \dots, N\}$
SM	SM attribute
C	SM capacitor attribute
s	Cluster current filter/plant
o	AC port current filter/plant
ϵ	Auxiliary term for compactness
dc	DC port
ac	AC port
g	AC grid
des	DES attribute
z	Circulating currents
cm	Common mode
0	DC frequency
1	AC fundamental frequency
h	Frequency components, $h \in \{0, 1\}$
avg	Average value
δ	Deviation from average value
d	Direct component in Clarke-Park space
q	Quadrature component in Clarke-Park space
ss	Steady state
u	Manipulated control variable
w	Control disturbance
$*$	Reference value
i	Current
v	Voltage

Superscripts

Σ	Σ component
Δ	Δ component
$\Sigma\Delta$	Set of Σ and Δ values
$+, -, 0$	Sequence components
p	Primary component
c	Compensatory component
nom	Nominal value
dc	DC compensation
ac	AC compensation
max	Maximum value

Abbreviations

MMC	Modular Multilevel Converter
DES	Distributed Energy System
DSBC	Double Star Bridge Cell
HVDC	High Voltage dc
SM	Submodule
BESS	Battery Energy Storage System
EV	Electric Vehicle
BMS	Battery Management System
CHB	Cascaded H-Bridge
SST	Solid State Transformer
PET	Power Electric Transformer
DAB	Dual Active Bridge
PV	Photovoltaic
MPPT	Maximum Power Point Tracking
FC	Fuel Cell
HBSM	Half-Bridge SM
FBSM	Full-Bridge SM
SoC	State of Charge
PS	Phase Shift
LS	Level Shift
PWM	Pulse Width Modulation

P	Proportional Controller
PI	Proportional Integral Controller
NLC	Nearest Level Control
MPC	Model Predictive Control
DPC	Decoupled Power Control
CPC	Coupled Power Control
SEC	Series Energy Control
PEC	Parallel Energy Control
KCL	Kirchoffs Current Law
KVL	Kirchoffs Voltage Law
PLL	Phase Locked Loop
LQR	Linear Quadratic Regulator
SLSQP	Sequential Least Squares Programming
PBF	Power Balance Factor
PIF	Power Imbalance Factor
PIR	Power Imbalance Region
PIS	Power Imbalance Space

B. FULL $\Sigma\Delta$ POWER EQUATIONS

$$\begin{bmatrix} P_{a,0}^\Sigma \\ P_{b,0}^\Sigma \\ P_{c,0}^\Sigma \end{bmatrix} = -R_s \begin{bmatrix} I_{za,0}^2 \\ I_{zb,0}^2 \\ I_{zc,0}^2 \end{bmatrix} + \frac{1}{2}V_{dc} \begin{bmatrix} I_{za,0} \\ I_{zb,0} \\ I_{zc,0} \end{bmatrix} \quad (\text{B.1})$$

$$\begin{bmatrix} P_{a,0}^\Delta \\ P_{b,0}^\Delta \\ P_{c,0}^\Delta \end{bmatrix} = V_{cm,0} \begin{bmatrix} I_{za,0} \\ I_{zb,0} \\ I_{zc,0} \end{bmatrix} \quad (\text{B.2})$$

$$(\text{B.3})$$

$$\begin{bmatrix} P_{a,1}^\Sigma \\ P_{b,1}^\Sigma \\ P_{c,1}^\Sigma \end{bmatrix} = -\frac{1}{2}R_s \left(\begin{bmatrix} I_{za,1}^2 \\ I_{zb,1}^2 \\ I_{zc,1}^2 \end{bmatrix} + \frac{1}{4} \begin{bmatrix} I_a^2 \\ I_b^2 \\ I_c^2 \end{bmatrix} \right) - \frac{1}{4}R_o \begin{bmatrix} I_a^2 \\ I_b^2 \\ I_c^2 \end{bmatrix} - \frac{1}{4}V_g \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \\ - \frac{1}{4}V_{cm,1} \begin{bmatrix} I_a \cos(\phi_{cm,1} - \theta_a) \\ I_b \cos(\phi_{cm,1} - \theta_b) \\ I_c \cos(\phi_{cm,1} - \theta_c) \end{bmatrix} \quad (\text{B.4})$$

$$\begin{bmatrix} P_{a,1}^\Delta \\ P_{b,1}^\Delta \\ P_{c,1}^\Delta \end{bmatrix} = \frac{1}{2} \begin{bmatrix} I_{za,1} \\ I_{zb,1} \\ I_{zc,1} \end{bmatrix} \odot \left((R_s + R_o) \begin{bmatrix} I_a \cos(\phi_{za,1} - \theta_a) \\ I_b \cos(\phi_{zb,1} - \theta_b) \\ I_c \cos(\phi_{zc,1} - \theta_c) \end{bmatrix} + \omega L_o \begin{bmatrix} I_a \sin(\phi_{za,1} - \theta_a) \\ I_b \sin(\phi_{zb,1} - \theta_b) \\ I_c \sin(\phi_{zc,1} - \theta_c) \end{bmatrix} \right. \\ \left. + V_g \begin{bmatrix} \cos(\phi_{za,1} - \theta_a) \\ \cos(\phi_{zb,1} - \theta_b) \\ \cos(\phi_{zc,1} - \theta_c) \end{bmatrix} + V_{cm,1} \begin{bmatrix} \cos(\phi_{za,1} - \phi_{cm,1}) \\ \cos(\phi_{zb,1} - \phi_{cm,1}) \\ \cos(\phi_{zc,1} - \phi_{cm,1}) \end{bmatrix} \right) \quad (\text{B.5})$$

$$\begin{bmatrix} P_{a,h}^\Sigma \\ P_{b,h}^\Sigma \\ P_{c,h}^\Sigma \end{bmatrix} = -\frac{1}{2}R_s \begin{bmatrix} I_{za,h}^2 \\ I_{zb,h}^2 \\ I_{zc,h}^2 \end{bmatrix}, \quad \forall h \notin \{0,1\} \quad (\text{B.6})$$

$$\begin{bmatrix} P_{a,h}^\Delta \\ P_{b,h}^\Delta \\ P_{c,h}^\Delta \end{bmatrix} = \frac{1}{2}V_{cm,h} \begin{bmatrix} I_{za,h} \\ I_{zb,h} \\ I_{zc,h} \end{bmatrix} \odot \begin{bmatrix} \cos(\phi_{za,h} - \phi_{cm,h}) \\ \cos(\phi_{zb,h} - \phi_{cm,h}) \\ \cos(\phi_{zc,h} - \phi_{cm,h}) \end{bmatrix}, \quad \forall h \notin \{0,1\} \quad (\text{B.7})$$

C. VECTORIZED MODEL IMPLEMENTATION IN PYTHON

Vectorized model

```
import numpy as np
from scipy.optimize import minimize
from scipy.optimize import LinearConstraint
import plotly.graph_objects as go
import time

f = 50
w = 2*np.pi*f

R_s = 0.96
R_o = 0.31
L_s = 5e-3
L_o = 1e-3

R_e = (R_s + 2*R_o)/2
L_e = (L_s + 2*L_o)/2
Z_s = np.sqrt((w*L_s)**2 + R_s**2)

P_nom = 600

V_g = 75
vC_ref = 48
N_sm = 4

Vdc = N_sm*vC_ref
zeta_dc = Vdc/(2*V_g)

kappa = 1

def gamma(lambda_D):
    vec = [0, 0, 0]
    for i in range(3):
        s_term = (1/3)*sum([lambda_D[j]**2
                             for j in range(3) if j!=i])
        p_term = (2/3)*np.multiply(*[lambda_D[j]
                                       for j in range(3) if j!=i])
        vec[i] = np.sqrt((lambda_D[i]**2+s_term-p_term).clip(min=0))
    return np.array(vec)

def lambda_S_avg(lambda_S):
    return sum(lambda_S)/3

def lambda_S_d(lambda_S):
    return lambda_S - lambda_S_avg(lambda_S)
```

```

def V_S1_dc(lambda_D):
    return -P_nom/(3*V_g)*Z_s*gamma(lambda_D)

def V_S1_ac(lambda_D):
    return V_S1_dc(lambda_D)

def V_D1_dc():
    term_1 = ((2*P_nom*Z_s)/(3*V_g))**2
    term_2 = 4*kappa*P_nom*R_e/3
    term_3 = V_g**2
    return np.sqrt(term_1 + term_2 + term_3)

def V_D1_ac(lambda_S):
    term_1 = (((2*P_nom*Z_s)/(3*V_g))**2) * \
        np.square(kappa + lambda_S_avg(lambda_S))
    term_2 = (4*P_nom*R_e/3)*(kappa + lambda_S_avg(lambda_S))
    term_3 = V_g**2
    return np.sqrt(term_1 + term_2 + term_3)

def zeta_ix_dc(lambda_S, lambda_D):
    num = np.abs(kappa - lambda_S) + \
        2*zeta_dc*np.sqrt( np.square(gamma(lambda_D)) + \
            2*np.abs(lambda_D) + 1)
    den = 1 + 2*zeta_dc
    return num/den

def zeta_vx_dc(lambda_S, lambda_D):
    num = (P_nom/(3*V_g))*( Z_s*np.abs(gamma(lambda_D)) + \
        (R_s/(2*zeta_dc))*np.abs(kappa - lambda_S) ) + \
        V_D1_dc()
    den = ( (R_s*P_nom)/(6*zeta_dc*V_g) + V_D1_dc() )
    return num/den

def zeta_ix_ac(lambda_S, lambda_D):
    num_sq_1 = np.square(gamma(lambda_D))
    num_sq_2 = np.square(kappa + lambda_S_avg(lambda_S))
    num_sq_3 = 2*np.abs( (kappa + \
        lambda_S_avg(lambda_S))*lambda_D )
    num = np.abs(kappa - lambda_S_d(lambda_S)) + \
        2*zeta_dc*np.sqrt( num_sq_1 + num_sq_2 + num_sq_3 )
    den = 1 + 2*zeta_dc
    return num/den

def zeta_vx_ac(lambda_S, lambda_D):
    num_1 = (P_nom/(3*V_g))*(Z_s*np.abs(gamma(lambda_D)) + \
        (R_s/(2*zeta_dc))*np.abs(kappa - \
            lambda_S_d(lambda_S)))
    num_2 = V_D1_ac(np.array(lambda_S))
    den = ( (R_s*P_nom)/(6*zeta_dc*V_g) + \

```

```

        V_D1_ac(np.array([0, 0, 0])) )
    return (num_1 + num_2)/den

```

Global overrating optimization problem

```

supply_mode = -1 ## 1, 0 or -1
kappa = -1

lambda_0 = np.array([-1, -1, -1, 1, -1, -1])
bound = 1
lb_S = -1*bound
ub_S = 1*bound
lb_D = -1*bound
ub_D = 1*bound
bnds = [(lb_S, ub_S)]*3 + [(lb_D, ub_D)]*3

# Cluster power boundaries
A_con = np.array([[1, 0, 0, 1, 0, 0],
                  [0, 1, 0, 0, 1, 0],
                  [0, 0, 1, 0, 0, 1],
                  [1, 0, 0, -1, 0, 0],
                  [0, 1, 0, 0, -1, 0],
                  [0, 0, 1, 0, 0, -1]])
con_power = LinearConstraint(A_con, lb=-bound, ub=bound)

## Supply or Load DES
con_supply = LinearConstraint(A_con, lb=0, ub=np.inf)
con_load = LinearConstraint(A_con, lb=-np.inf, ub=0)

cons = (con_power,)
if supply_mode > 0:
    cons += (con_supply,)
elif supply_mode < 0:
    cons += (con_load,)

## DES power boundary for Inhocerent DES
## Avoids exceeding compensated port nominal power
A_con_inc = np.array([[1, 1, 1, 0, 0, 0]])
con_des_power = LinearConstraint(A_con_inc, lb=-3*0,
                                ub=3*bound) ## Modes I - IV
con_des_power = LinearConstraint(A_con_inc, lb=-3*bound,
                                ub=3*0) ## Modes II - III
cons += (con_des_power,)

def zeta_i_dc_opt(lambda_):
    return -max(zeta_ix_dc(lambda_[3:], lambda_[3:]))

def zeta_i_ac_opt(lambda_):
    return -max(zeta_ix_ac(lambda_[3:], lambda_[3:]))

```

```

def zeta_v_dc_opt(lambda_):
    return -max(zeta_vx_dc(lambda_[0:3], lambda_[3:]))

def zeta_v_ac_opt(lambda_):
    return -max(zeta_vx_ac(lambda_[0:3], lambda_[3:]))

opt_prob = zeta_i_ac_opt
factor = zeta_ix_ac

lambda_0 = np.array([0, 0, 0, 1, -1, -1])

sol = minimize(opt_prob, lambda_0, args=(), method='SLSQP',
               bounds=bnds, constraints=cons).x
rounded_sol = np.array([round(l, 2) for l in sol])
print(rounded_sol)
print(factor(rounded_sol[0:3], rounded_sol[3:]))
print(max(factor(rounded_sol[0:3], rounded_sol[3:])))

```

PIF calculation

```

def Zeta(lambda_s, lambda_d):
    zeta_i = zeta_ix(lambda_s, lambda_d).max(axis=0)
    zeta_i[zeta_i <= 1] = 1
    zeta_i[zeta_i > 1] = 0
    return zeta_i

def mc_integrate(a, b, n, sub_sec=False, a_sec=0, b_sec=0, dim=6):
    if not sub_sec:
        rand = np.random.uniform(a, b, (dim, n))
        ## Incoherent. Not surpass compensated por nominal power
        ## speeds up, but not necessary as Zeta filters those points
        # rand = rand[:,rand[:,3,:].sum(axis=0) >= 0] ## DES supply
        # rand = rand[:,rand[:,3,:].sum(axis=0) <= 0] ## DES load
        s_rand = (rand[3:,:] + rand[:,3:])/2
        d_rand = (rand[3:,:] - rand[:,3:])/2
        domain = np.power(b-a, dim)
    else:
        rand_1 = np.random.uniform(a, b, (5, n))
        rand_2 = np.random.uniform(a_sec, b_sec, (1, n))
        rand = np.vstack([rand_1, rand_2])
        s_rand = (rand[3:,:] + rand[:,3:])/2
        d_rand = (rand[3:,:] - rand[:,3:])/2
        domain = np.power(b-a, dim-1)*(b_sec-a_sec)

    z_data = Zeta(s_rand, d_rand)
    z_mean = z_data.mean()

    return domain*z_mean

```

```

def mc_strat(a, b, n, sections=1, dim=6):
    delta = (b-a)/sections
    vol = 0
    for i in range(sections):
        a_sec = a+i*delta
        b_sec = a+(i+1)*delta

        vol += mc_integrate(a, b, n, sub_sec=True,
                             a_sec=a_sec, b_sec=b_sec, dim=dim)

    return vol

bound = 1
point_n = int(1e7)
mode = 1

## 0s to +- bound for Incoherent
if mode == 1:
    kappa = 1
    a = 0
    b = bound
    zeta_ix = zeta_ix_dc
if mode == 2:
    kappa = -1
    a = -bound
    b = 0
    zeta_ix = zeta_ix_dc
if mode == 3:
    kappa = 1
    a = -bound
    b = 0
    zeta_ix = zeta_ix_ac
if mode == 4:
    kappa = -1
    a = 0
    b = bound
    zeta_ix = zeta_ix_ac

t0 = time.time()
#vol = mc_integrate(a, b, point_n)
vol = mc_strat(a, b, point_n, sections=1000)
print(time.time() - t0)
print(vol)
PIF = vol/np.power(b-a, 6) ## *2 for Incoherent
print(PIF)

```