



PONTIFICIA UNIVERSIDAD CATÓLICA DE CHILE
ESCUELA DE INGENIERÍA

STABILITY ANALYSIS OF DISTRIBUTION NETWORK WITH DISTRIBUTED GENERATION AND SOFT-OPEN-POINT

GABRIEL MATÍAS YUSEFF CAMPUSANO

Thesis submitted to the Office of Research and Graduate Studies
in partial fulfillment of the requirements for the degree of
Master of Science in Engineering

Advisor:

JAVIER PEREDA TORRES

Santiago de Chile, January 2020

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*To my family, to Camila, to my friends
and to all those who have ever believed in me.*

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RESUMEN

Las redes de distribución eléctrica están en un periodo de transformación, en donde la inclusión y masificación de nuevas tecnologías, como la generación distribuida y los vehículos eléctricos, hacen que estas operen bajo una mayor estrés, aumentando la posibilidad de problemas, como lo son fluctuaciones de voltaje e incremento en los armónicos. En esta tesis se presentan algunas tecnologías nuevas desarrolladas para mantener y aumentar la calidad del servicio de las redes de distribución; y de estas se escoge como objeto de estudio los Soft-Open-Point (SOP), puesto que se establecen como una tecnología que permite controlar la transferencia de potencias entre dos puntos de la red previamente desconectados.

Esta tecnología ya ha sido probada y en esta tesis se presenta un estudio de estabilidad de una red de distribución que posee generación distribuida y un SOP. Para ello se presenta un sistema con los componentes descritos, a partir del cual se obtiene un modelo matemático linealizado en el espacio de estados. Este modelo es luego verificado por medio de simulaciones, donde se puede observar que converge al mismo punto que su modelo circuital con un bajo error. El modelamiento de la red se realiza de manera modular, en donde se separó en varios subsistemas, los cuales luego se unieron en un único sistema. Esto permite realizar análisis de cada subsistema de manera mas simple, además de facilitar la inclusión de nuevos subsistemas, en caso de estudiar una red mas compleja.

Finalmente se realiza un análisis de la estabilidad del modelo linealizado, se concluye que al escoger los parámetros adecuados, el sistema es localmente controlable hacia un punto de operación. Además, se realiza un análisis de parámetros críticos, de donde se pudo observar que el sistema es altamente sensible ante cambios en los parámetros de los controladores de corriente. Además, se concluyó que la inclusión de una resistencia de amortiguamiento es necesaria para el correcto funcionamiento del sistema.

Palabras claves: Redes de Distribución, Soft-Open-Point (SOP), control activo de potencia, Generación Distribuida (DG), analisis de estabilidad, vehiculos electricos, factores de participación.

ABSTRACT

The electric distribution networks are in a period of transformation, where the inclusion and massification of new technologies, such as distributed generation and electric vehicles, make them operate under greater stress, increasing the possibility of problems, such as voltage fluctuations and increase on harmonics. In this thesis some new technologies developed to maintain and increase the service quality of distribution networks are presented; and from these, Soft-Open-Point (SOP) is chosen as the object of study, since they are established as a technology that allows controlling the transfer of powers between two previously disconnected points of the network.

This technology has already been tested and in this thesis a stability study of a distribution network that has distributed generation and an SOP is made. For this, a system with the described components is presented, from which a linearized mathematical model in its state-space representation is obtained. This model is then verified by means of simulations, where it can be observed that it converges to the same point as its circuital model with low error. The modelling of the network is done in a modular way, where it is separated into several subsystems, which were then joined in a single system. This allows the analysis of each subsystem to be made in a simpler way, in addition to facilitating the inclusion of new subsystems, in case of studying a more complex network.

Finally, an analysis of the stability of the linearized model is carried out, it is concluded that by choosing the appropriate parameters, the system is locally controllable towards an operating point. In addition, an analysis of critical parameters is performed, from which it was observed that the system is highly sensitive to changes in the parameters of the current controllers. In addition, it was concluded that the inclusion of a damping resistor is necessary for the proper functioning of the system.

Keywords: Distribution networks, Soft-Open-Point (SOP), active power-flow control, Distributed Generation (DG), sensibility analysis, electric vehicles, participation factors.

NOMENCLATURE

Mathematical nomenclatures

$\mathbb{R}$	Set of the real numbers
$\mathbb{R}^n$	Euclidean space of dimension n
$\mathbb{R}^{n \times m}$	Set of $n \times m$ matrices with real coefficients
$0_{n \times m}$	An $n \times m$ matrix filled with zeros
$A_{(j)}$	j^{th} column of the matrix A
$A_{(j_1, j_2, \dots, j_n)}$	Sub matrix formed by the columns $j_1, j_2, \dots, j_n$ from the matrix A
$d\{x\}$	Denotes the direct component of the variable x in the dq reference frame
$q\{x\}$	Denotes the quadrature component of the variable x in the dq reference frame

Symbols

v	Voltage [V]
i	Current [A]
R	Resistance [Ω]
C	Capacitor [F]
L	Inductance [H]
f	Frequency [Hz]
t	Time [s]
x	State vector
$\dot{x}$	First derivative of the state vector x with respect to time
y	Output vector
u	Input vector
Δ	Denotes small-signal when prefix of any variable
θ	Angle [rad]
P	Active Power [W]

Q	Reactive Power [Var]
K_{pf}	Frequency gain in Classic Droop Controller
K_{qv}	Voltage gain in Classic Droop Controller
k_p	Active power gain in Droop Controller
k_q	Reactive power gain in Droop Controller
K_p	Proportional gain in PI Controller
K_i	Integral gain in PI Controller
k_{pr}	Proportional gain in PR Controller
k_{ir}	Integral gain in PR Controller
ω_r	Resonance frequency in PR Controller
ω_c	Central frequency in PR Controller
x_0	Equilibrium state
Σ	Topological system
A	State matrix in state-space representation
B	Input matrix in state-space representation
C	Output matrix in state-space representation
D	Feedthrough matrix in state-space representation
$\mathcal{W}, \mathcal{V}$	Subsets of $\mathbb{R}^n$
χ_u	Characteristic equation of a system Σ
PF	Participation factors matrix
p	Participation factor
ϕ_R	Right eigenvectors matrix
R_1	Line resistance between the substation and feeder [Ω]
L_1	Line inductance between the substation and feeder [H]
R_2	Line resistance between the feeder and PCC_{SOP} [Ω]
L_2	Line inductance between the feeder and PCC_{SOP} [H]
R_L	Load resistance [Ω]
L_L	Load inductance [H]
R_g	Coupling resistance [Ω]

L_g	Coupling inductor [H]
R_i	Filter resistance [Ω]
L_i	Filter inductor [H]
C_f	Filter capacitor [F]
R_d	Damping resistor [Ω]
C_V	Virtual Capacitor [F]
δ	Angle of a reference frame [$^\circ$]
T	Transformation matrix
J	Jacobian matrix
s	Laplace variable

States

x_1, x_2	Output of the integrals in PR Controller
v_i	Output voltage of inverter
i_i	LCL Filter input current
i_o	LCL Filter output current
v_c	LCL Filter capacitor voltage
i_1	Current from substation to PCC_{load}
i_2	Current from SOP to PCC_{load}
i_L	Current of the passive load
v_o	Output voltage of SOP
v_L	Voltage on the passive load
x_3	Output of integral in PI controller
v_{dc}	DC-Link capacitor voltage

Subscripts

a, b, c	Variable in the abc reference frame
dq	Variable in its local dq reference frame
DQ	Variable in the common dq reference frame

$\alpha\beta$	Variable in the $\alpha\beta$ reference frame
rms	Root mean square value of the variable
CC	Matrix of the Current Controller subsystem
LCL	Matrix of the LCL Filter subsystem
$delay$	Matrix of the Inverter subsystem
RL	Matrix of the Lines and Loads subsystem
VC	Matrix of the Virtual Capacitors subsystem
dc	Matrix of the DC-Link subsystem

Superscript

A	Variable in the side A
B	Variable in the side B
dg	Variable in the DG
inv	Matrix in the Inverse mode of operation
$*$	Denotes that the variable is a reference value

Abbreviations

ac	Alternate Current
dc	Direct Current
DG	Distributed Generation
DN	Distribution Network
DVR	Dynamic Voltage Restorer
ESR	Equivalent Series Resistor
EV	Electric Vehicles
FUN-LV	Flexible Urban Network - Low Voltage
LAC	Locally Asymptotically Controllable
NOP	Normally Open Point
MPPT	Maximum Power Point Tracker
PCC	Point of Common Coupling

PET	Power Electronic Transformer
PLL	Phase-Locked Loop
PV	Photovoltaic cell
PWM	Pulse Width Modulation
SOP	Soft-Open-Point

CHAPTER 1. INTRODUCTION

In 2015 the Government of Chile established that by 2050 the 70% of the total capacity of electrical generation should be through renewable energy resources (Chilean Government, 2017). This objective has become necessary to achieve, due to the increase of electrical demand and the continuous emissions of greenhouse gases produced by coal based electrical generation. By 2018, the electrical generation through renewable energy sources was about 35.000 *GWh*, which represent about 45% of the country (Ministry of Energy of Chile, 2019) and it is estimated that the objective will be accomplished. A way to accomplish this is to encourage on-site generation, refereed in this thesis as distributed generation (DG); which means that users generate their own electricity, mainly through renewable sources such as solar and wind power.

Another technology whose use is increasing and is set to make a great impact is electromobility, the global tendency is to increase the use of electric vehicles (EV), whether are electric cars, motorcycles, trains, etc. In 2018, over two million EVs were sold and it is expected that by 2040, over 50% of the global sales of vehicles will be electric (McKerracher et al., 2019). Furthermore, the electrification of the buses is being carried out in the world; in Chile by the end of 2019 there will be over 400 electric buses circulating in the public transport (Arce, 2019), which represent over 8.5% of the fleet in Santiago; with a goal that by 2022, it increases over 30% (Ministry of Energy of Chile, 2015).

As described above, DG and EV are technologies that will increase their use in Chile and the world. However, new technologies bring new problems, for example, photovoltaic generation can lead to voltage irregularities or even act as loads and the increase of electric vehicles leads to higher demands of power. Problems like these represent new challenges that must be solved in order to maintain and increase the reliability of distribution networks.

This chapter provides some necessary context for this thesis. First, an explanation of what a distribution network is, some of its topologies and typical issues it has. It also provides a definition to distributed generation and how much will penetrate into the distribution network. Then, some solutions that increase the reliability of distribution networks

are presented. Later, some of the notation used in later chapters is presented. Finally, the focus, objectives, hypothesis, methodology and structure of this thesis are presented.

1.1. Distribution Networks

According to Short (2004): “Electric power distribution is the portion of the power delivery infrastructure that takes the electricity from the highly meshed, high-voltage transmission circuits and delivers it to customers” (p.19). A distribution network connects the transmission line and the customers, this is made through a substation, which utilizes a set of transformers and different protection elements, such as breakers and voltage regulators.

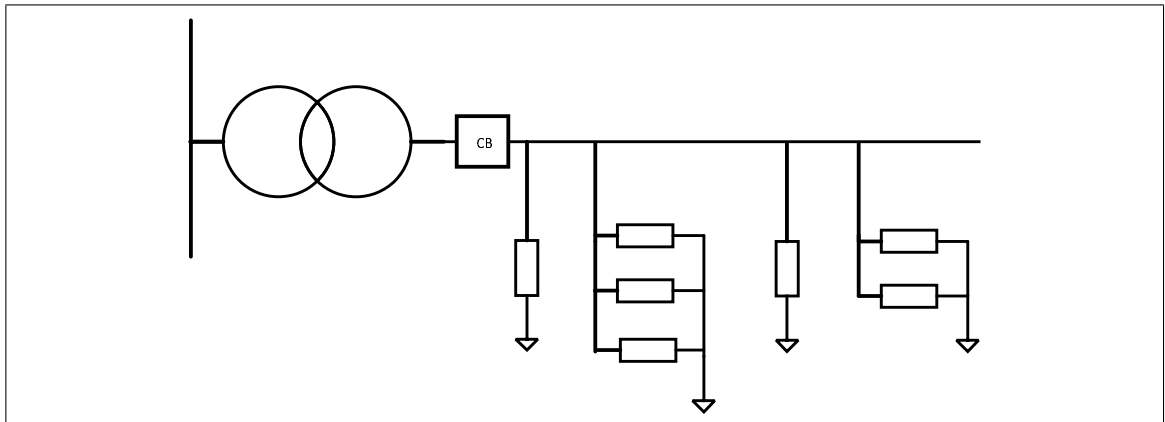


Figure 1.1. Radial DN

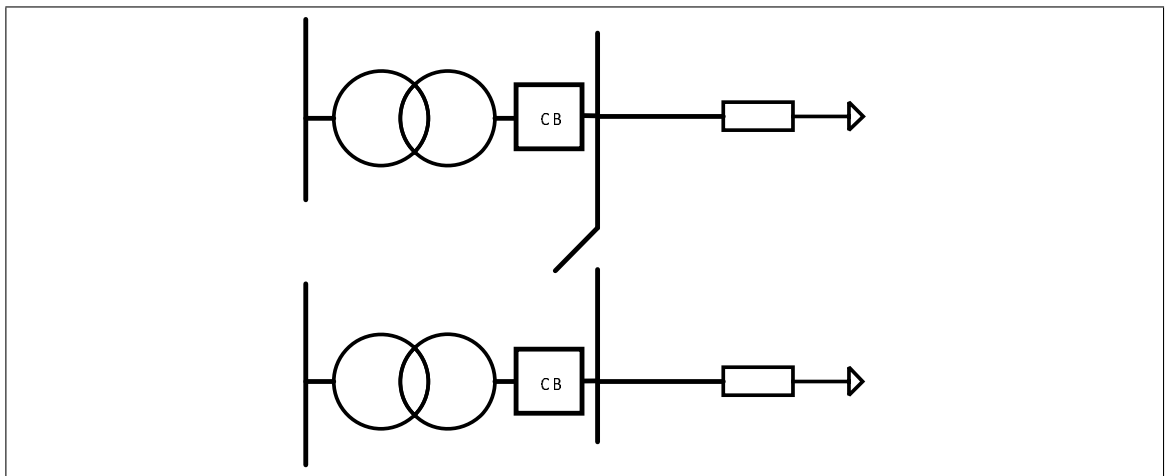


Figure 1.2. Parallel DN

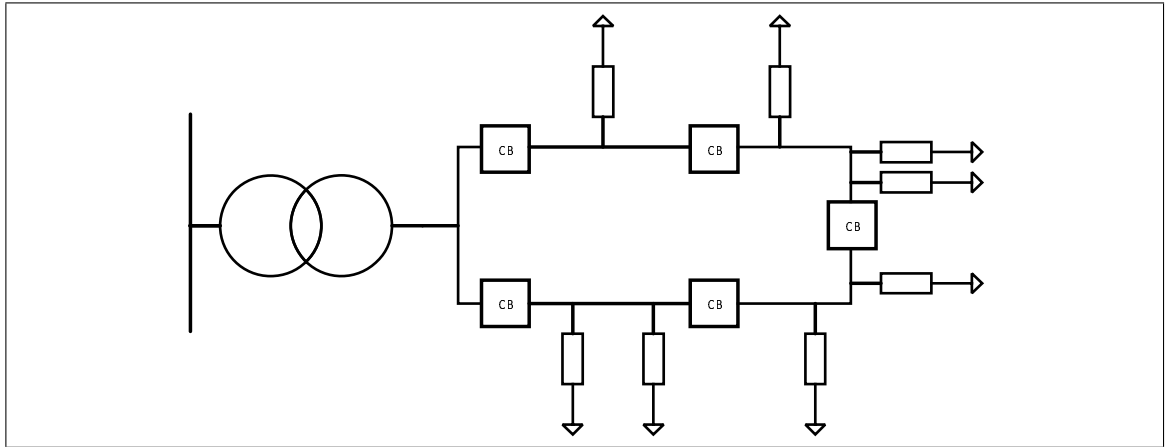


Figure 1.3. Ring DN

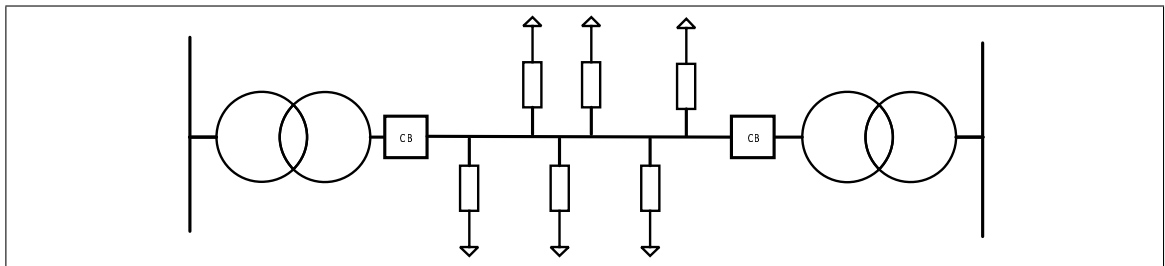


Figure 1.4. Mesh DN

The configuration of the distributed network affects its reliability in supplying energy to the customers. Some typical configuration are presented on figures 1.1-1.4, and explained below.

- **Radial:** This is the most simple structure, it connects a set of loads directly to the feeder as seen on Figure 1.1. This is the cheapest to implement, but in case of a fault in the feeder, the power supply will not be maintained.
- **Parallel:** In this structure there are several feeders that connect different parts of the network, in case of a fault in one of them, a switch will connect the feeders and will maintain the supply through only one transformer. This configuration is shown on Figure 1.2. This configuration increases reliability, but is more expensive due to the use of several transformers.

- **Ring:** In this structure a main feeder will be connected to various interconnected local feeders in a ring type configuration, as seen on Figure 1.3. If there is a fault in a local feeder, it will be reconnected to the main feeder through another path. This configuration reduces voltage drops, but it is more expensive than a Radial configuration, due to the increase of cables and protections.
- **Mesh:** A mesh structure is seen on Figure 1.4. In a mesh grid, the customers are connected to more than one power supply, this allows for more power to be delivered and increases the reliability of the network. The problems associated to this structure are its high cost, and the synchronization of the different sources, as all must meet the network regulations.

The problems that may arise in a distribution network can be related to external sources, such as vandalism, lightning and trees; or internal sources, increase of loads or malfunctions of elements. Some typical problems encountered on a distribution network are:

- **Faults:** An element fails to perform its duties, whether it is due to a short-circuit or a broken wire. The fault normally involves a short-circuit between a phase with another phase or between a phase and ground. A short-circuit produces a low impedance path, which leads to a very high current. A fault, for example, can lead to interruption of the supply and, in a worst case scenario, the explosion of the substation (Short, 2004).
- **Voltage irregularities:** The voltage in a distribution network is outside of its nominal value boundaries. An overvoltage can provoke a peak of current, decrease the life of the transformers, increase harmonic distortion of the voltage, etc. (Haider, Vaziri, & Phillips, 2006). Meanwhile, an under-voltage can increase the stress of certain components, due to an increase in the current needed to reach its nominal power.
- **Equipment issues:** The equipment in a distribution network is key to maintain its optimal performance. If a breaker does not open a circuit in case of a fault, this can lead to the problems described above. Similarly, the voltage regulator in a substation can solve the problems in case of over-voltage or under-voltage.

As technology progresses, there are new challenges that the distribution network must be prepared to deal with. One of these is the increase of electric vehicles, these correspond to new loads which require high current and, in consequence, the power demand of the network is increased. Another new challenge is distributed generation, even though on-site generation bring benefits to the customers and can take stress off from the network, a deficient control and a decentralized planning of these decreases the quality of the voltage.

1.2. Distributed Generation

In "*Distributed generation: a definition*", Ackermann (2001) recopilates different definitions of DG in order to form the following definition utilized in this thesis: "Distributed generation is an electric power source connected directly to the distribution network or on the customer site of the meter" (p.201). In other words, a DG source is a power source that is placed near the loads of a grid in order to supply directly to these.

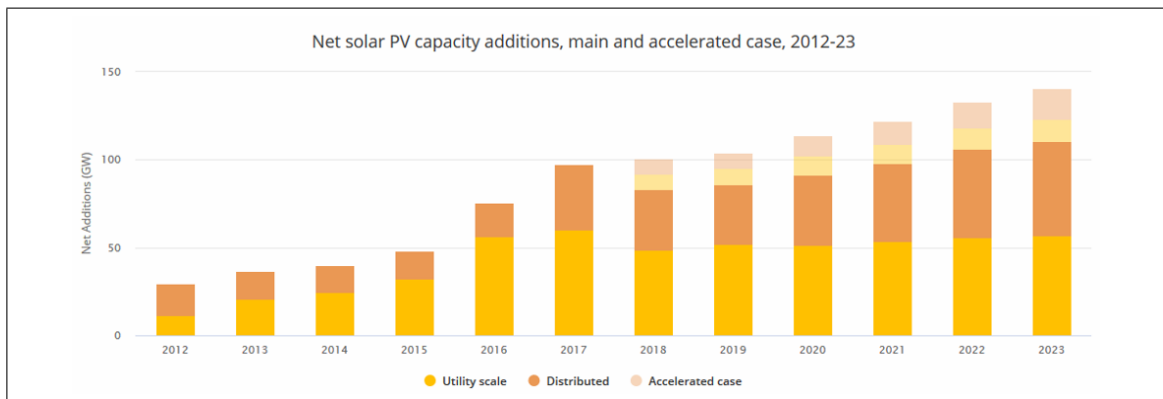


Figure 1.5. Net solar PV capacity, 2012-2023 (IEA 2018)

Distributed generation is commonly associated, but not restricted, to renewable energy sources, such as photovoltaic panels and wind turbines. As seen on figure 1.5, a great percentage of energy generated through PVs will be DG. In Chile, the current DG capacity is 23,26 MW and it is increasing at a rate of 1,1 MW per month (Valencia, 2019). It is expected that DG continues to grow due to new laws that encourage on-site generation, for example in the new Law of distributed generation (Law 20,571) it is regulated that the distributors can buy the excess of energy from clients (Chilean Government, 2017).

Distributed generation can lead to problems in a distribution network, such as increased harmonics, flicker, bad voltage regulation. These can be produced due to malfunctions in the DG sources, deficient controllers, their location or bad design (Ackermann & Knyazkin, 2002). Due to this, it is important to design accordingly to the network and, if possible, have a central controller that regulates different sectors of a grid and act rapidly in case of faults.

1.3. Solutions in Distribution Networks

As distribution network has become bigger and demand has increased, new strategies have been developed in order to face the challenges associated with new technologies and higher demands. Some typical solutions that have been implemented are:

- **Meshing of networks:** If a higher demand is presented, an existing distribution network can connect to another and form a meshed network. As shown in figure 1.4, in this configuration different substations support each other in order to supply the power demand.
- **More transformers:** A transformer is connected in parallel to another transformer in a substation. By doing this, the capacity of the substation is increased and allows it to supply a higher demand. The downfall of this solution is the high cost of the transformers, their size and that both must be identical in order to avoid circulation currents.
- **Taps in transformers:** The taps are different point of connection in a side of the transformer, each tap connects to a different number of turns in the winding and changes the turns ratio. This allows for control of the output voltage of the transformer and its biggest disadvantage is that it only changes the voltage in a discrete manner.

With the development of power electronics and communications, new and more complex structures for distribution networks have emerged. Microgrids allow for a distribution network to isolate from the utility grid and continue to supply in case of an emergency; meanwhile in a Smartgrid, there is communication between various of its elements, which brings faster response in case of problems and better regulation of the components.

Besides the restructuring of the distribution networks, in the field of power electronics there are new elements that can be connected to the grid in order to increase the quality of the voltage and permits faster and cheaper solutions to problems, these are presented in figures 1.6-1.12, and described below:

- **Power Electronic Transformers (PET):** It is a power electronic device that transfers power from the transmission line into the distribution line or between

distribution networks. It fulfills the same function as a traditional transformer, with lower cost (Wang, Guerrero, Chen, & Blaabjerg, 2010). As corresponds to an electronic converter, it provides real-time control to the output voltage, so it can quickly solve problems related to voltage irregularities. Although, this solution increases the flexibility of the network, the technology is still young and the efficiency of the EPTs are still lower than traditional transformers (Yue et al., 2018).

- **Battery Storage Support System:** Batteries can be connected in the substation in order to make peak shaving, this lessens the stress of the transformer by charging the battery when there is low demand, and utilizing that energy in high demand periods. This method increases the life of the transformer and reliability of the network, but the prices of the batteries must be lower in order to make this a more cost effective solution (Chaichana, 2019).
- **Reactive compensation:** These devices are placed near the loads of the distribution network and correspond to AC/DC converters connected to an isolated DC bus and the feeder (Gao & Awodele, 2015). The controllers regulate reactive power transfer, which allows compensation of harmonics and correct unbalances between the three-phases of the network.
- **Dynamic voltage restorer (DVR):** It is an energy storage device connected to the distribution network through a converter. This device is used to correct a voltage sag (a short period under-voltage), normally placed near a sensible load (Nielsen, Newman, Nielsen, & Blaabjerg, 2004). The controller in a DVR must be designed so that it detects and quickly corrects voltage sags.
- **Feeder regulator:** These correspond to power converters connected between the substation and one of its feeders; or in the middle of the feeder. This allows to control the voltage on the feeder in order to maintain it inside the constraints.
- **Soft-Open-Point (SOP):** An AC/DC/AC converted connected between two points, which can be open or Normally-Open-Point (NOP), in which there is a switch

between them. It regulates active and reactive power transfer between its terminals, this allows for voltage control, isolation and support between substations (Zhang et al., 2019).

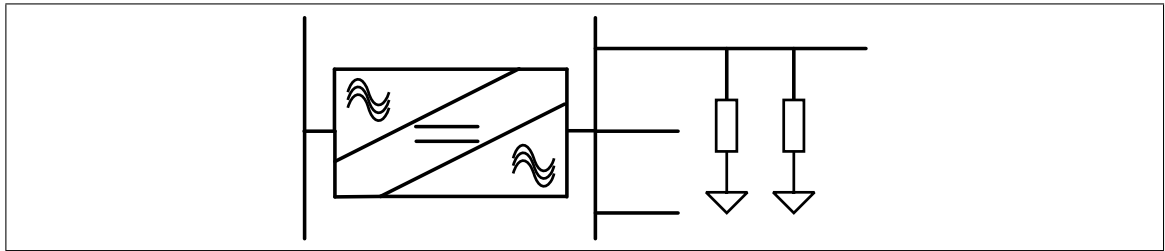


Figure 1.6. Power Electronic Transformer

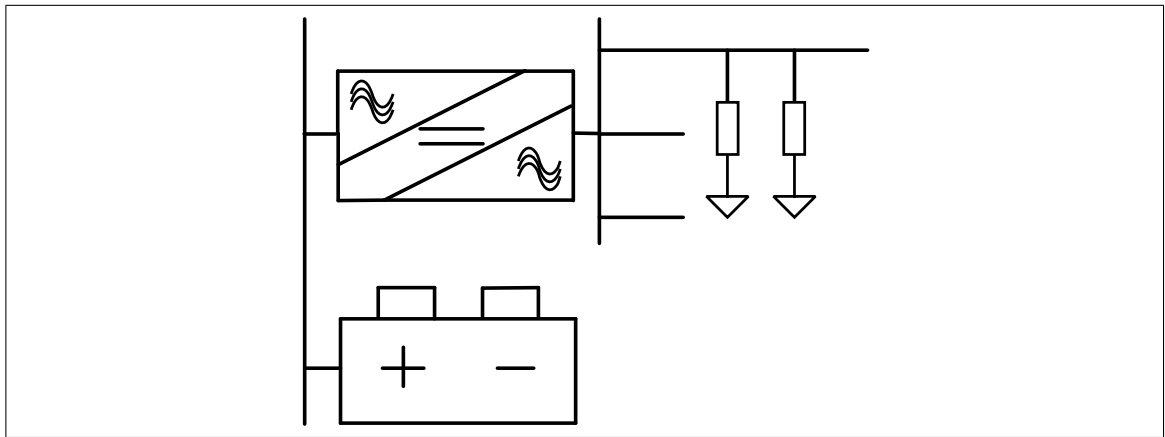


Figure 1.7. Battery Storage Support System

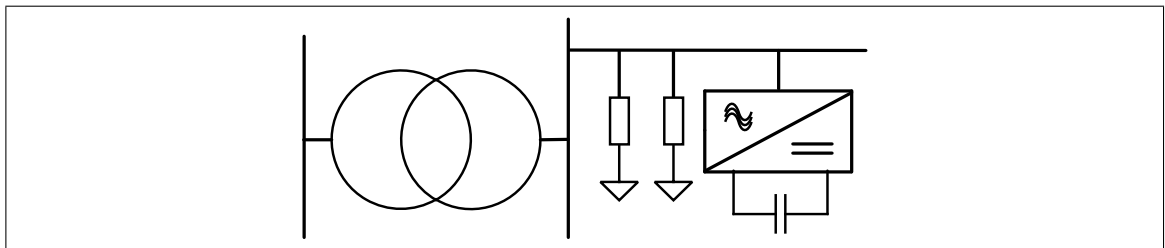


Figure 1.8. Reactive Compensator

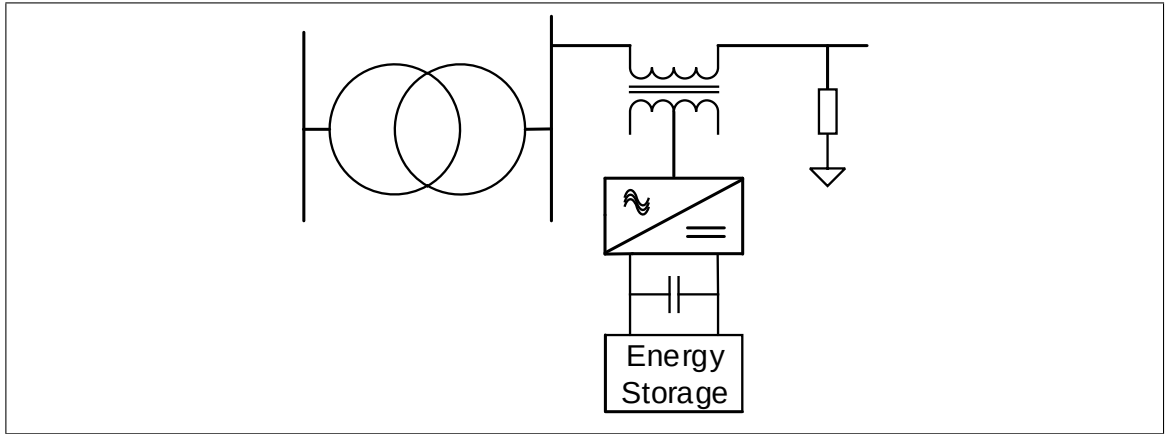


Figure 1.9. Dynamic Voltage Restorer

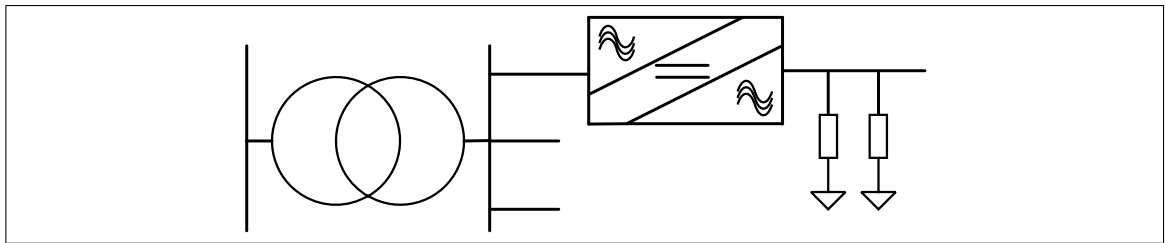


Figure 1.10. Feeder Regulator

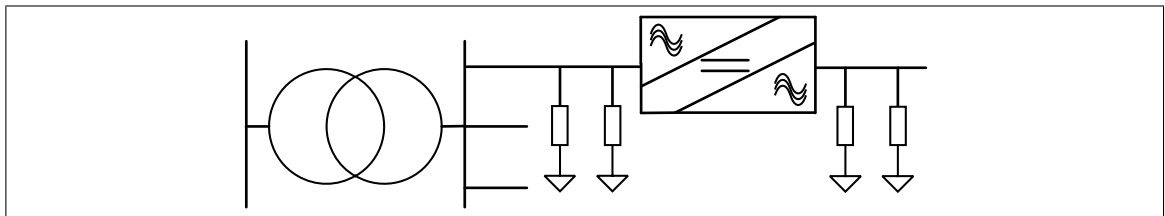


Figure 1.11. Mid-Feeder Regulator

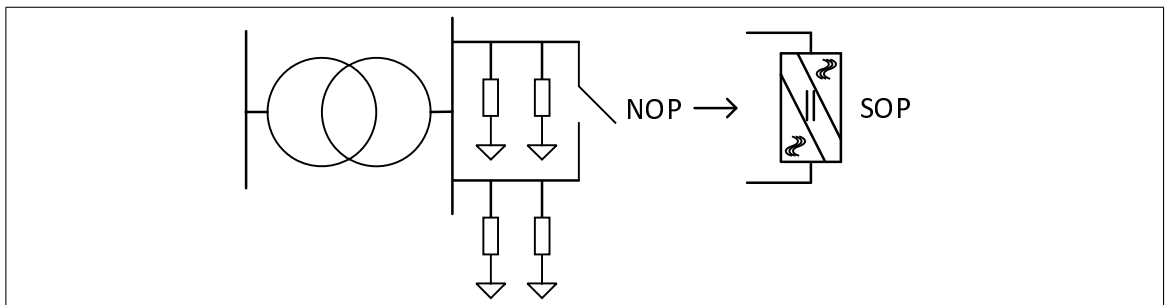


Figure 1.12. Soft-Open Point

1.4. Contribution

This thesis aims to further validate the use of SOP as a device that increases the reliability and flexibility of Distribution Networks. The benefits and viability of SOPs have been proven through studies and even implementation, but the stability of a distribution network with SOP and DG has not been studied. By modelling and analysing a distribution network with SOP and DG, this thesis can serve as both a validation of the use of SOPs and as a framework which can serve as a reference to study more complex systems.

1.4.1. Objectives

General objective:

Make a stability analysis of a distribution network that includes distributed generation and Soft-Open-Point. This will determine the stability of the system under typical values of electrical components and constant power DG. It will also find critical parameters and study the impact they have on the stability.

Specific objectives:

- i. Design a back-to-back Soft-Open-Point connected to the feeders of a distribution network that includes inverter-connected distributed generation.
- ii. Model the specified system through its differential equations into a state-space representation.
- iii. Linearize the model in order to study the system in the vicinity of an operation point.
- iv. Validate the model through simulations.
- v. Determine the stability of the linear model.
- vi. Analyze the system through sensibility analysis and participation factors. And determine the parameters that are the most critical to the stability of the system.
- vii. Find conditions to the critical parameters for which the system is stable.

1.4.2. Hypothesis

In order to achieve the general objective of this thesis, the following hypothesis must be confirmed:

H: A distribution network with DG and SOP contains at least one operation point, defined by the designer and different to the zero, to which the system converges from a vicinity.

1.4.3. Methodology

To achieve the different objectives it is necessary first to investigate different control strategies for SOPs, then design a control system that forces it to reach a reference point in the system.

The system will be divided into various subsystems and these must be modelled into their state-space representation with the use of Kirchhoff's Laws and the topology of the controllers; the non-linear equations will be linearized around an operation point. After that, the subsystems are used to form a single model that describes the dynamics of the whole system around the operation point; which is verified through simulations in Matlab/Simulink.

Then, the stability of the linearized system is determined by studying the Eigenvalues of the state matrix. After this, participation factors are used to determine which states are the most critical for the stability and a sensibility analysis of the parameters that influence these states are made in order to determine ranges of values in which the system is stable.

1.4.4. Thesis structure

Chapter 2 details the function and design of a SOP, this includes the power electronics and different control systems. It also presents the concepts of control theory used in this thesis.

In **Chapter 3**, the studied distribution network is presented. Then a state-space representation is obtained for its small-signal model, which is verified through simulations.

In **Chapter 4** the stability of the system is analyzed and conditions for it are determined. A sensitivity analysis of various parameters of the system and an analysis with participation factors are also presented.

Finally, **Chapter 5** contains the conclusions of this thesis.

CHAPTER 2. SOFT-OPEN-POINT

In Chapter 1, Soft-Open-Point (SOP) was presented as a power electronic solution that can regulate the transfer of power between two nodes of a distribution network. Through this device, two or more distribution network can be connected and transfer power in a regulated way. This increases the flexibility of the networks, as it allows support between them in case of a higher demand in one of these; regulate the voltages in the terminals to a reference point and, in case of a fault in one of the sides, it can disconnect the networks (Bloemink & Green, 2010).

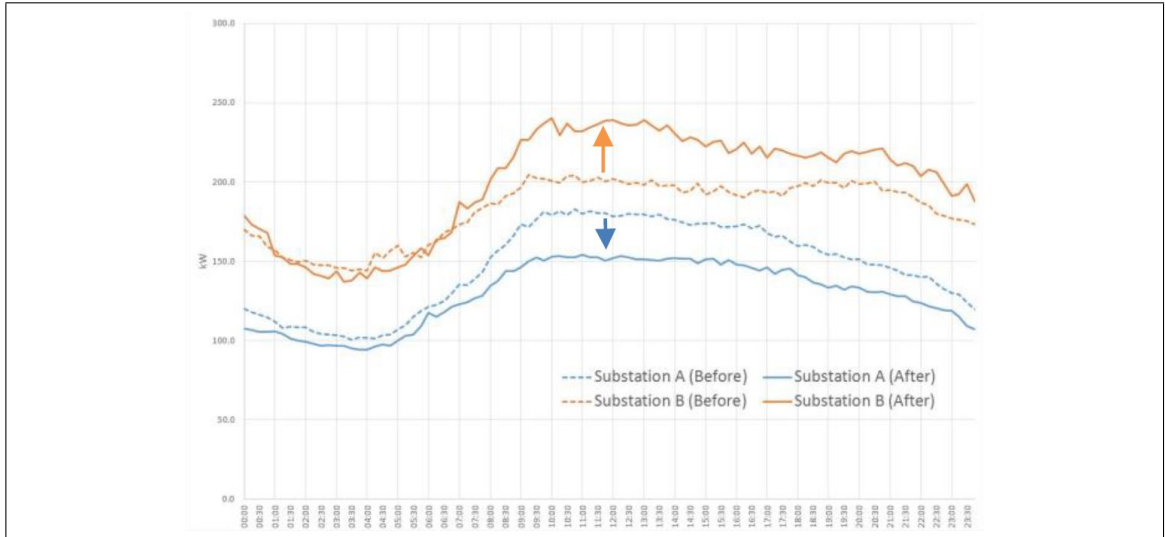


Figure 2.1. Power demand of two substations connected through a SOP (Newton et al., 2017)

This technology has been tested in the UK *Flexible Urban Network - Low Voltage* (FUN-LV) project, which consisted in increasing the flexibility of low-voltage distribution networks in the country, by installing three different types of electrical elements in 36 sites in the London and Brighton grids; 24 of these corresponded to SOPs, 12 were two-ports and the other 12 were three-ports (Newton, Lang, & Terry, 2017). In figure 2.1 the demand of two substations are seen, a SOP was connected between two points in their respective networks. In the figure, the dotted lines represent the demand of the substations before this, and the continuous lines are the demands while the SOP operates; it can be

seen an increase in the power demanded from substation B and a decrease in the demand from substation A, this is because the algorithm of the control system established support between the substations.

2.1. Power Electronics

Power electronics is a branch of electrical engineering that studies the conversion of energy using solid state electronics (semiconductors). For this, it studies the design of converters, which are made of passive components, semiconductors and controllers. The design and control of these allows for a controlled transference and transformation of electric energy. These are classified into four groups depending on whether the input and output currents are direct current (DC) or alternating current (AC). The classifications are: AC/AC converters, DC/DC converters, AC/DC converters (rectifiers) and DC/AC converters (inverters).

Figure 2.2 shows a simple three-phase inverter constituted by six transistors (S1-S6), that acts as switches, and an output current filter, composed by inductors. The goal of the inverter is to transfer power between an AC-side (nodes A , B , C) and a DC-side (capacitor C). By controlling the switching of the transistors, it regulates the power magnitudes and the direction of the transference.

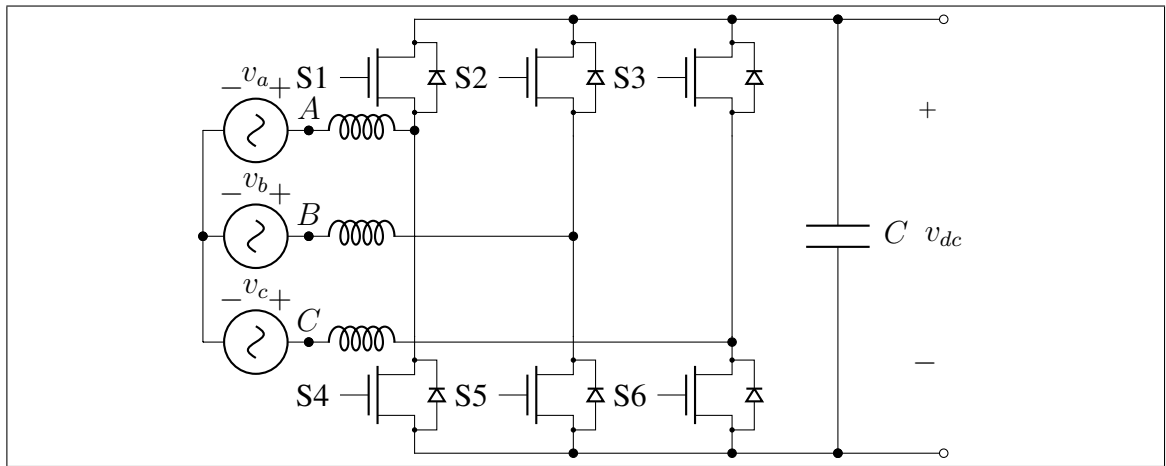


Figure 2.2. Three-phase inverter

Soft-Open-Point utilizes a back-to-back configuration in which two converters are connected through a DC-link, and it can be seen on figure 2.2. Commonly, one of the converters acts as an active rectifier and transfer power from the AC system to the DC-link in order to maintain its voltage; meanwhile the second converter works as an inverter to supply a load connected to its second terminal.

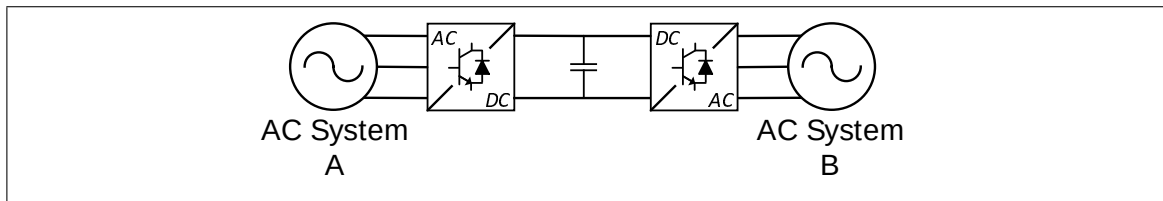


Figure 2.3. Back-to-Back converter

2.2. Controllers

A control system regulates the behaviour of a plant in order to achieve an objective. In an inverter this can be to establish a voltage or current in a point, or to set how much active and reactive power is transferred between its terminals. This usually is done by a cascade connection of different controllers that calculate individual aspect for which they are best suited (nested controllers).

Reference frames

The controllers of an inverter can be designed in one of three reference frames, in the natural reference frame (*abc*-frame) the variables are analysed in the time-domain; in the stationary reference frame ($\alpha\beta$ frame) the axes are fixed and the three-phase variables are represented as the sum of two rotating vectors; finally, in the synchronous reference frame (*dq*-frame) the variables are represented as continuous values by inserting them into a rotating frame.

In order to transform between the different reference frames, the Park and Clarke transforms are used. In this thesis, the transformation that will be used are power invariant, and are described in 2.1 and 2.2. Where x_{abc} , $x_{\alpha\beta\gamma}$, x_{dq0} are the variables represented in the *ABC*, *dq* and $\alpha\beta$ frames, respectively; $T_{\alpha\beta}$ is the power invariant Clarke transformation matrix and T_{dq} is the Park transformation matrix. If the three-phase system is balanced, the γ or 0 components are zero; as the system presented in Chapter 3 is balanced, the transformation matrices will not take these into account.

$$x_{\alpha\beta} = T_{\alpha\beta}x_{abc} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} x_a(t) \\ x_b(t) \\ x_c(t) \end{bmatrix} \quad (2.1)$$

$$x_{dq} = T_{dq}x_{abc} = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \end{bmatrix} \begin{bmatrix} x_a(t) \\ x_b(t) \\ x_c(t) \end{bmatrix} \quad (2.2)$$

As the dq frame is rotating at a certain angular speed ω , there exists a dependence between the *direct* and *quadrature* components of a variable represented in this frame. This can be seen in (2.3), where $f(x, u)$ and $g(x, u)$ are functions and $u \in \mathbb{R}^n$ is a set of variables.

$$\begin{aligned} d\{\dot{x}_{dq}\} &= f(x_{dq}, u) + \omega q\{x_{dq}\} \\ q\{\dot{x}_{dq}\} &= g(x_{dq}, u) - \omega d\{x_{dq}\} \end{aligned} \quad (2.3)$$

Droop controller

Droop controllers emulate the inertia of generators in order to control the inverter output voltage and frequency (Wu, Tang, Vasquez, & Guerrero, 2014). A typical design utilizes equations 2.4 and 2.5, which relates the output voltage and frequency, with the active and reactive power transmitted. The curves corresponding to these equations are seen in figure 2.4.

$$f = f_0 + K_{pf}(P_o - P) \quad (2.4)$$

$$V = V_0 + K_{qv}(Q_o - Q) \quad (2.5)$$

Where:

- f : Output frequency of the inverter.
- f_0 : Nominal frequency of the generator.
- P_o, Q_o : Set points of the active and reactive power.
- P, Q : Instantaneous active and reactive power transferred between the terminals of the inverter.
- K_{pf}, K_{qv} : Controllable parameters.

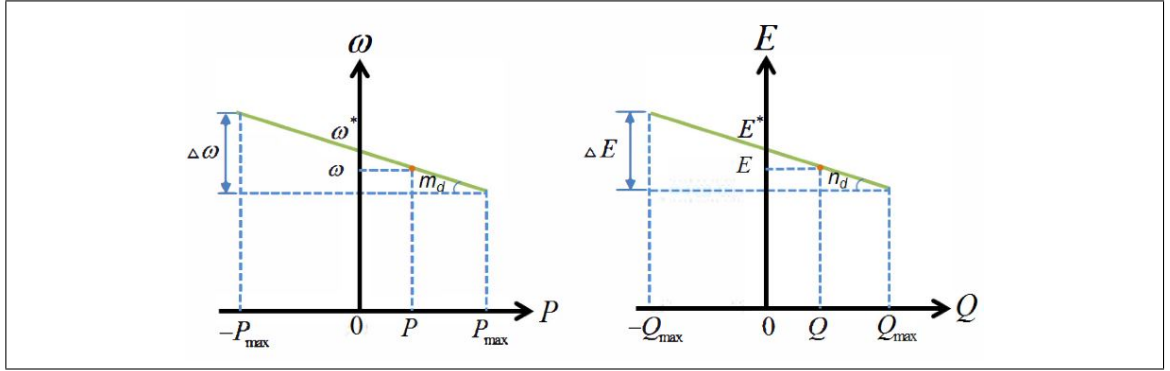


Figure 2.4. Power regulation characteristics with droop controller. (Wu et al., 2014)

Equations 2.6 and 2.7 show a Droop controller in the dq -frame. In these, the power to be transferred through the inverter is calculated with the difference between reference active (v_d^*) and reactive voltages (v_q^*), and the rms output voltage, calculated in equation 2.8

$$P^* = k_p(v_d^* - v_{rms}) \quad (2.6)$$

$$Q^* = k_q(v_q^* - v_{rms}) \quad (2.7)$$

$$v_{rms} = \sqrt{\frac{d\{v_{dq}\}^2 - q\{v_{dq}\}^2}{2}} \quad (2.8)$$

Power controller

A power controller utilizes active (P^*) and reactive (Q^*) power reference to calculate the output of the inverter. This is usually done with the Proportional-Integral controller seen on figure 2.5, in which the error between the power transferred by the inverter is used to calculate the inverter reference currents. The transfer function of each component of the controller is given by (2.9)

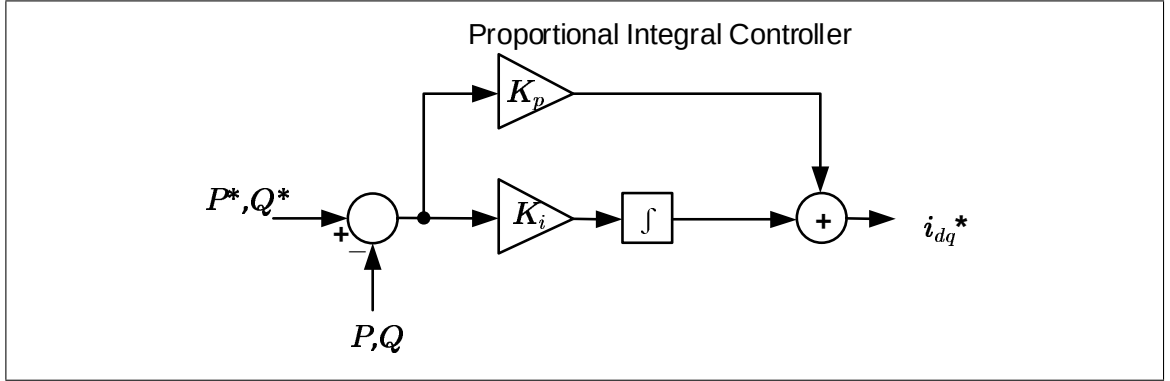


Figure 2.5. PI Power controller

$$H(s) = K_p + \frac{K_i}{s} \quad (2.9)$$

Equation 2.10 shows the definitions for active and reactive power in the dq -frame. Another configuration of power controller, utilizes these equations to calculate the active and reactive reference currents, from power references (P^*, Q^*) and the output voltage (v_{dq}) as in (2.11) and (2.12).

$$\begin{bmatrix} P \\ Q \end{bmatrix} = \begin{bmatrix} d\{v_{dq}\} & q\{v_{dq}\} \\ -q\{v_{dq}\} & d\{v_{dq}\} \end{bmatrix} \begin{bmatrix} d\{i_{dq}\} \\ q\{i_{dq}\} \end{bmatrix} \quad (2.10)$$

$$d\{i_{dq}^*\} = \frac{P^* d\{v_{dq}\} + Q^* q\{v_{dq}\}}{d\{v_{dq}\}^2 + q\{v_{dq}\}^2} \quad (2.11)$$

$$q\{i_{dq}^*\} = \frac{P^* q\{v_{dq}\} - Q^* d\{v_{dq}\}}{d\{v_{dq}\}^2 + q\{v_{dq}\}^2} \quad (2.12)$$

Current controller

Current controllers utilize a current reference as an input in order to calculate the necessary output voltage of the inverter. This is then given to a modulator, which controls the switching of the transistor in order to achieve this voltage in the terminal of the converter.

There are different types of current controllers, each one with their own advantages and disadvantages. One of these is the Proportional-Integral controller, which is designed in the dq -frame, is shown in figure 2.6 and the outputs are given by (2.13). As the dq -frame introduces a dependency (cross-coupling) between the active and reactive parts of a variable, in order to increase the performance of the controllers, decoupling terms are added at the end of the it.

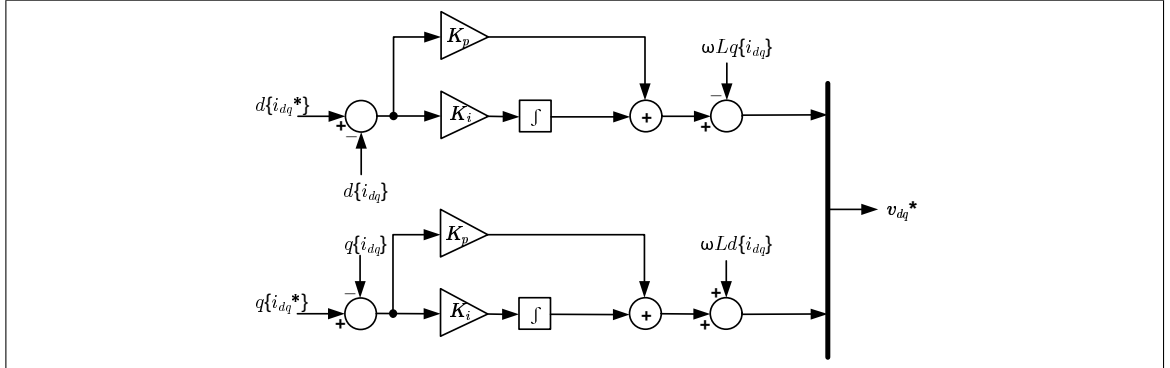


Figure 2.6. PI Current controller

$$H_{PI}(s) = K_p + \frac{K_i}{s} \quad (2.13)$$

Other commonly used is the Proportional-Resonant controller, this is designed in the abc -reference frame, it is shown in figure 2.7 and its transfer function is given by (2.14), this corresponds to a non-ideal PR, which is used in order to have a bigger bandwidth. This controller does not requires the variables to transform into another frame, and it has been demonstrated a better performance compared to the PI (Rhili, 2019; Hlali et al., 2019).

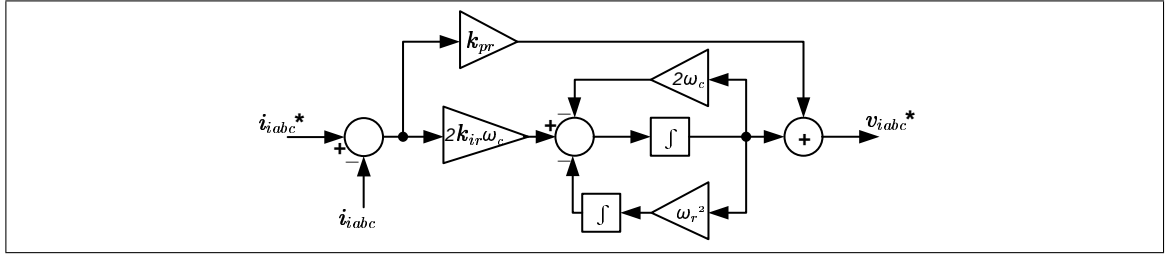


Figure 2.7. PR Current controller

$$H_{PR}(s) = \frac{v_{abc}^*}{i_{abc}^* - i_{abc}} = \frac{2K_{ir}\omega_c s}{s^2 + 2\omega_c s + \omega_r^2} + k_{pr} \quad (2.14)$$

2.3. Control Theory

Control theory is a field of mathematics that studies the control of dynamic systems. Through it, techniques has been developed to study these systems and determine various aspect of it, being stability one of the most important. If an operation point of a system is stable, it means that the energy and variables of the system converges to the operation point in time and, if subject to a small perturbation, the system remains in the surrounding of this point. Another important aspect that studies is the design of controllers, which forces the output of a system to follow a set point determined by an operator.

2.3.1. Stability

The dynamics of a system can be described by a set of first-order differential equations. This can be written as the vectorial representation (2.15), in which f is the set of differential equations that describe the system, x is the *state vector*, u is the *input vector*, y is the *output vector*, g is the set of equations of the output and t is time. The states variables of a system are internal variables of the system.

$$\dot{x} = f(x, u, t), y = g(x, u, t), \quad x \in \mathbb{R}^n, u \in \mathbb{R}^k, y \in \mathbb{R}^m \quad (2.15)$$

If the set of equations f and g are linear, the system is **linear**. If these are dependent of the time, it is called **time-variant**; if not, it is called **time-invariant**.

In "*Mathematical Control Theory: Deterministic Finite Dimensional Systems*", Sontag (1998) provides the following concepts that will be used in order to study the stability and controllability of the system that will be modelled.

Definition 5.5.2: Let Σ be a topological system and x_0 an equilibrium state. Then Σ is:

- **Locally asymptotically controllable to x_0** if for each neighbourhood $\mathcal{V}$ of x_0 there is some neighbourhood $\mathcal{W}$ of x_0 such that each $x \in \mathcal{W}$ can be asymptotically controlled to x_0 without leaving $\mathcal{V}$.
- **Globally asymptotically controllable to x_0** if it is locally asymptotically controllable and also every $x \in \mathcal{X}$ can be asymptotically controlled to x_0 .

(Sontag, 1998, p.224)

In the definition, $\mathcal{X} \subset \mathbb{R}^n$ is the set of all possible values for the states of the system.

The term *asymptotically controllable* means that the states of a system Σ can be driven to the desired states x_0 .

Proposition 5.5.6: Let $\Sigma = (A, B)$ be a continuous-time (respectively, discrete-time) time-invariant linear system. The following statements are then equivalent:

- 1 Σ is globally asymptotically controllable (to 0).
- 2 Σ is locally asymptotically controllable (to 0).
- 3 Each root of χ_u has negative real part (respectively, has magnitude less than 1).
- 4 There is some $F \in \mathbb{R}^{m \times n}$ such that $A + BF$ is Hurwitz (respectively, discrete-time Hurwitz).

(Sontag, 1998, p.226)

The term χ_u corresponds to the characteristic equation of the system Σ . In "*Power System Stability and Control*", (Kundur, 1993) states that the roots of a characteristic equation correspond to the eigenvalues of the A matrix of a time-invariant linear system. Therefore, the stability of a linear system can be determined by studying the eigenvalues of its A matrix.

Theorem 19: Assume that Σ is a time-invariant continuous-time system of class $\mathcal{C}^1$ (it is differentiable and its derivative is continuous),

$$\dot{x} = f(x, u)$$

with $X \subset \mathbb{R}^n$ and $U \subset \mathbb{R}^m$ open, and let (x_0, u_0) be an equilibrium pair, i.e.,

$$f(x_0, u_0) = 0$$

Assume that the linearization of Σ at (x_0, u_0) is asymptotically controllable. Then Σ is locally asymptotically controllable (to x_0). Moreover, there exists in that case a matrix

$F \in \mathbb{R}^{n \times m}$ such that the closed-loop system

$$\dot{x} = f_{cl}(x) := f(x, u_0 + F(x - x_0))$$

is locally asymptotically stable. (Sontag, 1998, p.247)

2.3.2. Linearization

Assume a non-linear time-variant system, described by

$$\dot{x}(t) = f(x, u), \quad y(t) = g(x, u), \quad x \in \mathbb{R}^n, u \in \mathbb{R}^k, y \in \mathbb{R}^m \quad (2.16)$$

If we determine an operation point (x_0, u_0, y_0) of the system and define

$$\Delta x(t) = x(t) - x_0, \Delta y(t) = y(t) - y_0, \Delta u(t) = u(t) - u_0$$

We can see that

$$\Delta \dot{x} = f(x_0 + \Delta x, u_0 + \Delta u) \quad (2.17)$$

$$\Delta y = g(x_0 + \Delta x, u_0 + \Delta u) - y_0 \quad (2.18)$$

If we make a Taylor expansion on (2.17) and (2.18), we get

$$\Delta \dot{x} = \frac{\partial f(x_0, u_0)}{\partial x} \Delta x + \frac{\partial f(x_0, u_0)}{\partial u} \Delta u + O(\|\Delta x\|^2) + O(\|\Delta u\|^2) \quad (2.19)$$

$$\Delta y = \frac{\partial g(x_0, u_0)}{\partial x} \Delta x + \frac{\partial g(x_0, u_0)}{\partial u} \Delta u + O(\|\Delta x\|^2) + O(\|\Delta u\|^2) \quad (2.20)$$

If we only maintain the first-order terms of (2.19) and (2.20), the system is **linearized** around an operation point. And it can be rewritten as

$$\Delta \dot{x} = A \Delta x + B \Delta u, \quad \Delta y = C \Delta x + D \Delta u \quad (2.21)$$

where the Jacobian matrices A , B , C and D are defined as

$$A = \frac{\partial f(x_0, u_0)}{\partial x} = \left[\left(\frac{\partial f_i(x_0, u_0)}{\partial x_j} \right)_{ij} \right] \in \mathbb{R}^{n \times n} \quad (2.22)$$

$$B = \frac{\partial f(x_0, u_0)}{\partial u} = \left[\left(\frac{\partial f_i(x_0, u_0)}{\partial u_j} \right)_{ij} \right] \in \mathbb{R}^{n \times k} \quad (2.23)$$

$$C = \frac{\partial g(x_0, u_0)}{\partial x} = \left[\left(\frac{\partial g_i(x_0, u_0)}{\partial x_j} \right)_{ij} \right] \in \mathbb{R}^{m \times n} \quad (2.24)$$

$$D = \frac{\partial g(x_0, u_0)}{\partial u} = \left[\left(\frac{\partial g_i(x_0, u_0)}{\partial u_j} \right)_{ij} \right] \in \mathbb{R}^{m \times k} \quad (2.25)$$

The linearization allows the analysis of a non-linear system around an operation point, this is valid as long as perturbations in Δx and Δu are small. If the perturbations are big, a new linearized model must be used in the new point of operation.

2.3.3. Participation factors

Participation factors are a measure of the participation of a state variable in a eigenvalue (or mode), and vice versa. It can be used to determine which states and, in consequence, which parameters are influencing the most critical poles of a system.

Given the following linear time-invariant system

$$\dot{x} = Ax + Bu, \quad x \in \mathbb{R}^n, u \in \mathbb{R}^m, A \in \mathbb{R}^{n \times n}, B \in \mathbb{R}^{n \times m}$$

equation (2.26) describes the participation factor p_{ki} as the participation of the k^{th} state into the i^{th} eigenvalue of A , as how much the eigenvalue λ_i changes to a change in the element a_{kk} of the matrix A .

In order to easily calculate the participation factors, (2.27) is used, in which ϕ_R is the matrix that contains the right eigenvectors of the A matrix. A condition is established in (2.28), in which the sum of all the participation factors of the i -th mode, must be 1.

$$p_{ki} = \frac{\partial \lambda_i}{\partial a_{kk}} \quad (2.26)$$

$$PF = \left[(\phi_R^{-1})_{ij}^T \cdot (\phi_R)_{ij} \right]_{ij}, \quad i = 1, 2, \dots, N, \quad j = 1, 2, \dots, N \quad (2.27)$$

$$\sum_{k=1}^N p_{ki} = 1, \quad \forall j = 1, 2, \dots, N \quad (2.28)$$

CHAPTER 3. SMALL-SIGNAL MODEL

This chapter will address the description and the modelling of a distributed network with distributed generation and a Soft-Open-Point. First the system analyzed will be presented and divided into different subsystems; then, each of these will be linearized, if needed, and modelled into its state-space representation, which then will be integrated in a single unified model; finally, a set of simulations will be presented, that will demonstrate the validity of the model.

3.1. System

The system is shown in figure 3.1. It is a distribution network that contains one substation with two feeders, there is a passive load connected to each feeder; the feeders are connected to a Soft-Open-Point in the Point of Common Coupling (PCC_{SOP}); the SOP correspond to a Back-to-Back inverter and a LCL filter, with a damping resistor connected in series to the filter capacitors; distributed generator (DG) is connected to one of the loads. The impedance of the lines are taken into account. The side of the system that does not have DG is labelled as Side A, and the side with DG is labelled Side B.

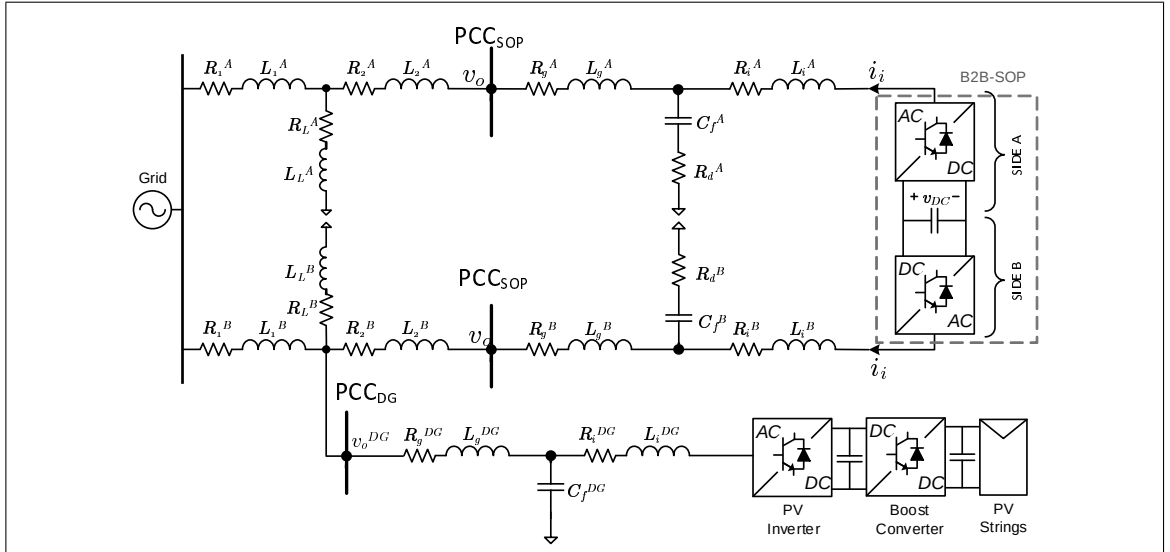


Figure 3.1. Model of the distribution network

The DG source in this case is represented as a PV string, but it can be any technology such as a diesel generator or wind turbines. The DG source is connected to a DC/DC converter in order to maintain a constant voltage in its DC-link; which is connected to the network through an inverter.

The system possesses a series of simplifications in order to make a general model that can be easily modified to suit any particular network. It is assumed that the three-phases are balanced, the PLL and the higher level control will not be taken into account and the DG will be modelled as a constant power supply.

Two modes of operation are determined for the SOP, in **normal mode** the inverter in side B is tasked to maintain the voltage in the DC-Link and the inverter in side A controls the active component of the voltage in its side. Meanwhile, in **inverse mode** the inverter in side A maintains the DC-link and the inverter in side B controls the active voltage in its side. Independent of the mode, both inverters also maintains the quadrature component of the voltage in their respective side.

The control loop of side A and side B, in normal mode, are shown in figures 3.2 and 3.3. Both control loops utilizes a droop controller to calculate the power transference between each inverter and the feeder; a power controller to calculate the reference currents needed and a PR current controller; this is chosen because of its higher efficiency and, if the system is unbalanced, additional control loops can be added to solve this, which give their own references that must be added in the *abc*-frame, thus a PR must be used after this sum. The loops only differ in how each droop controller calculates the active power reference.

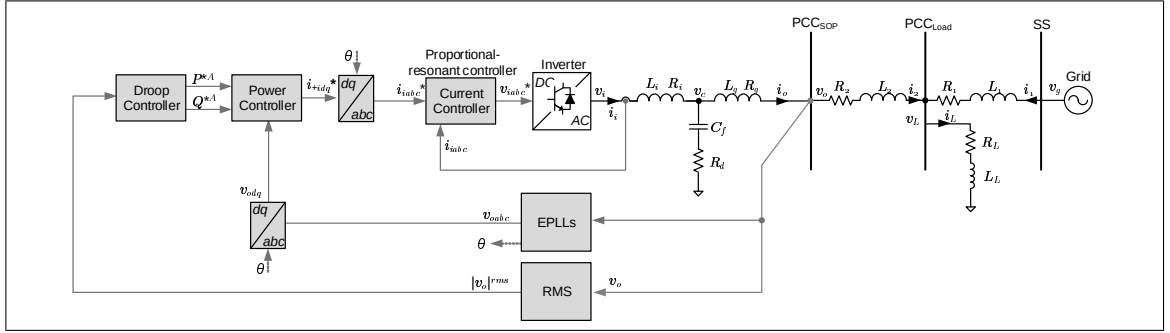


Figure 3.2. Side A control loop

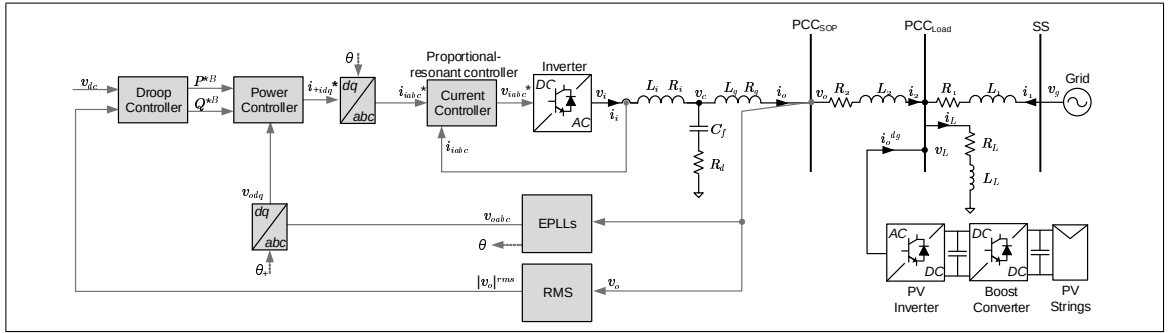


Figure 3.3. Side B control loop

3.2. Subsystem Models

In order to simplify the modelling of the system, this will be divided into the subsystems in figure 3.4. Each of these will be modelled in a state-space representation and then will be unified into a sole system. This form of modelling allows for greater flexibility, as the system is modularized, individual changes can be easily integrated into the model.

The system will be modelled for the normal mode of operation, and section 3.3.1 will address the necessary changes to model in inverse mode. As the components and controllers are almost the same on both sides, most of the subsystems will have the same state-space representation.

The inverters and LCL filter variables are defined in their own local dq reference frames, defined by the angle in the PCC_{SOP} of their respective side. The subsystems are integrated in a common DQ reference frame defined by the angle in the SS bus-bar. The matrices T^j are used to transform the variables from the local reference frames dq to

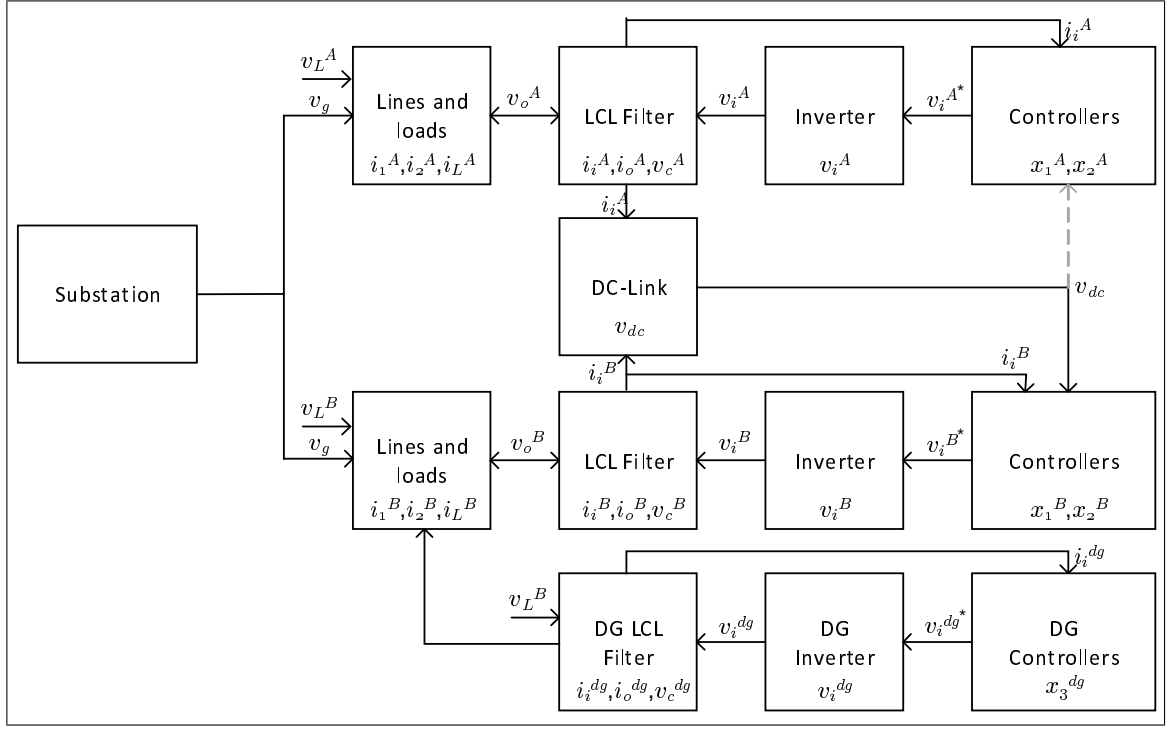


Figure 3.4. System represented as various subsystems

the common reference frame DQ . The definition of the matrix is given in (3.1), where $\delta^j \in \{A, B, dg\}$ is the angle in the PCC_{SOP} ; and the transformation in (3.2).

$$T^j = \begin{bmatrix} \cos(\delta^j) & \sin(\delta^j) \\ -\sin(\delta^j) & \cos(\delta^j) \end{bmatrix} \quad (3.1)$$

$$x_{DQ}^j = T^j x_{dq}^j \quad j \in \{A, B, dg\} \quad (3.2)$$

Both the droop and power controllers utilized are non-linear, so the system will be studied around an operation point, through a linearization. The operation point is determined by the expected values of the variables in normal conditions and desired references. The state vector in the operation point will be labeled as x_0 and the input vector as u_0 ; and the linearized states will be preceded by Δ .

3.2.1. Droop and Power Controllers

The droop controllers are (2.6) and (2.7), which are suited to the normal mode of operation. The inverter in side A follows (3.3) in order to maintain the direct voltage of its side; and the inverter in side B maintains the DC-link voltage through (3.4) and both inverters maintains the quadrature voltage of their respective side through (3.5).

$$P^{*A} = k_p^A \cdot (v_p^* - v_{o.rms}^A) \quad (3.3)$$

$$P^{*B} = k_p^B \cdot (v_{dc}^* - v_{dc}) \quad (3.4)$$

$$Q^{*j} = k_q^j \cdot (v_q^{*j} - v_{o.rms}^j), j \in \{A, B\} \quad (3.5)$$

Where

$$v_{o.rms}^j = \sqrt{\frac{(d\{v_{o.dq}^j\})^2 + (q\{v_{o.dq}^j\})^2}{3}}, \quad j \in \{A, B\}$$

The power controller to be utilized for both inverters in the SOP is given by (2.11) and (2.12). It calculates the output reference currents of the inverters with the power references given by the droop controller and the voltage in the PCC_{SOP}.

$$d\{i_{i.dq}^{*j}\} = \frac{P^* d\{v_{o.dq}^j\} + Q^* q\{v_{o.dq}^j\}}{d\{v_{o.dq}^j\}^2 + q\{v_{o.dq}^j\}^2}, \quad j \in \{A, B\} \quad (3.6)$$

$$q\{i_{i.dq}^{*j}\} = \frac{P^* q\{v_{o.dq}^j\} - Q^* d\{v_{o.dq}^j\}}{d\{v_{o.dq}^j\}^2 + q\{v_{o.dq}^j\}^2}, \quad j \in \{A, B\} \quad (3.7)$$

The jacobian matrices J^A and J^B will be obtained from the linearization of the controllers. The matrices are given by (3.8) and (3.9), the left part of the sum is the influence of the droop controller over the reference current and the right part of the sum is the influence of the power controller.

$$\begin{aligned}
J^A &= \left. \frac{di_{i.dq}^{*A}}{dv_{o.dq}^A} \right|_{(x_0, u_0)} \\
&= \left(\frac{\partial i_{i.dq}^{*A}}{\partial v_{o.dq}^A} + \frac{\partial i_{i.dq}^{*A}}{\partial [P^{*A} Q^{*A}]} \frac{\partial [P^{*A} Q^{*A}]}{\partial v_{o.dq}^A} \right) \Big|_{(x_0, u_0)} \\
&= J_{v_{o.dq}^{*A}}^{i_{i.dq}^A} + J_{PQ^A}^{i_{i.dq}^{*A}} \cdot J_{v_{o.dq}^A}^{PQ^A}
\end{aligned} \tag{3.8}$$

$$\begin{aligned}
J^B &= \left. \frac{di_{i.dq}^{*B}}{d[v_{o.dq}^B v_{dc}]} \right|_{(x_0, u_0)} \\
&= \left(\frac{\partial i_{i.dq}^{*B}}{\partial [v_{o.dq}^B v_{dc}]} + \frac{\partial i_{i.dq}^{*B}}{\partial [P^{*B} Q^{*B}]} \frac{\partial [P^{*B} Q^{*B}]}{\partial [v_{o.dq}^B v_{dc}]} \right) \Big|_{(x_0, u_0)} \\
&= J_{[v_{o.dq}^{*B} v_{dc}]}^{i_{i.dq}^B} + J_{PQ^B}^{i_{i.dq}^{*B}} \cdot J_{[v_{o.dq}^B v_{dc}]}^{PQ^B}
\end{aligned} \tag{3.9}$$

With the Jacobian matrices, the linearized system of the droop and power controllers are given by (3.10) and (3.11). The Jacobian matrix of side B is then separated into two submatrices.

$$[\Delta i_{i.dq}^{*A}] = J^A [\Delta v_{o.dq}^A] \tag{3.10}$$

$$[\Delta i_{i.dq}^{*B}] = J^B \begin{bmatrix} \Delta v_{o.dq}^B \\ \Delta v_{dc} \end{bmatrix} = J_{vo}^B [\Delta v_{o.dq}^B] + J_{dc}^B [\Delta v_{dc}] \tag{3.11}$$

The terms in equations (3.8) and (3.9) are detailed in (3.12) - (3.18). In all of these equations $j \in \{A, B\}$.

$$J^B = \begin{bmatrix} J_{vo}^B & J_{dc}^B \end{bmatrix} \quad J_{vo}^B = \begin{bmatrix} J_{(1)}^B & J_{(2)}^B \end{bmatrix} \quad J_{dc}^B = \begin{bmatrix} J_{(3)}^B \end{bmatrix} \tag{3.12}$$

$$J_{v_{o.dq}^{*A}}^{i_{i.dq}^A} = \begin{bmatrix} J_{v_{o.dq}^A}^{i_{i.dq}^A} & J_{v_{o.dq}^A}^{i_{i.dq}^A} \\ J_{v_{o.dq}^A}^{i_{i.dq}^A} & J_{v_{o.dq}^A}^{i_{i.dq}^A} \end{bmatrix} \quad J_{v_{o.dq}^{*B}}^{i_{i.dq}^B} = \begin{bmatrix} J_{v_{o.dq}^B}^{i_{i.dq}^B} & J_{v_{o.dq}^B}^{i_{i.dq}^B} & 0 \\ J_{v_{o.dq}^B}^{i_{i.dq}^B} & J_{v_{o.dq}^B}^{i_{i.dq}^B} & 0 \end{bmatrix} \tag{3.13}$$

$$J_{v_{o.dq}^j}^{i_{i.dq}^j} = -J_{v_{o.dq}^j}^{i_{i.dq}^j} = \frac{P^{*j}(q\{v_{o.dq}^j\}^2 - d\{v_{o.dq}^j\}^2) - 2Q^{*j}d\{v_{o.dq}^j\}q\{v_{o.dq}^j\}}{(d\{v_{o.dq}^j\}^2 + q\{v_{o.dq}^j\}^2)^2} \quad (3.14)$$

$$J_{v_{o.dq}^j}^{i_{i.dq}^j} = J_{v_{o.dq}^j}^{i_{i.dq}^j} = \frac{Q^{*j}(d\{v_{o.dq}^j\}^2 - q\{v_{o.dq}^j\}^2) - 2P^{*j}d\{v_{o.dq}^j\}q\{v_{o.dq}^j\}}{(d\{v_{o.dq}^j\}^2 + q\{v_{o.dq}^j\}^2)^2} \quad (3.15)$$

$$J_{v_{o.dq}^A}^{PQ^A} = \frac{-1}{\sqrt{3(d\{v_{o.dq}^A\}^2 + q\{v_{o.dq}^A\}^2)}} \begin{bmatrix} k_p^A \cdot d\{v_{o.dq}^A\} & k_p^A \cdot q\{v_{o.dq}^A\} \\ k_q^A \cdot d\{v_{o.dq}^A\} & k_q^A \cdot q\{v_{o.dq}^A\} \end{bmatrix} \quad (3.16)$$

$$J_{[v_{o.dq}^B \ v_{dc}]}^{PQ^B} = \begin{bmatrix} 0 & 0 & -B_0^{-1} \\ -\frac{k_q^B \cdot d\{v_{o.dq}^B\}}{B_0} & -\frac{k_q^B \cdot q\{v_{o.dq}^B\}}{B_0} & 0 \end{bmatrix} \quad (3.17)$$

$$B_0 = \sqrt{3(d\{v_{o.dq}^B\}^2 + q\{v_{o.dq}^B\}^2)}$$

$$J_{PQ^j}^{i_{i.dq}^j} = \frac{1}{d\{v_{o.dq}^j\}^2 + q\{v_{o.dq}^j\}^2} \begin{bmatrix} d\{v_{o.dq}^j\} & q\{v_{o.dq}^j\} \\ q\{v_{o.dq}^j\} & -d\{v_{o.dq}^j\} \end{bmatrix} \quad (3.18)$$

3.2.2. Current controller

The current controller of each inverter of the SOP corresponds to the proportional-resonant controller seen on figure 3.5, and its transfer function is given by (3.19). Even though this is designed in the abc -reference frame, it can be analysed in the dq -reference frame.

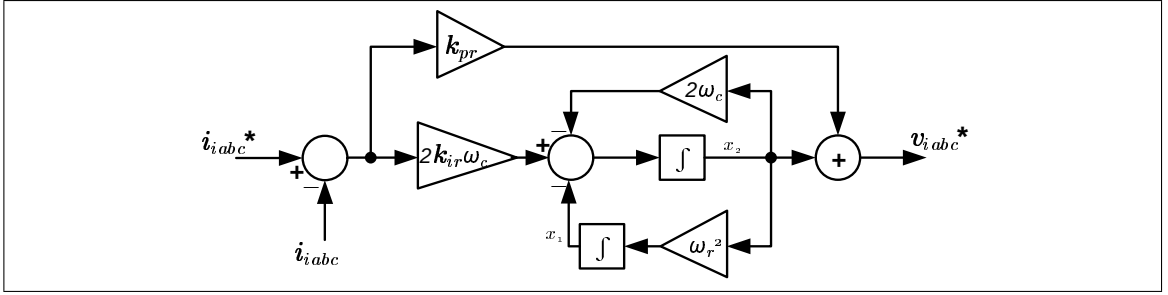


Figure 3.5. Proportional-Resonant controller of the inverter of the SOP.

$$H(s) = \frac{v_{abc}^{j*}}{i_{abc}^{j*} - i_{abc}^*} = \frac{2K_{ir}^j \omega_c^j s}{s^2 + 2\omega_c^j s + \omega_r^2} + k_{pr}^j, j \in \{A, B\} \quad (3.19)$$

If the outputs of the integrals are used as the state variables of the controller, the following equations are obtained.

$$v_{i.dq}^{*j} = x_{2.dq}^j + k_{pr}^j (i_{i.dq}^{*j} - i_{i.dq}^j), \quad j \in \{A, B\} \quad (3.20)$$

$$\dot{x}_{1.dq}^j = (\omega_r^j)^2 x_{2.dq}^j, \quad j \in \{A, B\} \quad (3.21)$$

$$\dot{x}_{2.dq}^j = -x_{1.dq}^j - 2\omega_c^j x_{2.dq}^j + 2k_{ir}^j \omega_c^j (i_{i.dq}^{*j} - i_{i.dq}^j), \quad j \in \{A, B\} \quad (3.22)$$

With (3.20), (3.21) and (3.22), the state-space model of the PR controller is made

$$\begin{bmatrix} \dot{\Delta x}_{1.dq}^j \\ \dot{\Delta x}_{2.dq}^j \end{bmatrix} = A_{CC}^j \begin{bmatrix} \Delta x_{1.dq}^j \\ \Delta x_{2.dq}^j \end{bmatrix} + B_{CC}^j \begin{bmatrix} \Delta i_{i.dq}^{*j} \\ \Delta i_{i.dq}^j \end{bmatrix}, \quad j \in \{A, B\} \quad (3.23)$$

$$\begin{bmatrix} \Delta v_{i.dq}^{*j} \end{bmatrix} = C_{CC}^j \begin{bmatrix} x_{1.dq}^j \\ x_{2.dq}^j \end{bmatrix} + D_{CC}^j \begin{bmatrix} v_{i.dq}^{*j} \\ v_{i.dq}^j \end{bmatrix}, \quad j \in \{A, B\} \quad (3.24)$$

Where

$$A_{CC}^j = \begin{bmatrix} 0 & \omega & (\omega_r^j)^2 & 0 \\ -\omega & 0 & 0 & (\omega_r^j)^2 \\ -1 & 0 & -2\omega_c^j & \omega \\ 0 & -1 & -\omega & -2\omega_c^j \end{bmatrix}, \quad j \in \{A, B\} \quad (3.25)$$

$$B_{CC}^j = 2k_{ir}^j \omega_c^j \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}, \quad j \in \{A, B\} \quad (3.26)$$

$$C_{CC}^j = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad j \in \{A, B\} \quad (3.27)$$

$$D_{CC}^j = k_{pr}^j \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}, \quad j \in \{A, B\} \quad (3.28)$$

3.2.3. LCL Filter

An LCL filter is chosen due to its better attenuation of the switching harmonics produced in the inverters. A damping resistor is added in series to the capacitor in order to provide greater stability to the system (Kahlane et al., 2014).

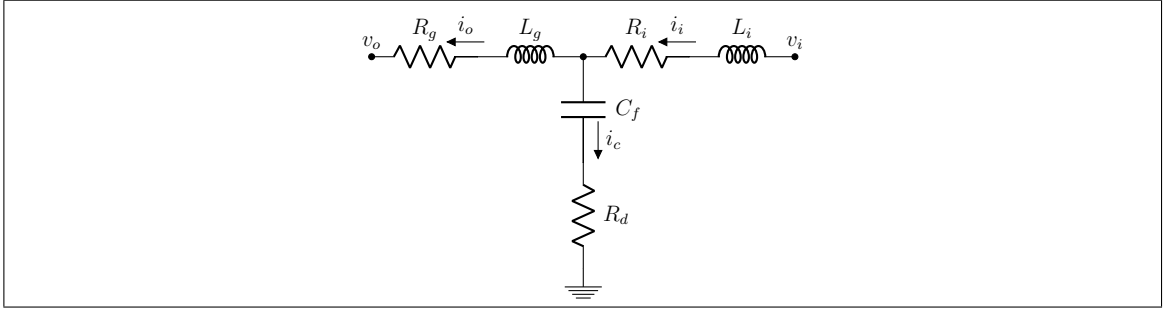


Figure 3.6. LCL filter with damping resistor

The equations of the LCL filter in figure 3.6 are given by:

$$i_{i,dq}^j = -\frac{R_i^j}{L_i^j} i_{i,dq}^j + \frac{1}{L_i^j} (v_{i,dq}^j - v_{c,dq}^j), \quad j \in \{A, B\} \quad (3.29)$$

$$i_{o,dq}^j = -\frac{R_g^j}{L_g^j} i_{o,dq}^j + \frac{1}{L_g^j} (v_{c,dq}^j - v_{o,dq}^j), \quad j \in \{A, B\} \quad (3.30)$$

$$v_{c,dq}^j = \left(\frac{1}{sC_f^j} + R_d^j \right) i_{c,dq}^j, \quad j \in \{A, B\} \quad (3.31)$$

$$i_{c,dq}^j = i_{i,dq}^j - i_{o,dq}^j, \quad j \in \{A, B\} \quad (3.32)$$

It must be taken into account that s is the Laplace operator that represents a derivative with respect of time

$$s = \frac{\partial}{\partial t} \quad (3.33)$$

If (3.32) is used in (3.31) and then we get the derivatives from (3.29), (3.30) and (3.31)

$$i_{i,dq}^j = -\frac{R_i^j}{L_i^j} i_{i,dq}^j + \frac{1}{L_i^j} (v_{i,dq}^j), \quad j \in \{A, B\} \quad (3.34)$$

$$\dot{i}_{o.dq}^j = -\frac{R_g^j}{L_g^j} i_{o.dq}^j + \frac{1}{L_g^j} (v_{c.dq}^j - v_{o.dq}^j), \quad j \in \{A, B\} \quad (3.35)$$

$$\dot{v}_{c.dq}^j = R_d^j (\dot{i}_{i.dq}^j - \dot{i}_{o.dq}^j) + \frac{1}{C_f^j} (i_{i.dq}^j - i_{o.dq}^j), \quad j \in \{A, B\} \quad (3.36)$$

From (3.34), (3.35) and (3.36), we can get the state-space representation of the LCL filter with damping resistor.

$$\begin{bmatrix} \Delta \dot{i}_{i.dq}^j \\ \Delta \dot{i}_{o.dq}^j \\ \Delta \dot{v}_{c.dq}^j \end{bmatrix} = A_{LCL}^j \begin{bmatrix} \Delta i_{i.dq}^j \\ \Delta i_{o.dq}^j \\ \Delta v_{c.dq}^j \end{bmatrix} + B_{LCL}^j \begin{bmatrix} \Delta v_{i.dq}^j \\ \Delta v_{o.dq}^j \end{bmatrix}, \quad j \in \{A, B\} \quad (3.37)$$

$$A_{LCL}^j = \begin{bmatrix} -\frac{R_i^j}{L_i^j} & \omega & 0 & 0 \\ -\omega & -\frac{R_i^j}{L_i^j} & 0 & 0 \\ 0 & 0 & -\frac{R_g^j}{L_g^j} & \omega \\ 0 & 0 & -\omega & -\frac{R_g^j}{L_g^j} \\ \frac{1}{C_f^j} - \frac{R_d^j R_i^j}{L_i^j} & R_d^j \omega & -\frac{1}{C_f^j} + \frac{R_d^j R_g^j}{L_g^j} & -R_d^j \omega \\ -R_d^j \omega & \frac{1}{C_f^j} - \frac{R_d^j R_i^j}{L_i^j} & R_d^j \omega & -\frac{1}{C_f^j} + \frac{R_d^j R_g^j}{L_g^j} \end{bmatrix}, \quad j \in \{A, B\} \quad (3.38)$$

$$\begin{bmatrix} -\frac{1}{L_i^j} & 0 \\ 0 & -\frac{1}{L_i^j} \\ \frac{1}{L_g^j} & 0 \\ 0 & \frac{1}{L_g^j} \\ -\frac{R_d^j}{L_i^j} - \frac{R_d^j}{L_g^j} & \omega \\ -\omega & -\frac{R_d^j}{L_i^j} - \frac{R_d^j}{L_g^j} \end{bmatrix}, \quad j \in \{A, B\}$$

$$B_{LCL}^j = \begin{bmatrix} \frac{1}{L_i^j} & 0 & 0 & 0 \\ 0 & \frac{1}{L_i^j} & 0 & 0 \\ 0 & 0 & -\frac{1}{L_g^j} & 0 \\ 0 & 0 & 0 & -\frac{1}{L_g^j} \\ \frac{R_d^j}{L_i^j} & 0 & \frac{R_d^j}{L_g^j} & 0 \\ 0 & \frac{R_d^j}{L_i^j} & 0 & \frac{R_d^j}{L_g^j} \end{bmatrix}, \quad j \in \{A, B\} \quad (3.39)$$

3.2.4. Inverter

The inverters of the SOP and the DG are represented using an average first-order model, this model describes the inverter as a first-order function, in which the output voltage is related to an input reference voltage through a delay (Queval & Ohsaki, 2012).

The transfer function of the inverter is given by (3.40), where T_s is a delay that takes into account the computation time of the controllers, the PWM and the delay of the inverter.

$$H(s) = \frac{v_{i.dq}^j}{v_{i.dq}^{*j}} = \frac{1}{1 + 1.5T_s^j s}, \quad j \in \{A, B, dg\} \quad (3.40)$$

Through (3.40) the derivative of the output voltage of the inverter can be obtained:

$$\dot{v}_i^j = \frac{1}{1.5T_s^j} (v_{i.dq}^j - v_{i.dq}^{*j}), \quad j \in \{A, B, dg\} \quad (3.41)$$

Finally, from (3.41), the state-space model of the inverter is given by (3.42).

$$\begin{bmatrix} \Delta \dot{v}_{i.dq}^j \end{bmatrix} = A_{delay}^j \begin{bmatrix} v_{i.dq}^j \end{bmatrix} + B_{delay}^j \begin{bmatrix} v_{i.dq}^{*j} \end{bmatrix}, \quad j \in \{A, B, dg\} \quad (3.42)$$

Where

$$A_{delay}^j = \begin{bmatrix} -\frac{1}{1.5T_s^j} & \omega \\ -\omega & -\frac{1}{1.5T_s^j} \end{bmatrix}, \quad j \in \{A, B, dg\} \quad (3.43)$$

$$B_{delay}^j = \begin{bmatrix} \frac{1}{1.5T_s^j} & 0 \\ 0 & \frac{1}{1.5T_s^j} \end{bmatrix}, \quad j \in \{A, B, dg\} \quad (3.44)$$

3.2.5. Distributed Generation (DG)

The control loop of the DG is shown in figure 3.7, it utilizes the same Power controller as the SOP (equations (3.6) and (3.7)) and a PI current controller (figure 2.6), operating in the dq reference frame. As PV strings are being used as an example, the active power reference is given by a maximum power point tracker (MPPT) and the reactive power reference is set to 0. For simplicity, the MPPT and the DC/DC converter are not considered in the model, eliminatin specific behaviours introduced by the PV system and representing the DG as a generic technology.

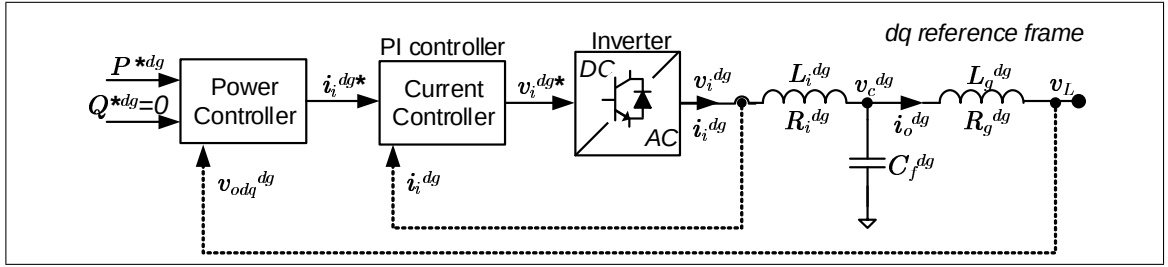


Figure 3.7. Control loop of the DG

The power controller equations are given by

$$d\{i_{idg}^{*dg}\} = \frac{P^{*dg}d\{v_{L.dq}^B\} + Q^{*dg}q\{v_{L.dq}^B\}}{(d\{v_{L.dq}^B\})^2 + (q\{v_{L.dq}^B\})^2} \quad (3.45)$$

$$q\{i_{idg}^{*dg}\} = \frac{P^{*dg}q\{v_{L.dq}^B\} - Q^{*dg}d\{v_{L.dq}^B\}}{(d\{v_{L.dq}^B\})^2 + (q\{v_{L.dq}^B\})^2} \quad (3.46)$$

As in the SOP, the power controller is non-linear, so it is linearized and expressed in a the Jacobian matrix (3.48)

$$J^{dg} = \frac{di_{idg}^{*dg}}{dv_{L.dq}^B} = \begin{bmatrix} J_{11}^{dg} & J_{12}^{dg} \\ J_{21}^{dg} & J_{22}^{dg} \end{bmatrix} \quad (3.47)$$

Where the values of the matrix are

$$J_{11}^{dg} = -J_{22}^{dg} = \frac{P^{*dg}((q\{v_{L.dq}^B\})^2 - (d\{v_{L.dq}^B\})^2) - 2Q^{*dg}d\{v_{L.dq}\}q\{v_{L.dq}\}}{((d\{v_{L.dq}\})^2 + (q\{v_{L.dq}\})^2)^2} \quad (3.48)$$

$$J_{12}^{dg} = J_{21}^{dg} = \frac{Q^{*dg}((d\{v_{L.dq}^B\})^2 - (q\{v_{L.dq}^B\})^2) - 2P^{*dg}d\{v_{L.dq}\}q\{v_{L.dq}\}}{((d\{v_{L.dq}\})^2 + (q\{v_{L.dq}\})^2)^2} \quad (3.49)$$

And the small-signal model of the power controller is expressed in (3.50).

$$\begin{bmatrix} \Delta i_{i.dq}^{dg} \end{bmatrix} = J^{dg} \begin{bmatrix} \Delta v_{L.dq}^B \end{bmatrix} \quad (3.50)$$

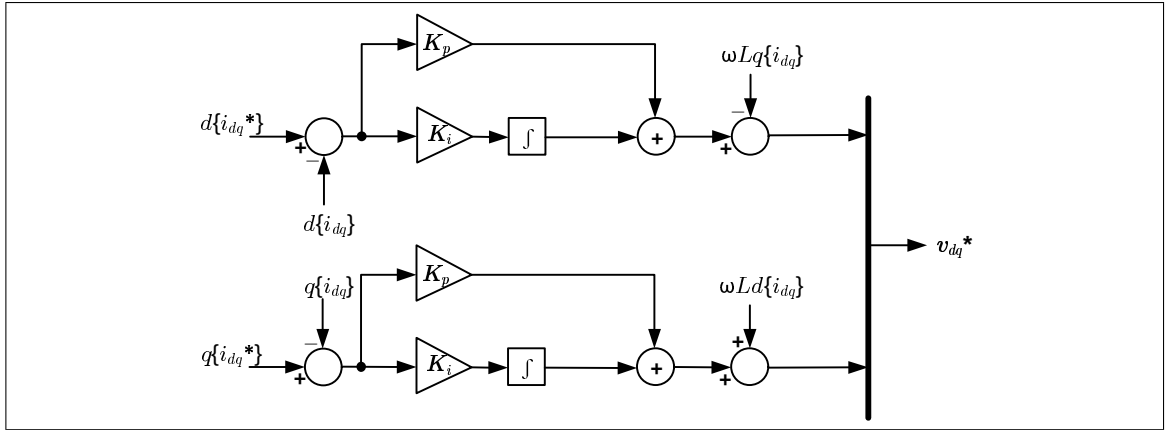


Figure 3.8. PI current controller of the DG

The current controller of the DG is the standard decoupled PI controller seen in figure 3.8, where the outputs are given by (3.51) and (3.52). By establishing the output of the integral as the state $x_{3.dq}$, the model of the current controller is given by (3.53) and (3.54).

$$d\{v_{dq}^*\} = (K_p + \frac{K_i}{s})(d\{i_{dq}^*\} - d\{i_{dq}\}) - \omega L q\{i_{dq}\} \quad (3.51)$$

$$q\{v_{dq}^*\} = (K_p + \frac{K_i}{s})(q\{i_{dq}^*\} - q\{i_{dq}\}) + \omega L d\{i_{dq}\} \quad (3.52)$$

$$\begin{bmatrix} \Delta \dot{x}_{3.dq}^{dg} \end{bmatrix} = A_{CC}^{dg} \begin{bmatrix} \Delta x_{3.dq}^{dg} \end{bmatrix} + B_{CC}^{dg} \begin{bmatrix} \Delta \dot{l}_{i.dq}^{*dg} \\ \Delta \dot{l}_{i.dq}^{dg} \end{bmatrix} \quad (3.53)$$

$$\begin{bmatrix} \Delta v_{i.dq}^{*dg} \end{bmatrix} = C_{CC}^{dg} \begin{bmatrix} \Delta x_{3.dq}^{dg} \end{bmatrix} + D_{CC}^{dg} \begin{bmatrix} \Delta \dot{l}_{i.dq}^{*dg} \\ \Delta \dot{l}_{i.dq}^{dg} \end{bmatrix} \quad (3.54)$$

Where the matrices are

$$A_{CC}^{dg} = \begin{bmatrix} 0 & \omega \\ -\omega & 0 \end{bmatrix} \quad B_{CC}^{dg} = k_i^{dg} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \quad (3.55)$$

$$C_{CC}^{dg} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad D_{CC}^{dg} = \begin{bmatrix} k_p^{dg} & 0 & -k_p^{dg} & -\omega L_i^{dg} \\ 0 & k_p^{dg} & \omega L_i^{dg} & -k_p^{dg} \end{bmatrix} \quad (3.56)$$

Finally, the LCL filter of the DG is identical, except for the damping resistor. The equations of the filter are the same and are given by (3.38) and (3.39), but the value of R_d^j , $j \in \{A, B\}$ is set to 0. So the state-space representation of the LCL filter of the DG is given by (3.57), (3.58) and (3.59)

$$\begin{bmatrix} \Delta \dot{l}_{i.dq}^{dg} \\ \Delta \dot{l}_{o.dq}^{dg} \\ \Delta \dot{v}_{c.dq}^{dg} \end{bmatrix} = A_{LCL}^{dg} \begin{bmatrix} \Delta l_{i.dq}^{dg} \\ \Delta l_{o.dq}^{dg} \\ \Delta v_{c.dq}^{dg} \end{bmatrix} + B_{LCL}^{dg} \begin{bmatrix} \Delta v_{i.dq}^{dg} \\ \Delta v_{L.dq}^B \end{bmatrix} \quad (3.57)$$

$$A_{LCL}^{dg} = \begin{bmatrix} -\frac{R_i^{dg}}{L_i^{dg}} & \omega & 0 & 0 & -\frac{1}{L_i^{dg}} & 0 \\ -\omega & -\frac{R_i^{dg}}{L_i^{dg}} & 0 & 0 & 0 & -\frac{1}{L_i^{dg}} \\ 0 & 0 & -\frac{R_g^{dg}}{L_g^{dg}} & \omega & \frac{1}{L_g^{dg}} & 0 \\ 0 & 0 & -\omega & -\frac{R_g^{dg}}{L_g^{dg}} & 0 & \frac{1}{L_g^{dg}} \\ \frac{1}{C_f^{dg}} & 0 & -\frac{1}{C_f^{dg}} & 0 & 0 & \omega \\ 0 & \frac{1}{C_f^{dg}} & 0 & -\frac{1}{C_f^{dg}} & -\omega & 0 \end{bmatrix} \quad (3.58)$$

$$B_{LCL}^{dg} = \begin{bmatrix} \frac{1}{L_i^{dg}} & 0 & 0 & 0 \\ 0 & \frac{1}{L_i^{dg}} & 0 & 0 \\ 0 & 0 & -\frac{1}{L_g^{dg}} & 0 \\ 0 & 0 & 0 & -\frac{1}{L_g^{dg}} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (3.59)$$

3.2.6. Passive lines and load

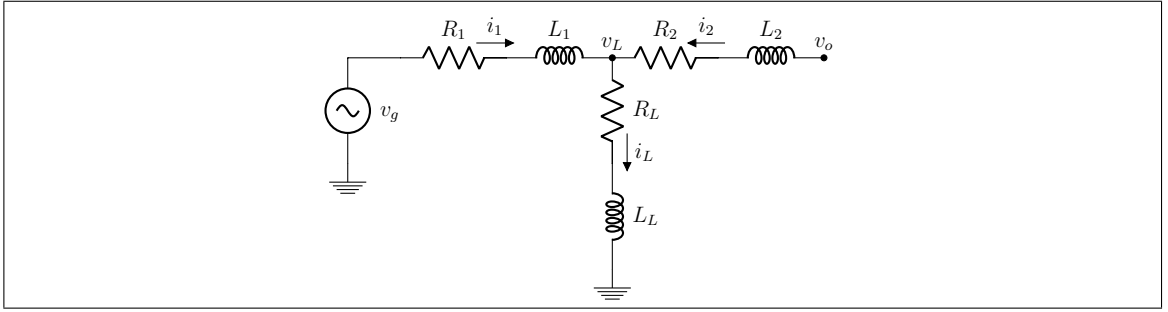


Figure 3.9. Load and lines connected to the feeder of side A

Figure 3.9 shows the impedance of the load and the lines connected to the feeder of the side A of substation, although the feeder in side B possess an extra connection to the DG, this does not directly influences the states of this subsystem, so the model will be the same. From the figure, we can obtain the following equations:

$$i_{1.DQ}^j = \frac{1}{R_1^j + sL_1^j} (v_{g.DQ}^j - v_{L.DQ}^j), \quad j \in \{A, B\} \quad (3.60)$$

$$i_{2.DQ}^j = \frac{1}{R_2^j + sL_2^j} (v_{o.DQ}^j - v_{L.DQ}^j), \quad j \in \{A, B\} \quad (3.61)$$

$$i_{L.DQ}^j = \frac{1}{R_L^j + sL_L^j} v_{L.DQ}^j, \quad j \in \{A, B\} \quad (3.62)$$

From equations (3.60), (3.61) and (3.62) we can get the derivatives of the currents

$$\dot{i}_{1.DQ}^j = -\frac{R_1^j}{L_1^j} i_{1.DQ}^j + \frac{1}{L_1^j} (v_{g.DQ}^j - v_{L.DQ}^j), \quad j \in \{A, B\} \quad (3.63)$$

$$\dot{i}_{2.DQ}^j = -\frac{R_2^j}{L_2^j} i_{2.DQ}^j + \frac{1}{L_2^j} (v_{o.DQ}^j - v_{L.DQ}^j), \quad j \in \{A, B\} \quad (3.64)$$

$$\dot{i}_{L.DQ}^j = -\frac{R_L^j}{L_L^j} i_{L.DQ}^j + \frac{1}{L_L^j} v_{L.DQ}^j, \quad j \in \{A, B\} \quad (3.65)$$

Equations (3.63), (3.64) and (3.65) are now written in matricial form.

$$\begin{bmatrix} \Delta \dot{i}_{1.DQ}^j \\ \Delta \dot{i}_{2.DQ}^j \\ \Delta \dot{i}_{L.DQ}^j \end{bmatrix} = A_{RL}^j \begin{bmatrix} \Delta i_{1.DQ}^j \\ \Delta i_{2.DQ}^j \\ \Delta i_{L.DQ}^j \end{bmatrix} + B_{RL}^j \begin{bmatrix} \Delta v_{g.DQ}^j \\ \Delta v_{o.DQ}^j \\ \Delta v_{L.DQ}^j \end{bmatrix}, \quad j \in \{A, B\} \quad (3.66)$$

$$A_{RL}^j = \begin{bmatrix} -\frac{R_1^j}{L_1^j} & \omega & 0 & 0 & 0 & 0 \\ -\omega & -\frac{R_1^j}{L_1^j} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{R_2^j}{L_2^j} & \omega & 0 & 0 \\ 0 & 0 & -\omega & -\frac{R_2^j}{L_2^j} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{R_L^j}{L_L^j} & \omega \\ 0 & 0 & 0 & 0 & -\omega & -\frac{R_L^j}{L_L^j} \end{bmatrix}, \quad j \in \{A, B\} \quad (3.67)$$

$$B_{RL}^j = \begin{bmatrix} \frac{1}{L_1^j} & 0 & 0 & 0 & -\frac{1}{L_1^j} & 0 \\ 0 & \frac{1}{L_1^j} & 0 & 0 & 0 & -\frac{1}{L_1^j} \\ 0 & 0 & \frac{1}{L_2^j} & 0 & -\frac{1}{L_2^j} & 0 \\ 0 & 0 & 0 & \frac{1}{L_2^j} & 0 & -\frac{1}{L_2^j} \\ 0 & 0 & 0 & 0 & \frac{1}{L_L^j} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{L_L^j} \end{bmatrix}, \quad j \in \{A, B\} \quad (3.68)$$

3.2.7. Virtual capacitance

So far, the voltages of the PCC_{SOP} ($v_{o,dq}^{\{A,B\}}$) and the loads ($v_{L,dq}^{\{A,B\}}$) are used as inputs of various subsystems, but their dynamics are not defined by a first-order differential equation. Pogaku, Prodanovic, & Green (2007) in *"Modelling, Analysis and Testing of Autonomous Operation of an Inverter-Based Microgrid"* utilize large non-existent resistors between nodes without defined voltages and ground, in order to incorporate their voltages into the model. The value of these **virtual resistors** are very large, so that it does not affect on the dynamics of the system.

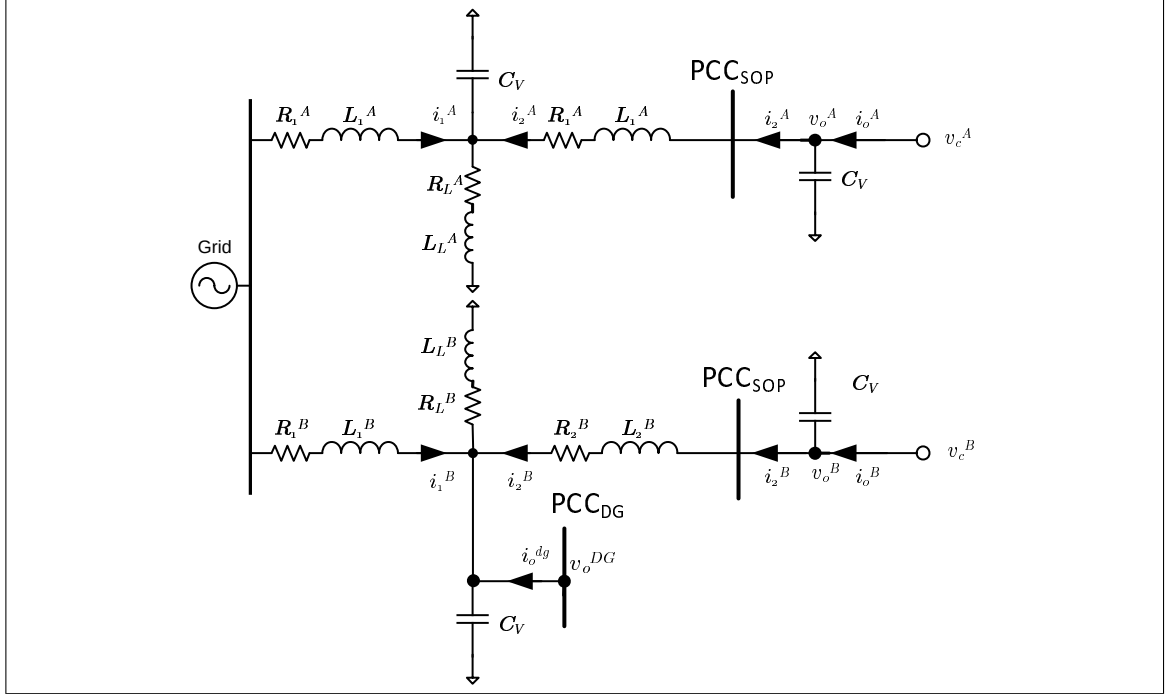


Figure 3.10. Virtual capacitors connected to the system

Instead of using virtual resistors, **virtual capacitors** are used to define first-order differential equations, where the voltages of the nodes are necessary, but are not defined. These capacitors are taken from the modelling the transmission lines, where a small parasitic capacitor is connected between the lines and ground (Steer, 2010). The value of these capacitors must be very small, so they do not affect the system. The connections are

shown in figure 3.10 and the dynamics equations voltages of $v_{o.DQ}^j$ and $v_{L.DQ}^j, j \in \{A, B\}$ are given by (3.69), (3.70) and (3.71).

$$\dot{v}_{o.DQ}^j = \frac{i_{o.DQ}^j - i_{2.DQ}^j}{C_V}, \quad j \in \{A, B\} \quad (3.69)$$

$$\dot{v}_{L.DQ}^A = \frac{i_{2.DQ}^A + i_{g.DQ}^A - i_{L.DQ}^A}{C_V} \quad (3.70)$$

$$\dot{v}_{L.DQ}^B = \frac{i_{o.DQ}^{dg} + i_{2.DQ}^A + i_{g.DQ}^A - i_{L.DQ}^A}{C_V} \quad (3.71)$$

Then, the equations are written in matricial form

$$\begin{bmatrix} \dot{v}_{o.DQ}^A \\ \dot{v}_{L.DQ}^A \end{bmatrix} = A_{VC}^A \begin{bmatrix} v_{o.DQ}^A \\ v_{L.DQ}^A \end{bmatrix} + B_{VC}^A \begin{bmatrix} i_{1.DQ}^A \\ i_{2.DQ}^A \\ i_{o.DQ}^A \\ i_{L.DQ}^A \end{bmatrix} \quad (3.72)$$

$$\begin{bmatrix} \dot{v}_{o.DQ}^B \\ \dot{v}_{L.DQ}^B \end{bmatrix} = A_{VC}^B \begin{bmatrix} v_{o.DQ}^B \\ v_{L.DQ}^B \end{bmatrix} + B_{VC}^B \begin{bmatrix} i_{1.DQ}^B \\ i_{2.DQ}^B \\ i_{o.DQ}^B \\ i_{L.DQ}^B \\ i_{o.DQ}^{dg} \end{bmatrix} \quad (3.73)$$

$$A_{VC}^j = \begin{bmatrix} 0 & \omega & 0 & 0 \\ -\omega & 0 & 0 & 0 \\ 0 & 0 & 0 & \omega \\ 0 & 0 & -\omega & 0 \end{bmatrix}, \quad j \in \{A, B\} \quad (3.74)$$

$$B_{VC}^A = \frac{1}{C_V} \begin{bmatrix} 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & -1 \end{bmatrix} \quad (3.75)$$

$$B_{VC}^B = \frac{1}{C_V} \begin{bmatrix} 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 1 \end{bmatrix} \quad (3.76)$$

3.2.8. DC-Link

The DC-Link capacitor voltage dynamics depends on the current that circulates between the inverters of the SOP and the feeders, as seen in figure. Therefore, the dynamic of the DC-Link voltage is established in (3.77).

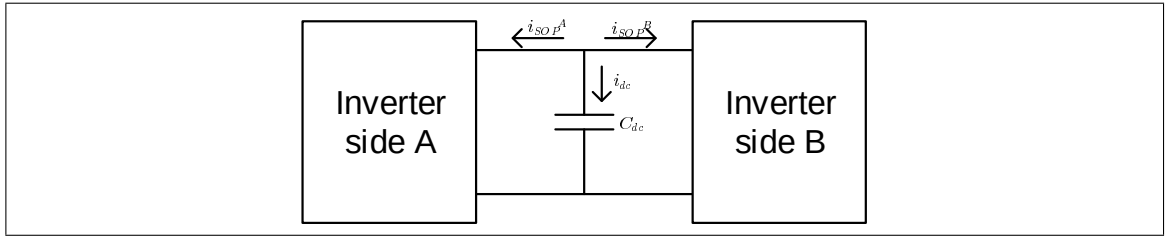


Figure 3.11. DC-Link currents.

$$\dot{v}_{dc} = \frac{1}{C} i_{dc} = -\frac{1}{C_{dc}} (i_{SOP}^A + i_{SOP}^B) \quad (3.77)$$

(Queval & Ohsaki, 2012), utilize a transformation in order to express the voltage of the DC-Link from the direct components of the currents, which gives (3.78). From it, the state-space representation is obtained in (3.79).

$$\dot{v}_{dc} = -\frac{3}{4C_{dc}} (d\{i_{i.dq}^A\} + d\{i_{i.dq}^B\}) \quad (3.78)$$

$$\begin{bmatrix} \Delta \dot{v}_{dc} \end{bmatrix} = A_{dc} \begin{bmatrix} \Delta v_{dc} \end{bmatrix} + B_{dc} \begin{bmatrix} \Delta i_{i.dq}^A \\ \Delta i_{i.dq}^B \end{bmatrix} \quad (3.79)$$

$$A_{dc} = \begin{bmatrix} 0 \end{bmatrix} \quad (3.80)$$

$$B_{dc} = \begin{bmatrix} -\frac{3}{4C_{dc}} & 0 & -\frac{3}{4C_{dc}} & 0 \end{bmatrix} \quad (3.81)$$

3.3. System Small-Signal Model

The state variables of the system are organized in table 3.1 and the subsystems models are summarized in table 3.2, in both tables $j \in \{A, B\}$.

Table 3.1. State Variables of the System

	Description	State variable
SOP	Output of integral 1 (PR controller)	$\Delta x_{1.dq}^j$
	Output of integral 2 (PR controller)	$\Delta x_{2.dq}^j$
	Output voltage of inverter	$\Delta v_{i.dq}^j$
	LCL filter input current	$\Delta i_{i.dq}^j$
	LCL filter output current	$\Delta i_{o.dq}^j$
	LCL filter capacitor voltage	$\Delta v_{c.dq}^j$
Lines and Load	Current from substation to PCC_{Load}	$\Delta i_{1.DQ}^j$
	Current from SOP to PCC_{Load}	$\Delta i_{2.DQ}^j$
	Current of the passive load	$\Delta i_{L.DQ}^j$
	Output voltage of SOP	$\Delta v_{o.DQ}^j$
	Voltage of the passive load	$\Delta v_{L.DQ}^j$
DG	Output of integral (PI controller)	$\Delta x_{3.dq}^{dg}$
	Output voltage of inverter	$\Delta v_{i.dq}^{dg}$
	LCL filter input current	$\Delta i_{i.dq}^{dg}$
	LCL filter output current	$\Delta i_{o.dq}^{dg}$
	LCL filter capacitor voltage	$\Delta v_{c.dq}^{dg}$
DC Link	DC-Link capacitor voltage	Δv_{dc}

Table 3.2. State-space representation of the subsystems

Subsystem	State-space representation
Droop and power controllers	$[\Delta i_{i.dq}^{*A}] = J^A [\Delta v_{o.dq}^A]$
	$[\Delta i_{i.dq}^{*B}] = J_{vo}^B [\Delta v_{o.dq}^B] + J_{dc}^B [\Delta v_{dc}]$
Current controllers	$\begin{bmatrix} \Delta \dot{x}_{1.dq}^j \\ \Delta \dot{x}_{2.dq}^j \end{bmatrix} = A_{CC}^j \begin{bmatrix} \Delta x_{1.dq}^j \\ \Delta x_{2.dq}^j \end{bmatrix} + B_{CC}^j \begin{bmatrix} \Delta i_{i.dq}^{*j} \\ \Delta i_{i.dq}^j \end{bmatrix}$
	$[\Delta v_{o.dq}^{*j}] = C_{CC}^j \begin{bmatrix} \Delta x_{1.dq}^j \\ \Delta x_{2.dq}^j \end{bmatrix} + D_{CC}^j \begin{bmatrix} \Delta i_{i.dq}^{*j} \\ \Delta i_{i.dq}^j \end{bmatrix}$
Inverter model	$[\Delta \dot{v}_{o.dq}^j] = A_{delay}^j [v_{i.dq}^j] + B_{delay}^j [v_{i.dq}^{*j}]$
LCL Filter	$\begin{bmatrix} \Delta \dot{i}_{i.dq}^j \\ \Delta \dot{i}_{o.dq}^j \\ \Delta \dot{v}_{c.dq}^j \end{bmatrix} = A_{LCL}^j \begin{bmatrix} \Delta i_{i.dq}^j \\ \Delta i_{o.dq}^j \\ \Delta v_{c.dq}^j \end{bmatrix} + B_{LCL}^j \begin{bmatrix} \Delta v_{i.dq}^j \\ \Delta v_{o.dq}^j \end{bmatrix}$
Loads and lines	$\begin{bmatrix} \Delta \dot{i}_{1.DQ}^j \\ \Delta \dot{i}_{2.DQ}^j \\ \Delta \dot{i}_{L.DQ}^j \end{bmatrix} = A_{RL}^j \begin{bmatrix} \Delta i_{1.DQ}^j \\ \Delta i_{2.DQ}^j \\ \Delta i_{L.DQ}^j \end{bmatrix} + B_{RL}^j \begin{bmatrix} \Delta v_{g.DQ}^j \\ \Delta v_{o.DQ}^j \\ \Delta v_{L.DQ}^j \end{bmatrix}$
Virtual Capacitors	$\begin{bmatrix} \dot{v}_{o.DQ}^A \\ \dot{v}_{L.DQ}^A \end{bmatrix} = A_{VC}^A \begin{bmatrix} v_{o.DQ}^A \\ v_{L.DQ}^A \end{bmatrix} + B_{VC}^A \begin{bmatrix} i_{1.DQ}^A \\ i_{2.DQ}^A \\ i_{o.DQ}^A \\ i_{L.DQ}^A \end{bmatrix}$
	$\begin{bmatrix} \dot{v}_{o.DQ}^B \\ \dot{v}_{L.DQ}^B \end{bmatrix} = A_{VC}^B \begin{bmatrix} v_{o.DQ}^B \\ v_{L.DQ}^B \end{bmatrix} + B_{VC}^B \begin{bmatrix} i_{1.DQ}^B \\ i_{2.DQ}^B \\ i_{o.DQ}^B \\ i_{L.DQ}^B \\ i_{o.DQ}^{dg} \end{bmatrix}$
DG Power controller	$[\Delta i_{i.dq}^{dg}] = J^{dg} [\Delta v_{L.dq}^B]$
DG Current controller	$[\Delta \dot{x}_{3.dq}^{dg}] = A_{CC}^{dg} [\Delta x_{3.dq}^{dg}] + B_{CC}^{dg} \begin{bmatrix} \Delta i_{i.dq}^{*dg} \\ \Delta i_{i.dq}^{dg} \end{bmatrix}$
	$[\Delta v_{i.dq}^{*dg}] = C_{CC}^{dg} [\Delta x_{3.dq}^{dg}] + D_{CC}^{dg} \begin{bmatrix} \Delta i_{i.dq}^{*dg} \\ \Delta i_{i.dq}^{dg} \end{bmatrix}$
DG Inverter model	$[\Delta \dot{v}_{o.dq}^{dg}] = A_{delay}^{dg} [v_{i.dq}^{dg}] + B_{delay}^{dg} [v_{i.dq}^{*dg}]$
DG LCL Filter	$\begin{bmatrix} \Delta \dot{i}_{i.dq}^{dg} \\ \Delta \dot{i}_{o.dq}^{dg} \\ \Delta \dot{v}_{c.dq}^{dg} \end{bmatrix} = A_{LCL}^{dg} \begin{bmatrix} \Delta i_{i.dq}^{dg} \\ \Delta i_{o.dq}^{dg} \\ \Delta v_{c.dq}^{dg} \end{bmatrix} + B_{LCL}^{dg} \begin{bmatrix} \Delta v_{i.dq}^{dg} \\ \Delta v_{L.dq}^{dg} \end{bmatrix}$

The states are grouped in the vectors Δx^A , Δx^B and Δx^{dg} , which are described in (3.82) and (3.83); then the final state vector ΔX is formed from these and stated in (3.84).

$$\Delta x^j = \begin{bmatrix} \Delta x_{1.dq}^j & \Delta x_{2.dq}^j & \Delta v_{i.dq}^j & \Delta i_{i.dq}^j & \Delta i_{o.dq}^j & \Delta v_{c.dq}^j \\ \Delta i_{1.DQ}^j & \Delta i_{2.DQ}^j & \Delta i_{L.DQ}^j & \Delta v_{o.DQ}^j & \Delta v_{L.DQ}^j \end{bmatrix}', \quad j \in \{A, B\} \quad (3.82)$$

$$\Delta x^{dg} = \begin{bmatrix} \Delta x_{3.dq}^{dg} & \Delta v_{i.dq}^{dg} & \Delta i_{i.dq}^{dg} & \Delta i_{o.dq}^{dg} & v_{c.dq}^{dq} \end{bmatrix}' \quad (3.83)$$

$$\Delta X = \begin{bmatrix} \Delta x^A & \Delta x^B & \Delta x^{dg} & \Delta v_{dc} \end{bmatrix}' \quad (3.84)$$

From the equations of the subsystems in table 3.2 and the state vectors, the state-space representation of each side of the system are described in (3.85) and (3.86). Equations (3.87)-(3.91) details the dynamics of side A; and (3.92) - (3.95), the dynamics of side B.

$$\Delta \dot{x}^A = A^A \Delta x^A + B^A \begin{bmatrix} \Delta v_{g.DQ} \end{bmatrix} \quad (3.85)$$

$$\Delta \dot{x}^B = A^B \Delta x^B + B^B \begin{bmatrix} \Delta v_{g.DQ} \\ \Delta v_{dc} \end{bmatrix} \quad (3.86)$$

$$A^A = \begin{bmatrix} A_{CC}^A & 0_{4 \times 2} & B_{CC(2)}^A & 0_{4 \times 10} & A_{auxA1} \\ B_{delay}^A C_{CC}^A & A_{delay}^A & B_{delay}^A D_{CC(2)}^A & 0_{2 \times 10} & A_{auxA2} \\ 0_{6 \times 2} & B_{LCL(1)}^A & A_{LCL}^A & 0_{6 \times 6} & A_{auxA3} \\ 0_{6 \times 4} & 0_{6 \times 4} & 0_{6 \times 4} & A_{RL}^A & B_{RL(2,3)}^A \\ 0_{4 \times 8} & B_{VC(1)}^A T^A & 0_{4 \times 2} & B_{VC(2,3,4)}^A & A_{VC}^A \end{bmatrix} \quad (3.87)$$

$$A_{auxA1} = \begin{bmatrix} B_{CC(1)}^A J^A (T^A)^{-1} & 0_{4 \times 2} \end{bmatrix} \quad (3.88)$$

$$A_{auxA2} = \begin{bmatrix} B_{delay}^A D_{CC(1)}^A J^A (T^A)^{-1} & 0_{2 \times 2} \end{bmatrix} \quad (3.89)$$

$$A_{auxA3} = \begin{bmatrix} B_{LCL(2)}^A (T^A)^{-1} & 0_{6 \times 2} \end{bmatrix} \quad (3.90)$$

$$B^A = \begin{bmatrix} 0_{12 \times 2} \\ B_{RL(1)}^A \\ 0_{4 \times 2} \end{bmatrix} \quad (3.91)$$

$$A^B = \begin{bmatrix} A_{CC}^B & 0_{4 \times 2} & B_{CC(2)}^B & 0_{4 \times 10} \\ B_{delay}^B C_{CC}^B & A_{delay}^B & B_{delay}^B D_{CC(2)}^B & 0_{2 \times 10} \\ 0_{6 \times 4} & B_{LCL(1)}^B & A_{LCL}^B & 0_{6 \times 6} \\ 0_{6 \times 4} & 0_{6 \times 4} & 0_{6 \times 4} & A_{RL}^B \\ 0_{4 \times 8} & B_{VC(1)}^B T^B & 0_{4 \times 2} & B_{VC(2,3,4)}^B \\ 0_{2 \times 5} & 0_{2 \times 5} & 0_{2 \times 5} & 0_{2 \times 5} \\ 0_{2 \times 5} & 0_{2 \times 5} & 0_{2 \times 5} & 0_{2 \times 5} \\ 0_{6 \times 5} & 0_{6 \times 5} & 0_{6 \times 5} & 0_{6 \times 5} \end{bmatrix} \quad (3.92)$$

$$\begin{bmatrix} B_{CC(1)}^B J_{dq}(T^B)^{-1} & 0_{4 \times 4} & 0_{4 \times 4} & 0_{2 \times 4} \\ B_{delay}^B D_{CC(1)}^B J_{dq}(T^B)^{-1} & 0_{2 \times 4} & 0_{2 \times 4} & 0_{2 \times 4} \\ B_{LCL(2)}^B (T^B)^{-1} & 0_{6 \times 4} & 0_{6 \times 4} & 0_{6 \times 4} \\ B_{RL(2)}^B & B_{RL(3)}^B & 0_{6 \times 5} & 0_{6 \times 5} \\ A_{VC}^B & 0_{4 \times 6} & B_{VC(5)}^B T^{dg} & 0_{4 \times 2} \\ B_{CC(1)}^{dg} J^{dg}(T^{dg})^{-1} & A_{CC}^{dg} & 0_{2 \times 2} & A_{auxB1} \\ B_{delay}^{dg} D_{CC(1)}^{dg} J^{dg}(T^{dg})^{-1} & B_{delay}^{dg} C_{CC}^{dg} & A_{delay}^{dg} & A_{auxB2} \\ B_{LCL(2)}^{dg} (T^{dg})^{-1} & 0_{6 \times 2} & B_{LCL(1)}^{dg} & A_{LCL}^{dg} \end{bmatrix}$$

$$A_{auxB1} = \begin{bmatrix} B_{CC(2)}^{dg} & 0_{2 \times 4} \end{bmatrix} \quad (3.93)$$

$$A_{auxB2} = \begin{bmatrix} B_{delay}^{dg} D_{CC(2)}^{dg} & 0_{2 \times 4} \end{bmatrix} \quad (3.94)$$

$$B^B = \begin{bmatrix} 0_{4 \times 2} & B_{CC(1)}^B J_{DC} \\ 0_{2 \times 2} & B_{delay}^B D_{CC(1)}^B J_{DC} \\ 0_{6 \times 2} & 0_{6 \times 2} \\ B_{RL(1)}^B & 0_{6 \times 2} \\ 0_{14 \times 2} & 0_{14 \times 2} \end{bmatrix} \quad (3.95)$$

Finally, the state-space representation of the system is described in (3.96). And the matrices are detailed through (3.97) - (3.103).

$$\Delta \dot{X} = A \Delta X + B \left[\Delta v_{g-DQ} \right] \quad (3.96)$$

$$A = \begin{bmatrix} A^A & 0_{22 \times 32} & 0_{22 \times 1} \\ 0_{32 \times 22} & A^B & X^0 \\ X^1 & X^2 & X^3 \end{bmatrix} \quad (3.97)$$

$$X^0 = \begin{bmatrix} B_{CC(1,2)}^B J_{dc}^B \\ B_{delay}^B D_{CC(1)}^B J_{dc}^B \\ 0_{26 \times 2} \end{bmatrix} \quad (3.98)$$

$$X^1 = \begin{bmatrix} 0_{1 \times 6} & B_{dc(1)}^B & 0_{1 \times 14} \end{bmatrix} \quad (3.99)$$

$$X^2 = \begin{bmatrix} 0_{1 \times 6} & B_{dc(2)}^B & 0_{1 \times 24} \end{bmatrix} \quad (3.100)$$

$$X^3 = 0 \quad (3.101)$$

$$B = \begin{bmatrix} B^A \\ B^{B2} \\ 0_{1 \times 2} \end{bmatrix} \quad (3.102)$$

$$B^{B2} = \begin{bmatrix} 0_{2 \times 12} \\ B_{RL(1,2)}^B \\ 0_{2 \times 14} \end{bmatrix} \quad (3.103)$$

3.3.1. Inverse mode model

The stability on the inverse mode of operation will also be proven, as the SOP must be able to operate in both modes. As stated previously, in this mode the inverter in side A is tasked to maintain the voltage of the DC-link and the inverter of side B maintains a voltage reference in its PCC. In order to do this, equations (3.3) and (3.4) are swapped and the new equations for the active power of the droop controllers are (3.104) and (3.105). The rest of the controllers and elements of the circuits remain the same.

$$P^{*A.inv} = k_p^A \cdot (v_{dc} - v_{dc}^*) \quad (3.104)$$

$$P^{*B.inv} = k_p^B \cdot (v_p^* - v_{o_{rms}}^A) \quad (3.105)$$

As the only change in the control system was the swapping of equations, this is reflected by switching the Jacobian matrices, then the linearized system of the droop and power controller for each side are given by (3.106) and (3.107).

$$\begin{bmatrix} \Delta i_{i.dq}^{*A} \end{bmatrix} = J^{A.inv} \begin{bmatrix} \Delta v_{o.dq}^A \\ \Delta v_{dc} \end{bmatrix} = J_{vo}^{A.inv} \begin{bmatrix} \Delta v_{o.dq}^A \end{bmatrix} + J_{dc}^{A.inv} \begin{bmatrix} \Delta v_{dc} \end{bmatrix} \quad (3.106)$$

$$\begin{bmatrix} \Delta i_{i.dq}^{*B} \end{bmatrix} = J^{B.inv} \begin{bmatrix} \Delta v_{o.dq}^B \end{bmatrix} \quad (3.107)$$

Where the Jacobian of side A is separated as in (3.108). And the matrices are given by (3.109) - (3.116).

$$J^{A.inv} = \begin{bmatrix} J_{vo}^{A.inv} & J_{dc}^{A.inv} \end{bmatrix} \quad (3.108)$$

$$\begin{aligned}
J^{A.inv} &= \frac{di_{i.dq}^{*A}}{d[v_{o.dq}^A v_{dc}]} \Big|_{(x_0, u_0, y_0)} \\
&= \left(\frac{\partial i_{i.dq}^{*A}}{\partial [v_{o.dq}^A v_{dc}]} + \frac{\partial i_{i.dq}^{*A}}{\partial [P^{*A} Q^{*A}]} \frac{\partial [P^{*A} Q^{*A}]}{\partial [v_{o.dq}^A v_{dc}]} \right) \Big|_{(x_0, u_0, y_0)} \\
&= J_{[v_{o.dq}^{*A} v_{dc}]}^{i_{i.dq}^{*A}} + J_{PQ^A}^{i_{i.dq}^{*A}} \cdot J_{[v_{o.dq}^A v_{dc}]}^{PQ^A}
\end{aligned} \tag{3.109}$$

$$\begin{aligned}
J^{B.inv} &= \frac{di_{i.dq}^{*B}}{dv_{o.dq}^B} \Big|_{(x_0, u_0, y_0)} \\
&= \left(\frac{\partial i_{i.dq}^{*B}}{\partial v_{o.dq}^B} + \frac{\partial i_{i.dq}^{*B}}{\partial [P^{*B} Q^{*B}]} \frac{\partial [P^{*B} Q^{*B}]}{\partial v_{o.dq}^B} \right) \Big|_{(x_0, u_0, y_0)} \\
&= J_{v_{o.dq}^{*B}}^{i_{i.dq}^{*B}} + J_{PQ^B}^{i_{i.dq}^{*B}} \cdot J_{v_{o.dq}^B}^{PQ^B}
\end{aligned} \tag{3.110}$$

$$J_{v_{o.dq}^{*A}}^{i_{i.dq}^A} = \begin{bmatrix} J_{v_{o.dq}^{*A}11}^{i_{i.dq}^A} & J_{v_{o.dq}^{*A}12}^{i_{i.dq}^A} & 0 \\ J_{v_{o.dq}^{*A}21}^{i_{i.dq}^A} & J_{v_{o.dq}^{*A}22}^{i_{i.dq}^A} & 0 \end{bmatrix} \quad J_{v_{o.dq}^{*B}}^{i_{i.dq}^B} = \begin{bmatrix} J_{v_{o.dq}^{*B}11}^{i_{i.dq}^B} & J_{v_{o.dq}^{*B}12}^{i_{i.dq}^B} \\ J_{v_{o.dq}^{*B}21}^{i_{i.dq}^B} & J_{v_{o.dq}^{*B}22}^{i_{i.dq}^B} \end{bmatrix} \tag{3.111}$$

$$J_{v_{o.dq}^{*j}11}^{i_{i.dq}^j} = -J_{v_{o.dq}^{*j}22}^{i_{i.dq}^j} = \frac{P^{*j}((q\{v_{o.dq}^j\})^2 - (d\{v_{o.dq}^j\})^2) - 2Q^{*j}d\{v_{o.dq}^j\}q\{v_{o.dq}^j\}}{((d\{v_{o.dq}^j\})^2 + (q\{v_{o.dq}^j\})^2)^2}, \quad j \in \{A, B\} \tag{3.112}$$

$$J_{v_{o.dq}^{*j}12}^{i_{i.dq}^j} = J_{v_{o.dq}^{*j}21}^{i_{i.dq}^j} = \frac{Q^{*j}((d\{v_{o.dq}^j\})^2 - (q\{v_{o.dq}^j\})^2) - 2P^{*j}d\{v_{o.dq}^j\}q\{v_{o.dq}^j\}}{((d\{v_{o.dq}^j\})^2 + (q\{v_{o.dq}^j\})^2)^2}, \quad j \in \{A, B\} \tag{3.113}$$

$$J_{[v_{o.dq}^A v_{dc}]}^{PQ^A} = \begin{bmatrix} 0 & 0 & -A_0^{-1} \\ -\frac{k_q^A \cdot d\{v_{o.dq}^A\}}{A_0} & -\frac{k_q^A \cdot q\{v_{o.dq}^A\}}{A_0} & 0 \end{bmatrix} \tag{3.114}$$

$$A_0 = \sqrt{3(d\{v_{o.dq}^B\})^2 + q\{v_{o.dq}^B\}^2}$$

$$J_{v_{o.dq}^B}^{PQB} = \frac{-1}{\sqrt{3(d\{v_{o.dq}^B\}^2 + q\{v_{o.dq}^B\}^2)}} \begin{bmatrix} k_p^B \cdot d\{v_{o.dq}^B\} & k_p^B \cdot q\{v_{o.dq}^B\} \\ k_q^B \cdot d\{v_{o.dq}^B\} & k_q^B \cdot q\{v_{o.dq}^B\} \end{bmatrix} \quad (3.115)$$

$$J_{PQ^j}^{i_{i.dq}^j} = \frac{1}{d\{v_{o.dq}^j\}^2 + q\{v_{o.dq}^j\}^2} \begin{bmatrix} d\{v_{o.dq}^j\} & q\{v_{o.dq}^j\} \\ q\{v_{o.dq}^j\} & -d\{v_{o.dq}^j\} \end{bmatrix} \quad (3.116)$$

Then, the new state-space representation for each side is given by (3.117) and (3.118).

$$\Delta \dot{x}^A = A^{A.inv} \Delta x^A + B^{A.inv} \begin{bmatrix} \Delta v_{g.DQ} \\ \Delta v_{dc} \end{bmatrix} \quad (3.117)$$

$$\Delta \dot{x}^B = A^{B.inv} \Delta x^B + B^{B.inv} \begin{bmatrix} \Delta v_{g.DQ} \end{bmatrix} \quad (3.118)$$

The matrices of side A are given by (3.119) to (3.123); and (3.124) - (3.127) are the matrices of side B.

$$A^{A.inv} = \begin{bmatrix} A_{CC}^A & 0_{4 \times 2} & B_{CC(2)}^A & 0_{4 \times 10} & A_{auxA1.inv} \\ B_{delay}^A C_{CC}^A & A_{delay}^A & B_{delay}^A D_{CC(2)}^A & 0_{2 \times 10} & A_{auxA2.inv} \\ 0_{6 \times 2} & B_{LCL(1)}^A & A_{LCL}^A & 0_{6 \times 6} & A_{auxA3.inv} \\ 0_{6 \times 4} & 0_{6 \times 4} & 0_{6 \times 4} & A_{RL}^A & B_{RL(2,3)}^A \\ 0_{4 \times 8} & B_{VC(1)}^A T^A & 0_{4 \times 2} & B_{VC(2,3,4)}^A & A_{VC}^A \end{bmatrix} \quad (3.119)$$

$$A_{auxA1.inv} = \begin{bmatrix} B_{CC(1)}^A J_{vo}^{A.inv} (T^A)^{-1} & 0_{4 \times 2} \end{bmatrix} \quad (3.120)$$

$$A_{auxA2.inv} = \begin{bmatrix} B_{delay}^A D_{CC(1)}^A J_{vo}^{A.inv} (T^A)^{-1} & 0_{2 \times 2} \end{bmatrix} \quad (3.121)$$

$$A_{auxA3.inv} = \begin{bmatrix} B_{LCL(2)}^A (T^A)^{-1} & 0_{6 \times 2} \end{bmatrix} \quad (3.122)$$

$$B^{A.inv} = \begin{bmatrix} 0_{4 \times 2} & B_{CC(1)} J_{dc}^{A.inv} \\ 0_{2 \times 2} & B_{delay}^A D_{CC(1)}^A J_{dc}^{A.inv} \\ 0_{6 \times 2} & 0_{6 \times 1} \\ B_{RL(1)}^A & 0_{6 \times 1} \\ 0_{4 \times 2} & 0_{4 \times 1} \end{bmatrix} \quad (3.123)$$

$$A^{B.inv} = \begin{bmatrix} A_{CC}^B & 0_{4 \times 2} & B_{CC(2)}^B & 0_{4 \times 10} \\ B_{delay}^B C_{CC}^B & A_{delay}^B & B_{delay}^B D_{CC(2)}^B & 0_{2 \times 10} \\ 0_{6 \times 4} & B_{LCL(1)}^B & A_{LCL}^B & 0_{6 \times 6} \\ 0_{6 \times 4} & 0_{6 \times 4} & 0_{6 \times 4} & A_{RL}^B \\ 0_{4 \times 8} & B_{VC(1)}^B T^B & 0_{4 \times 2} & B_{VC(2,3,4)}^B \\ 0_{2 \times 5} & 0_{2 \times 5} & 0_{2 \times 5} & 0_{2 \times 5} \\ 0_{2 \times 5} & 0_{2 \times 5} & 0_{2 \times 5} & 0_{2 \times 5} \\ 0_{6 \times 5} & 0_{6 \times 5} & 0_{6 \times 5} & 0_{6 \times 5} \end{bmatrix} \quad (3.124)$$

$$\begin{bmatrix} B_{CC(1)}^B J^{B.inv} (T^B)^{-1} & 0_{4 \times 4} & 0_{4 \times 4} & 0_{2 \times 4} \\ B_{delay}^B D_{CC(1)}^B J^{B.inv} (T^B)^{-1} & 0_{2 \times 4} & 0_{2 \times 4} & 0_{2 \times 4} \\ B_{LCL(2)}^B (T^B)^{-1} & 0_{6 \times 4} & 0_{6 \times 4} & 0_{6 \times 4} \\ B_{RL(2)}^B & B_{RL(3)}^B & 0_{6 \times 5} & 0_{6 \times 5} \\ A_{VC}^B & 0_{4 \times 6} & B_{VC(5)}^B T^{dg} & 0_{4 \times 2} \\ B_{CC(1)}^{dg} J^{dg} (T^{dg})^{-1} & A_{CC}^{dg} & 0_{2 \times 2} & A_{auxB1.inv} \\ B_{delay}^{dg} D_{CC(1)}^{dg} J^{dg} (T^{dg})^{-1} & B_{delay}^{dg} C_{CC}^{dg} & A_{delay}^{dg} & A_{auxB2.inv} \\ B_{LCL(2)}^{dg} (T^{dg})^{-1} & 0_{6 \times 2} & B_{LCL(1)}^{dg} & A_{LCL}^{dg} \end{bmatrix}$$

$$A_{auxB1.inv} = \begin{bmatrix} B_{CC(2)}^{dg} & 0_{2 \times 4} \end{bmatrix} \quad (3.125)$$

$$A_{auxB2.inv} = \begin{bmatrix} B_{delay}^{dg} D_{CC(2)}^{dg} & 0_{2 \times 4} \end{bmatrix} \quad (3.126)$$

$$B^{B.inv} = \begin{bmatrix} 0_{12 \times 2} \\ B_{RL(1)}^B \\ 0_{14 \times 2} \end{bmatrix} \quad (3.127)$$

Finally the state-space representation of the system when the SOP is functioning in inverse mode is (3.128). And the matrices are given by (3.129) - (3.135).

$$\Delta \dot{X} = A^{inv} \Delta X + B^{inv} [\Delta v_{g.DQ}] \quad (3.128)$$

$$A^{inv} = \begin{bmatrix} A^{A.inv} & 0_{22 \times 32} & 0_{22 \times 1} \\ 0_{32 \times 22} & A^{B.inv} & X^{0.inv} \\ X^{1.inv} & X^{2.inv} & X^{3.inv} \end{bmatrix} \quad (3.129)$$

$$X^{0.inv} = \begin{bmatrix} B_{CC(1,2)}^B J_{DC} \\ B_{delay}^B D_{CC(1)}^A J_{dc}^A \\ 0_{26 \times 2} \end{bmatrix} \quad (3.130)$$

$$X^{1.inv} = \begin{bmatrix} 0_{1 \times 6} & B_{DC(1)}^A & 0_{1 \times 14} \end{bmatrix} \quad (3.131)$$

$$X^{2.inv} = \begin{bmatrix} 0_{1 \times 6} & B_{DC(2)}^A & 0_{1 \times 24} \end{bmatrix} \quad (3.132)$$

$$X^{3.inv} = 0 \quad (3.133)$$

$$B^{inv} = \begin{bmatrix} B^{A2} \\ B^B \\ 0_{1 \times 2} \end{bmatrix} \quad (3.134)$$

$$B^{A2} = \begin{bmatrix} 0_{2 \times 12} \\ B_{RL(1,2)}^A \\ 0_{2 \times 14} \end{bmatrix} \quad (3.135)$$

3.4. Validation of the model through simulation.

In this section, a validation of the model for the normal mode of operation is made. Two models in Simulink were made, one utilizes the equations obtained in this chapter; and the second utilizes models of the components. In order to validate the mathematical model, both simulations must reach the same values in time.

The values of the components were chosen according to an experimental setup, which is describe in appendix A; and the parameters of the controllers through were obtained through analysis. The value of the controller parameters are in table 3.3, the components in table 3.4; in both tables $j \in \{A, B\}$. And the values of the operation point are listed in table 3.5.

Table 3.3. Parameters of the Controllers

	Description	Parameter	Value
SOP	Current Controller	k_{ir}^j	10000
		k_{pr}^j	50
		ω_c^j	$2\pi \cdot 0.1$
		ω_r^j	$2\pi \cdot 50$
	Droop Controller	k_P^A	$+100 [W/V]$
		k_P^B	$-100 [W/V]$
		k_Q^j	$100 [W/V]$
		v_P^*	$230 [V]$
		v_Q^*	$230 [V]$
		v_{DC}^*	$600 [V]$
DG (dg)	Current Controller	k_i^{dg}	100
		k_p^{dg}	1
	Power Controller	P^{*dg}	$10 [kW]$
		Q^{*dg}	$0 [kW]$

Table 3.4. Parameters of the System

	Description	Parameter	Value
SOP	Filter capacitor	C_f^j	24.6 [μF]
	Damping resistor	R_d^j	10 [Ω]
	Filter inductor	L_i^j	2.3 [mH]
		R_i^j	0.1 [Ω]
	Coupling inductor	L_g^j	0.93 [mF]
		R_g^j	0.03 [Ω]
	Switching frequency	f_s^j	10 [kHz]
DG (dg)	Filter capacitor	C^{dg}	8.8 [μF]
	Filter Inductor	L_f^{dg}	2.3 [mH]
		R_f^{dg}	0.1 [Ω]
	Coupling Inductor	L_c^{dg}	0.93 [mF]
		R_c^{dg}	0.03 [Ω]
Feeder	SS-load line	R_1^j	0.47 [Ω]
		L_1^j	1.2 [mF]
	SOP-load line	R_2^j	0.1 [Ω]
		L_2^j	1.2 [mF]
	Passive Load	R_L^j	16 [Ω]
		L_L^j	0.01 [mF]
	Grid frequency	ω	$2\pi \cdot 50$ [rad/s]
	Virtual Capacitor	C_V	1 [pF]

Table 3.5. Operation point values

Parameter	Side A value	Side B value
$x_{1.dq}^j$	$\begin{bmatrix} 4.7 \times 10^7 & 6 \times 10^6 \end{bmatrix}$	$\begin{bmatrix} 4.6 \times 10^7 & -3.8 \times 10^5 \end{bmatrix}$
$x_{2.dq}^j$	$\begin{bmatrix} 480 & 60 \end{bmatrix}$	$\begin{bmatrix} 472 & -3.8 \end{bmatrix}$
$v_{i.dq}^j$	$\begin{bmatrix} 392 & -11.2 \end{bmatrix}$	$\begin{bmatrix} 400 & -5.9 \end{bmatrix}$
$i_{i.dq}^j$	$\begin{bmatrix} 1.37 & -1.1 \end{bmatrix}$	$\begin{bmatrix} -1.3 & -0.32 \end{bmatrix}$
$i_{o.dq}^j$	$\begin{bmatrix} 1.4 & -2.2 \end{bmatrix}$	$\begin{bmatrix} -1.3 & -0.8 \end{bmatrix}$
$v_{c.dq}^j$	$\begin{bmatrix} 392 & -8.6 \end{bmatrix}$	$\begin{bmatrix} 400 & -2.5 \end{bmatrix}$
$i_{1.DQ}^j$	$\begin{bmatrix} 22.9 & 2.9 \end{bmatrix}$	$\begin{bmatrix} 1.3 & 3.1 \end{bmatrix}$
$i_{2.DQ}^j$	$\begin{bmatrix} 1.4 & -1.3 \end{bmatrix}$	$\begin{bmatrix} -1.3 & -0.8 \end{bmatrix}$
$i_{L.DQ}^j$	$\begin{bmatrix} 24.3 & 0 \end{bmatrix}$	$\begin{bmatrix} 25 & 1.2 \end{bmatrix}$
$v_{o.DQ}^j$	$\begin{bmatrix} 390 & -9 \end{bmatrix}$	$\begin{bmatrix} 400 & -2.1 \end{bmatrix}$
$v_{L.DQ}^j$	$\begin{bmatrix} 398 & -9.3 \end{bmatrix}$	$\begin{bmatrix} 400 & -1.7 \end{bmatrix}$
Parameter	Value	
$x_{3.dq}^{dg}$	$\begin{bmatrix} 397.3 & 46.7 \end{bmatrix}$	
$v_{i.dq}^{dg}$	$\begin{bmatrix} 396.7 & 64.4 \end{bmatrix}$	
$i_{i.dq}^{dg}$	$\begin{bmatrix} 25.1 & 1.5 \end{bmatrix}$	
$i_{o.dq}^{dg}$	$\begin{bmatrix} 25.2 & 0.4 \end{bmatrix}$	
$v_{c.dq}^{dg}$	$\begin{bmatrix} 398 & 4 \end{bmatrix}$	
v_{dc}	593.3	
$v_{g.DQ}$	$\begin{bmatrix} 400 & 0 \end{bmatrix}$	

Table 3.6. Simulation parameters.

Parameter	Value
Software	Matlab/Simulink R2018a
Simulation time	0.3[s]
Solver Circuital Simulation	ode8 (Dormant-Prince)
Step-size Circuit Simulation	10^{-5} [s]
Solver Mathematical Simulation	ode45 (Dormant-Prince)
Step-size Mathematical Simulation	Variable

A Simulink model of the system was made as a benchmark utilizing blocks from the Simscape library and the values listed on table 3.3 and table 3.4. A second model utilized the state-space model (3.96) in order to simulate the system in the vicinity of the operation point.

The results of the simulations are shown in figures 3.12 - 3.15. The parameters of the simulation are listed in table 3.6. Figure 3.12 and 3.13 show the voltages and currents of the load and the output of the SOP in side A and B in the DQ -frame. Figure 3.14 shows the output current and power of the DG. And figure 3.15 shows the voltage in the DC-Link.

As stated in (3.3) - (3.5) and the values in table 3.3, the controller was tasked to set the values of the output voltages of the SOP to 230V and the DC-link voltage to 600V. Figures 3.12, 3.13 and 3.15, shows that the controllers achieve these reference values. And figure 3.14 shows that the DG controller injects 10kW to the feeder of side B successfully. Table 3.7 shows the percentual errors for each of the variables analysed; it can be seen that the mean error is 1.68%; and the maximum error is given by the reactive power delivered by the DG, with a value of 17%. This error is consequence on how the reactive power is calculated, as it is given by (3.136); the errors of the components are also multiplied, thus giving a bigger error. The values of the reactive power injected by the DG are $-448,34VAr$ and $-368.77VAr$ in the circuital and the mathematical model,

respectively; which are both small compared to the active power transferred ($10000kW$), so this error does not invalidates the model.

$$Q = d\{v_{dq}\}q\{i_{dq}\} - q\{v_{dq}\}d\{i_{dq}\} \quad (3.136)$$

The state-space model initial values were set in a vicinity of the operation point and are presented in table 3.5. In figures 3.12 - 3.15, it can be seen that the state-space model converges to the circuitual simulation, thus validating the model presented in this chapter. The errors are probably due to the linealization around an specific operation point, as this operation only utilized first term of a Taylor expansion.

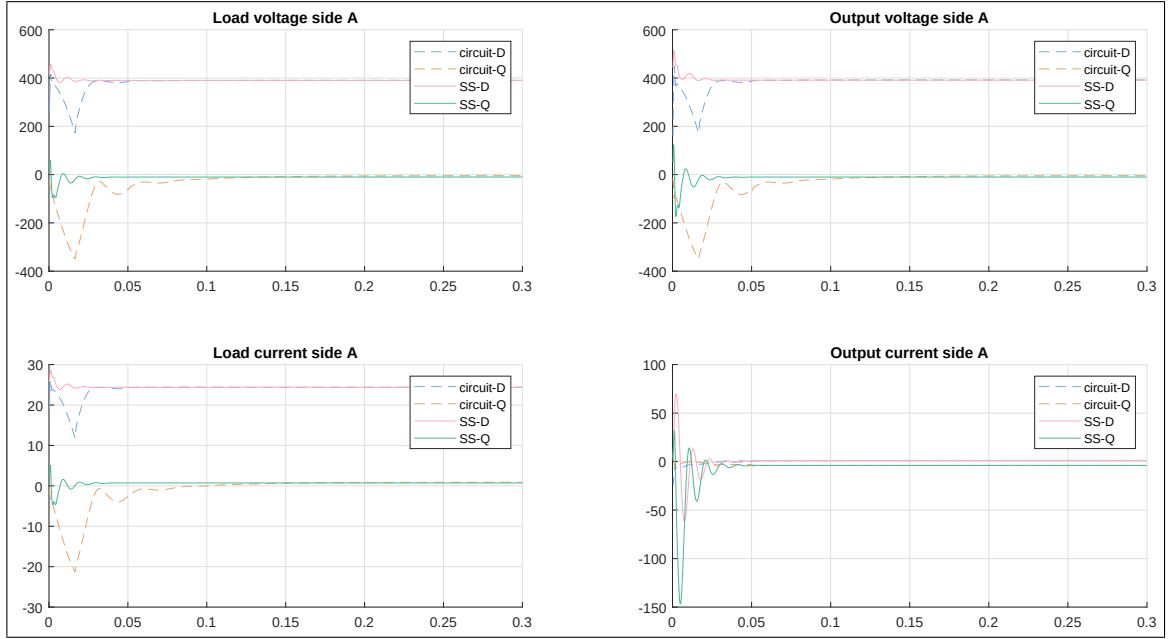


Figure 3.12. Side A simulation results.

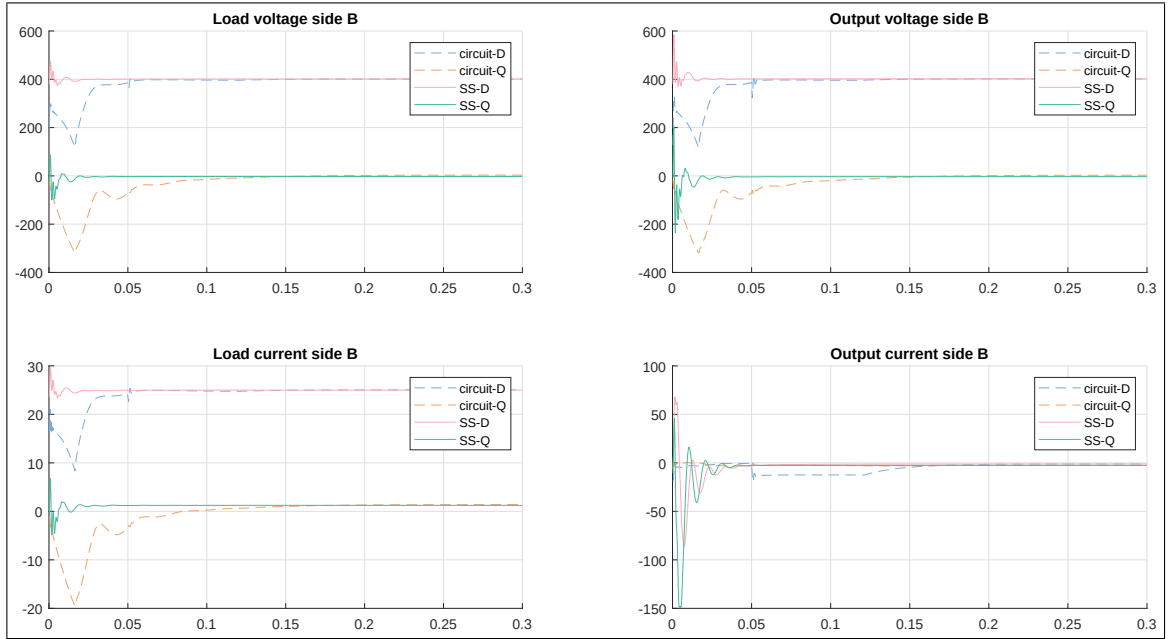


Figure 3.13. Side B simulation results.

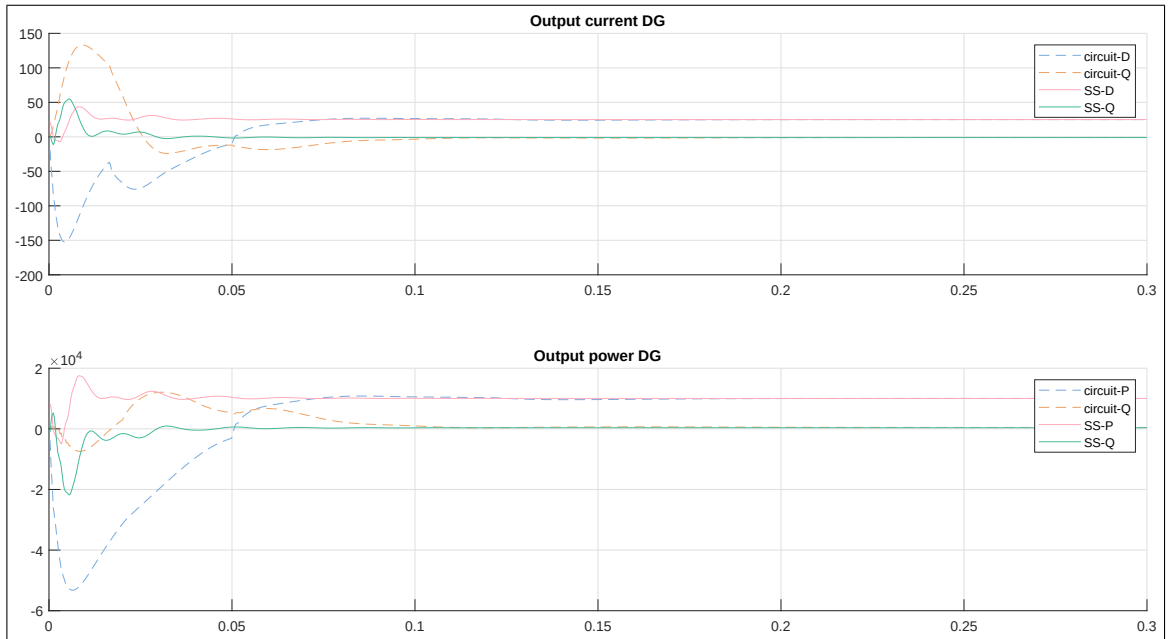


Figure 3.14. DG simulation results.

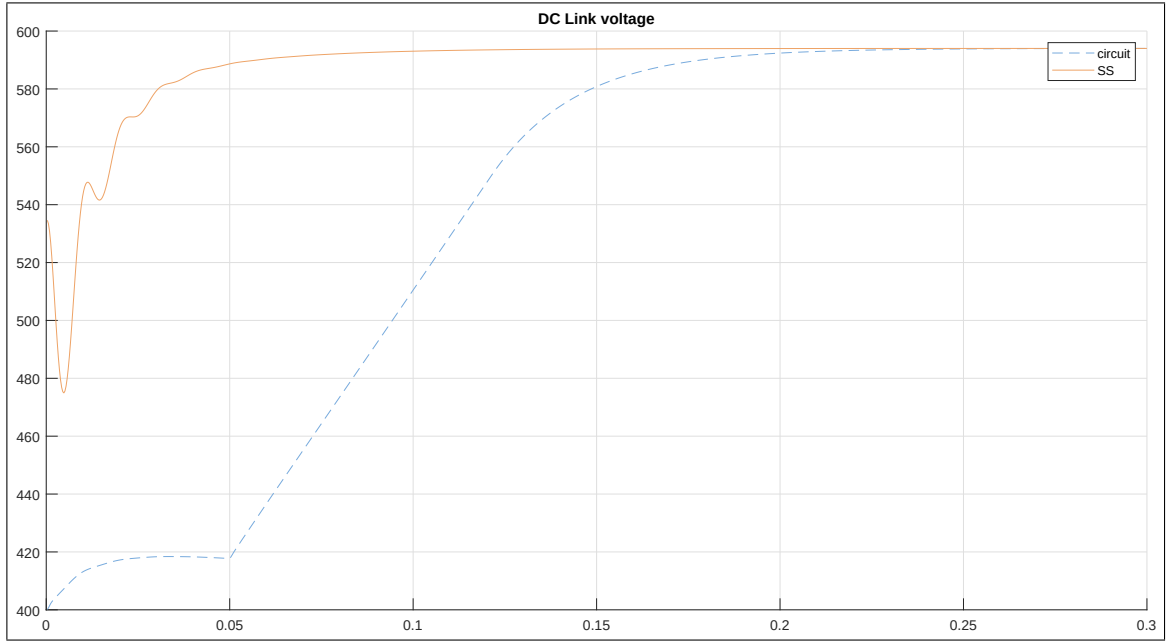


Figure 3.15. DC-Link voltage simulation result.

Table 3.7. Perceptual errors in variables.

Variable	Error (%)
v_{Lrms}^A	0.02%
v_{Lrms}^B	0.005%
$v_{o.rms}^A$	0.2%
$v_{o.rms}^B$	0.01%
i_{Lrms}^A	0.05%
i_{Lrms}^B	0.15%
$i_{o.rms}^A$	1.8%
$i_{o.rms}^B$	0.16%
$i_{o.rms}^B$	0.16%
$i_{o.rms}^{dg}$	0.01%
v_{dc}	0.004%
P^{dg}	0.02%
Q^{dg}	17%

CHAPTER 4. STABILITY ANALYSIS

In this chapter the stability analysis of the model obtained in Chapter 3 is performed. First, the stability of the system is determined with the concepts seen in Chapter 2. Then, the parameters that are most critical to the stability will be found with the use of participation factors. Finally, sensibility analysis are made to the critical parameters in order to find ranges of values in which the system remains stable.

4.1. System stability

Figure 4.1 and figure 4.3, show the eigenvalues of the system in normal mode and inverse mode, respectively; meanwhile figure 4.2 and figure 4.4 show the lower frequency eigenvalues of the modes. It can be seen that all the eigenvalues of the system, in both modes of operation, have negative real part, then by **Proposition 5.5.6**, the linearized systems (3.96) and (3.128) are globally asymptotically controllable to the operation point x_0 . Therefore, by **Theorem 19** the system is locally asymptotically controllable to x_0 for both modes of operation. This means that with the controller utilized, it is possible to find parameters that makes the equilibrium point x_0 locally asymptotically stable.

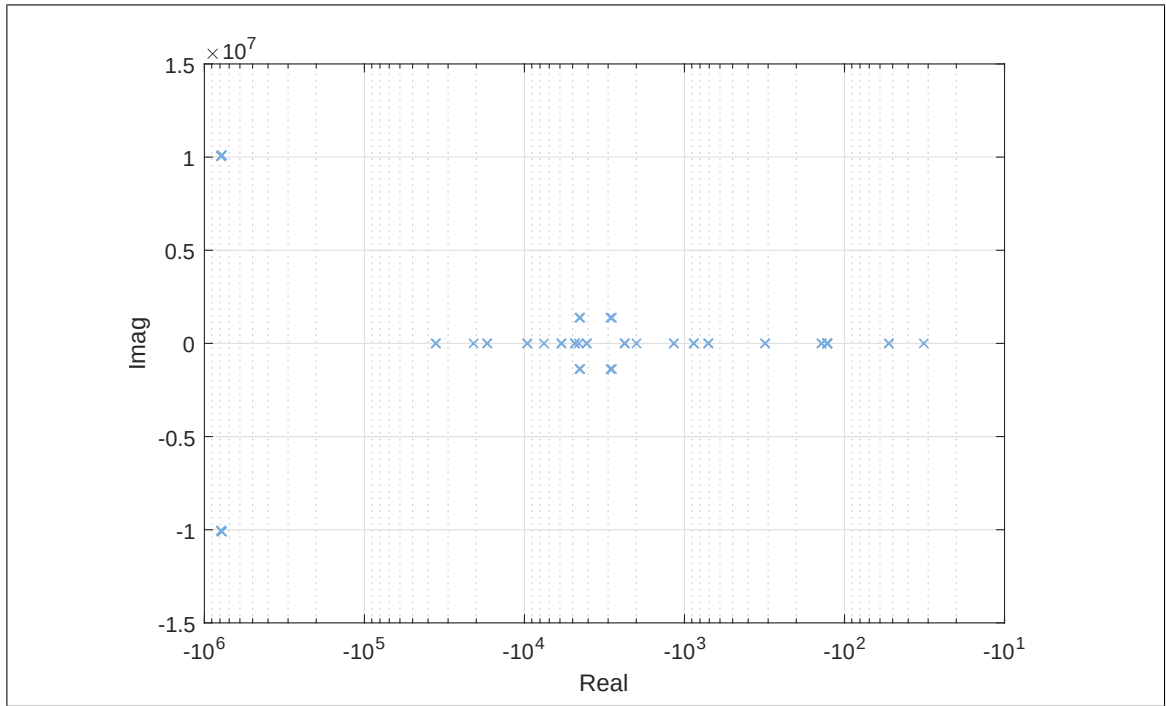


Figure 4.1. Eigenvalues of the model in normal mode.

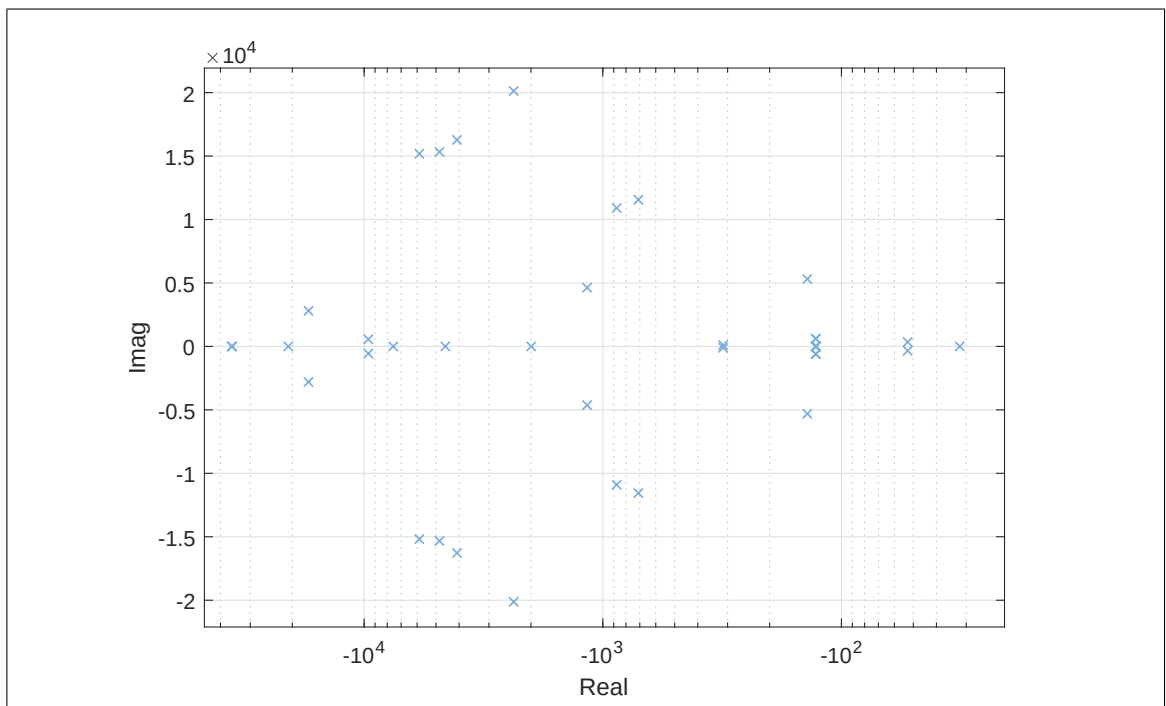


Figure 4.2. Lower frequency eigenvalues in normal mode.

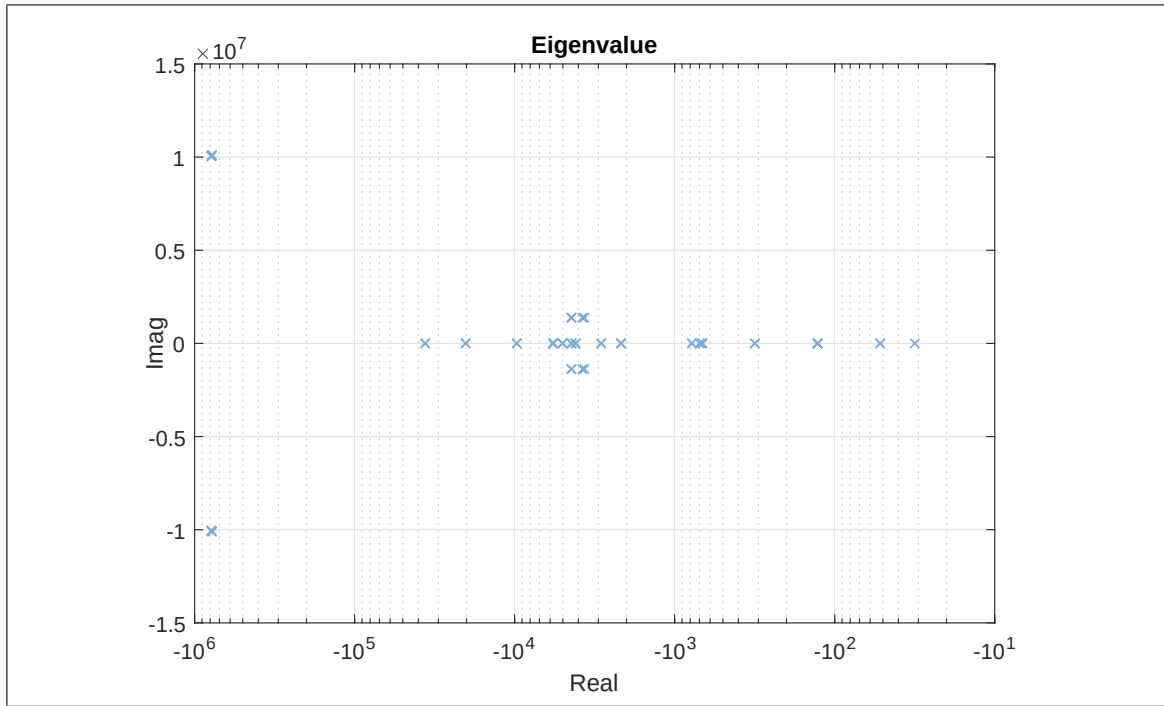


Figure 4.3. Eigenvalues of the model in inverse mode.

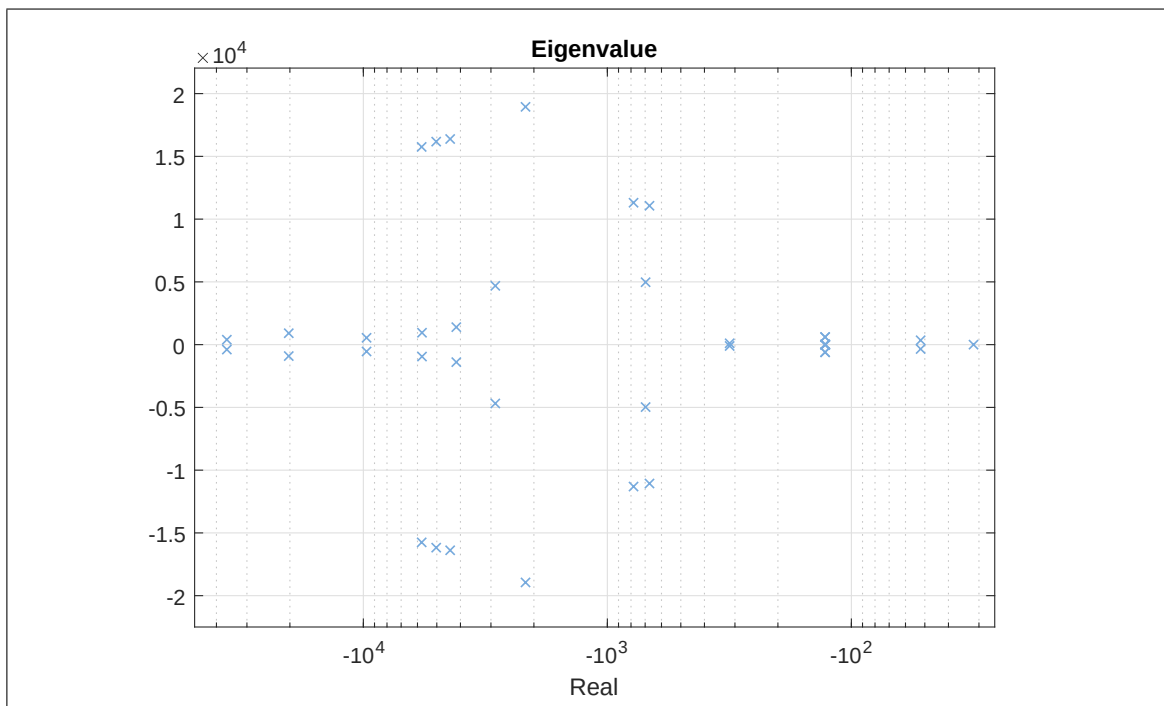


Figure 4.4. Lower frequency eigenvalues in inverse mode.

4.2. Participation factors and sensibility analysis

As stated in Chapter 2, participation factors are used to study how a state influences over an eigenvalue and viceversa. Figures 4.5 and 4.6 show how the eigenvalues are grouped according to which states influence them, these are detailed in table 4.1.

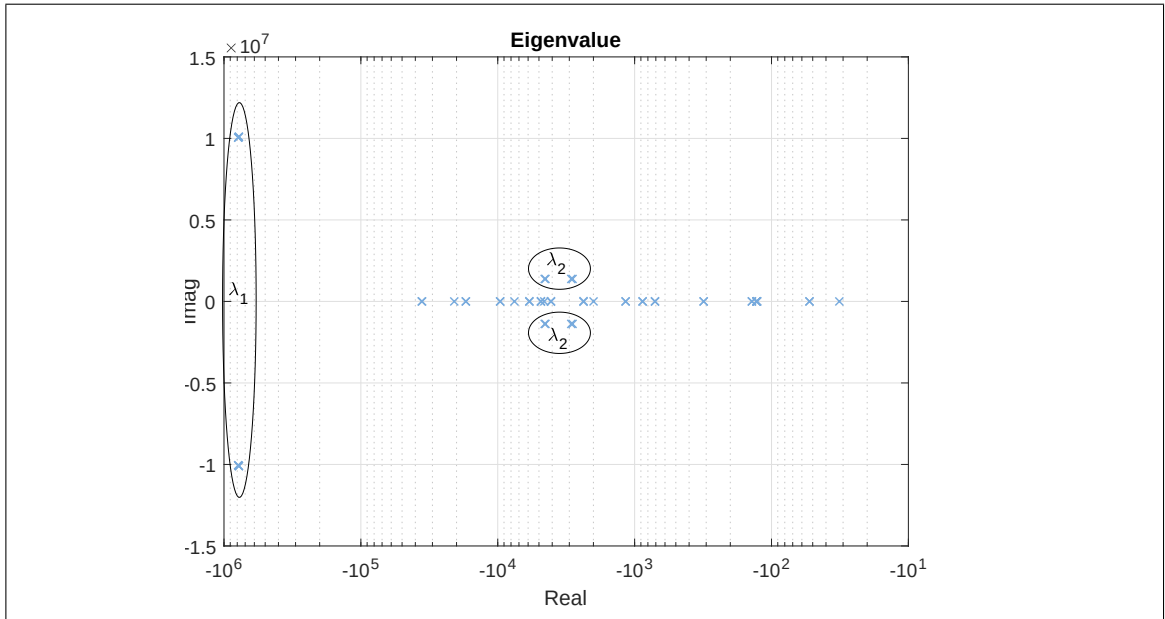


Figure 4.5. Participation factors of the higher frequency eigenvalues.

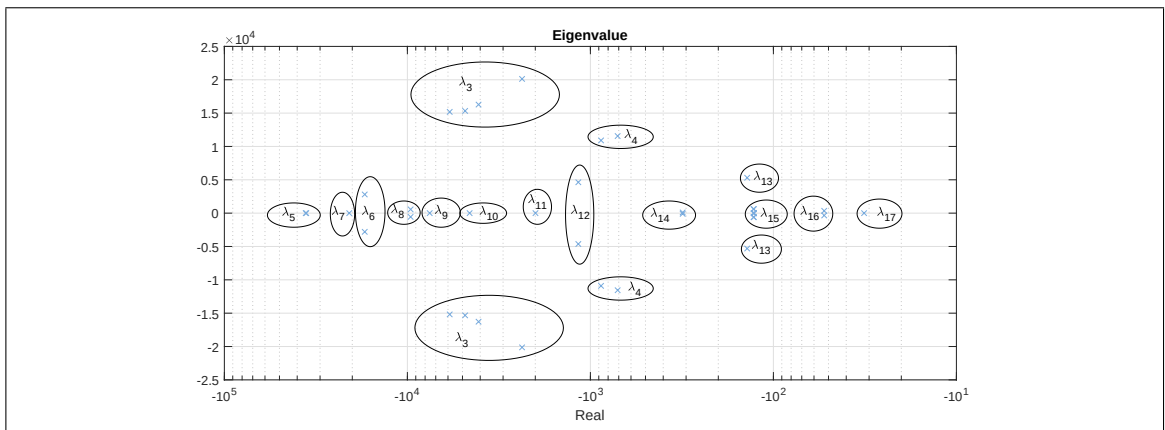


Figure 4.6. Participation factors of the lower frequency eigenvalues.

Table 4.1. Eigenvalues groups and parameters, $j \in \{A, B\}$.

Group	Dominant Subsystem	States	Parameters	Frequency
λ_1	Lines and Loads	v_L^j, i_L^j	$R_L^j, L_L^j, R_1^j, L_1^j, R_2^j, L_2^j$	$10MHz$
λ_2	Various	v_o^j, v_o^j, i_2^j	$C_V, R_2^j, L_2^j, L_g^j, R_g^j$	$1.4MHz$
λ_3	LCL Filter and Inverter	v_i^j, i_i^j, v_o^j	L_i^j, R_i^j, C_f^j, f_s	$[15, 20]kHz$
λ_4	DG LCL Filter	$v_c^{dg}, i_i^{dg}, i_o^{dg}$	$L_i^{dg}, R_i^{dg}, C_f^{dg}, L_g^{dg}, R_g^{dg}$	$[10, 11]kHz$
λ_5	Side B Lines and Loads	i_o^{dg}, i_1^B, i_2^B	$C_V, R_2^B, L_2^B, R_g^B, L_g^B$	$0Hz$
λ_6	Various	i_1^A, v_c^A, i_i^A	$R_1^A, L_1^A, R_i^A, L_i^A, C_f^A$	$2.8kHz$
λ_7	Side A Lines and Loads	i_1^A, i_2^A	$R_1^A, L_1^A, R_2^A, L_2^A$	$0Hz$
λ_8	DG Inverter	v_i^{dg}	$f_s^{dg}, k_p^{dg}, k_i^{dg}, L_i^{dg}$	$520Hz$
λ_9	Side B LCL Filter and Inverter	v_c^B, v_i^B	$C_f^B, L_i^B, R_i^B, f_s^B$	$0Hz$
λ_{10}	Side B Lines and Loads	v_c^B, i_1^B, i_2^B	$C_f^B, R_d^B, R_1^B, L_1^B, R_2^B, L_2^B$	$0Hz$
λ_{11}	Side A Lines and Loads	i_1^A, i_2^A	$R_1^A, L_1^A, R_2^A, L_2^A$	$0Hz$
λ_{12}	Side A LCL Filter	v_c^A, i_o^A, i_1^A	$C_f^A, R_d^A, R_i^A, L_i^A, R_1^A, L_1^A$	$4.6kHz$
λ_{13}	Side B LCL Filter	v_c^B, i_o^B, i_2^B	$C_f^B, R_d^B, R_i^B, L_i^B, R_2^B, L_2^B$	$5.3kHz$
λ_{14}	Various	i_i^{dg}, i_1^B	$R_i^{dg}, L_i^{dg}, R_1^B, L_1^B$	$105Hz$
λ_{15}	Side A and B Current Controllers	x_1^j, x_2^j	$k_{ir}^j, k_{pr}^j, \omega_c^j, \omega_r^j, k_p^j, k_q^j$	$[22, 605]Hz$
λ_{16}	DG Current Controller	x_3^{dg}	$k_p^{dg}, k_i^{dg}, L_i^{dg}$	$347Hz$
λ_{17}	DC-Link	v_{dc}	C_{dc}	$0Hz$

From Table 4.1 it can be seen that the most critical groups are $\lambda_{13}, \lambda_{15}, \lambda_{16}$ and λ_{17} . By seeing the states in the table, it can be stated that the most critical subsystems for the stability are the LCL filter of side B (λ_{13}), the SOP power, droop and current controllers (λ_{15}), the DG current controller (λ_{16}) and the DC-link (λ_{17}).

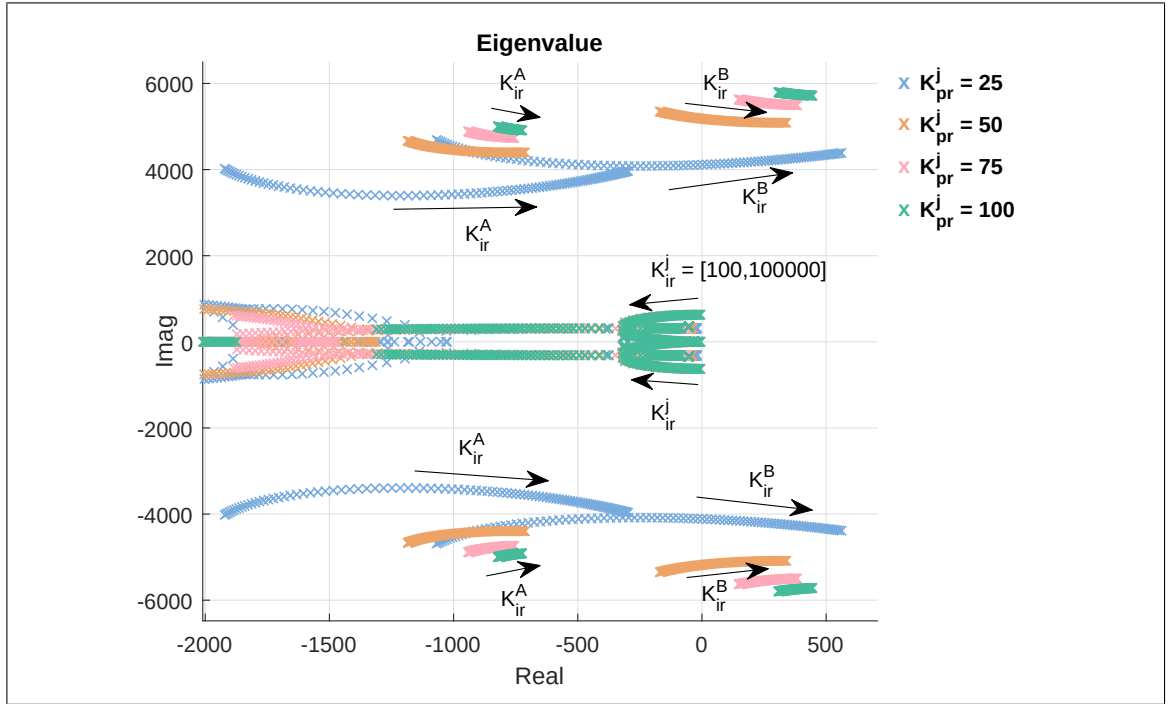


Figure 4.7. Eigenvalue traces of PR Current controller. $j \in \{A, B\}$

A sensibility analysis was made to some of the parameters of the critical subsystems. Figure 4.7 shows the eigenvalues traces of groups λ_4 and $\lambda_{12} - \lambda_{17}$ when the parameters of the SOP current controllers varies in the normal mode of operation. It can be seen that the system is more sensitive to the proportional parameters. It is also noted that by increasing both values, the higher frequency poles tends to the right-half side, which decreases their damping; and the lower frequency poles goes to the left side, increasing their damping.

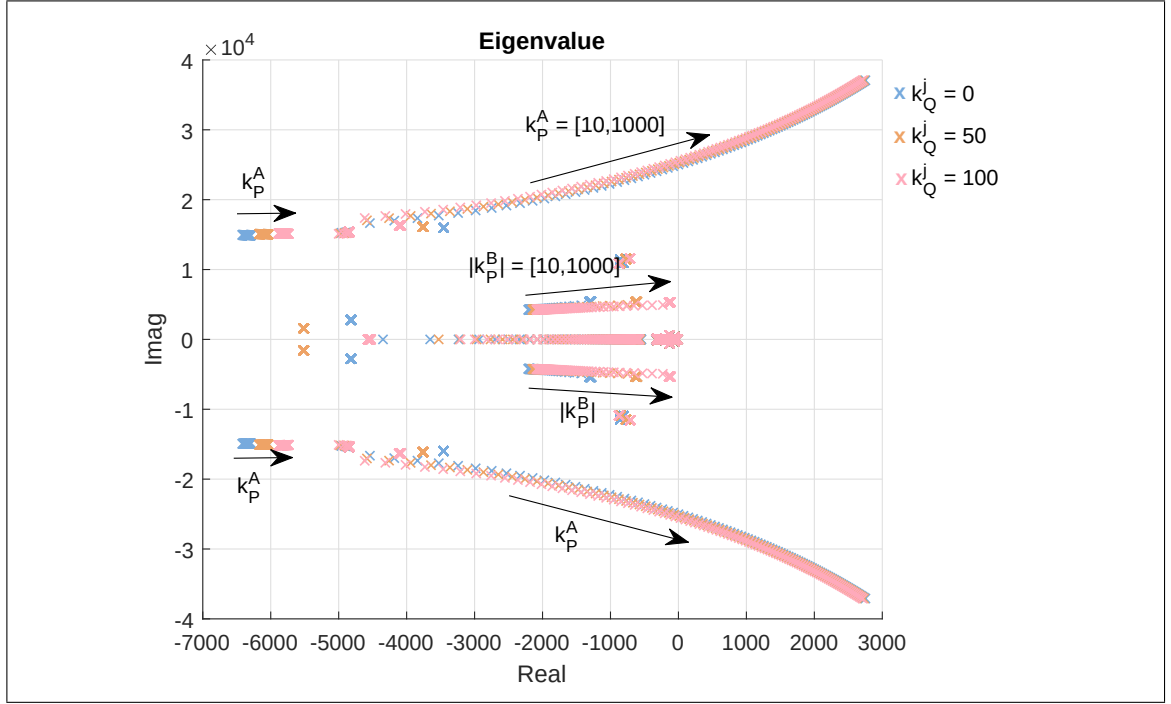


Figure 4.8. Eigenvalue traces of Droop controller. $j \in \{A, B\}$

In figure 4.8 the traces show the change in groups λ_3, λ_4 and $\lambda_{10} - \lambda_{17}$, when the Droop controllers parameters changes in the normal mode of operation. It is important to note that the value of $k_p^j, j \in \{A, B\}$ must be negative for the side that maintains the DC-link, as the trace is made in normal mode, the value of k_p^B is negative and the trace is made with its absolute value. As is seen on the figure, by increasing the value of either parameter, the system is closer to become unstable, and active power parameter has more influence on the stability. It is also important to note that k_p^A has a bigger effect on the stability than k_p^B . As these parameters indicate how much active and reactive power is transferred between the SOP and the feeders, these must be chosen carefully in order to achieve a faster response without becoming unstable. It is also noted that the system allows a higher value of active power than reactive power to be transferred through the SOP.

Table 4.2. Critical values of Droop and PR controllers.

Parameter	Normal mode	Inverse mode
	(Side B regulates DC link)	(Side A regulates DC link)
k_{pr}^A	(0, 95000]	(0, 7000]
k_{pr}^B	(0, 60]	(0, 106000]
k_{ir}^A	(0, 270000]	(0, 140000]
k_{ir}^B	(0, 40000]	(0, 230000]
k_p^A	(0, 310]	$[-6300, 0)$
k_p^B	$[-4800, 0)$	(0, 330]
k_q^A	(0, 310]	(0, 120]
k_q^B	(0, 220]	(0, 1000]

In Table 4.2 the stability conditions for the parameters of the controllers are shown; each of these are established when the other parameters remained constant. It is observed that in inverse mode the parameters have bigger ranges, allowing for a more flexible operation.

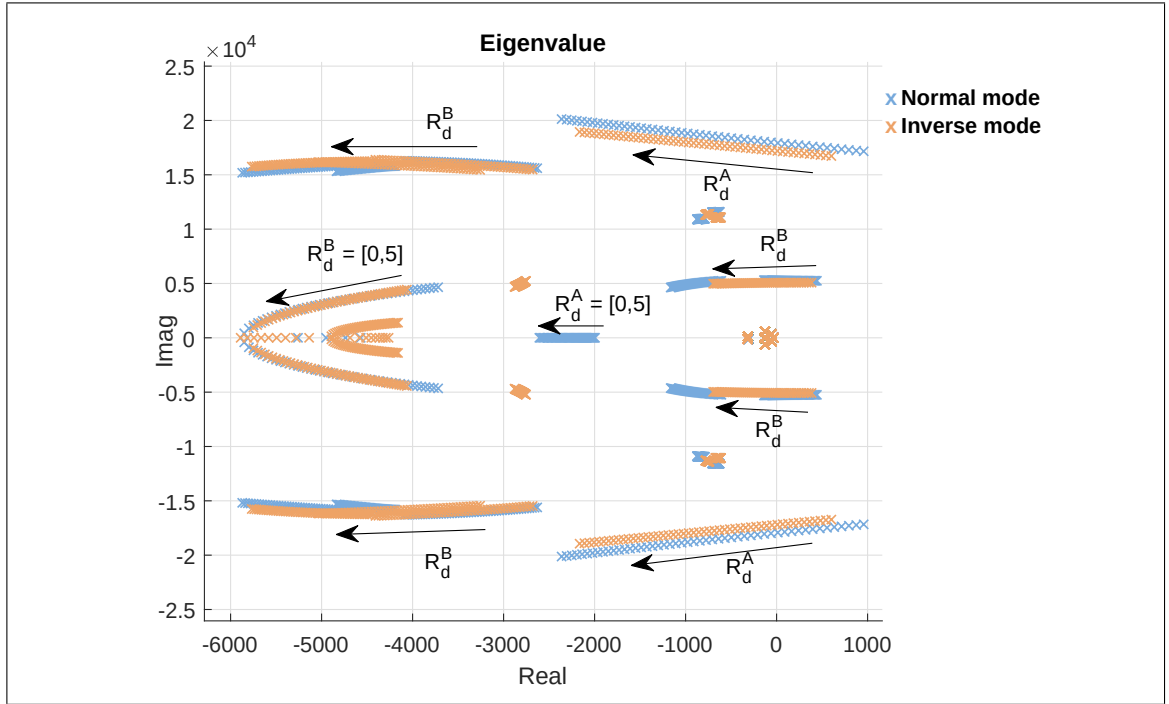


Figure 4.9. Eigenvalue trace of Damping resistor.

Figure 4.9 shows the eigenvalue traces of groups λ_3, λ_4 and $\lambda_{10} - \lambda_{17}$ when the damping resistor of the SOP LCL Filters varies for both modes. It can be seen that an increment of the resistor increases the damping of the poles and improves the stability of the system. Although, a bigger resistor leads to a higher power dissipation on the filter. The trace also shows that the inverse mode requires a smaller damping resistor to stabilize the system. The minimum values of R_d in order to achieve stability are 3.6Ω and 1.5Ω for normal and inverse mode, respectively. The values of the resistors are small and produces little losses in the system; and, depending on the material, the damping resistor function can be replaced with the equivalent series resistor (ESR) of the filter capacitor, as the value of this is compressed between $[0.1, 30]\Omega$.

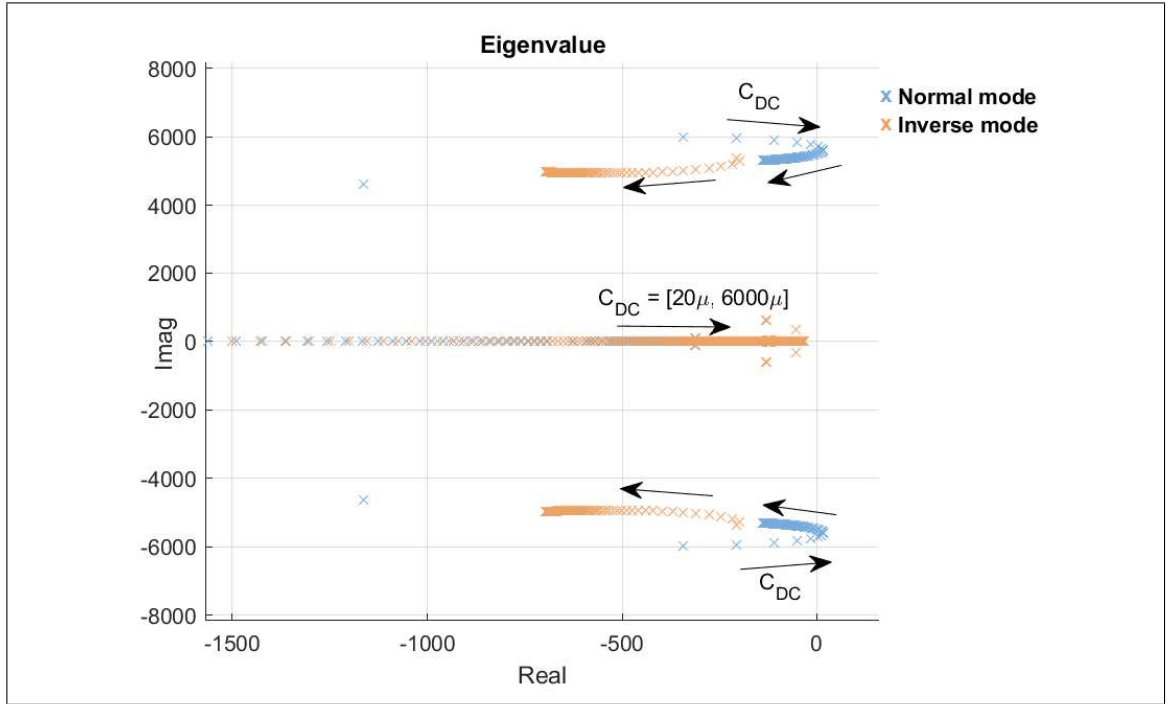


Figure 4.10. Eigenvalue trace of DC Link Capacitor.

Figure 4.10 shows the eigenvalue traces when the DC-Link capacitor value varies. From this analysis, it can be seen that the system in normal mode is unstable if $C_{dc} = [50, 100]\mu F$; in reverse mode, the value of the capacitor does not make the system unstable. Therefore, the value of the capacitor must be bigger than $0.1mF$.

CHAPTER 5. CONCLUSIONS

In this thesis a stability analysis was made to a distribution network (DN) that contains passive loads, inverter-connected distributed generation (DG) and a Soft-Open-Point (SOP), corresponding to a Back-to-Back converter connected between two feeders. In Chapters 1 and 2, new technologies were presented to solve different problems in distribution networks; the SOP was chosen to further study due to the increase in reliability the provides to the network and the simple design of its controllers. It was also presented a test made in the UK network; which demonstrates the feasibility of the SOP and how it increases the reliability of the network.

In Chapter 3 a linearized model of a DN with SOP and DG was obtained and was verified through simulations. The modelling of the system was made by dividing it into various subsystems and finding the state-space representation of each of them; then, these were integrated into one model. This modularized method of modelling allows for an easier access to each subsystem and a simpler integration of new subsystems, i.e more DG sources. So this thesis can be seen as a template to use to model similar systems.

In chapter 4 a stability analysis of the small-signal model was made, from which it was proven that by choosing appropriate parameters, a distribution network with SOP and DG can be locally asymptotically controllable to an established operation point, thus proving hypothesis H . Meaning that a SOP can be utilized by a central controller to establish that certain variables of the network must converge to a point, increasing the flexibility and reliability of it. By choosing the right parameters, the model was proven to be LAC for two modes of operation, **normal mode** and **inverse mode**, in each of these, one of the inverters of the SOP acts as an active load to its feeder, while the other acts as a controlled voltage power source; both inverters control the transfer of active of reactive power between the feeders.

From participation factors it was determined that the critical parameters of the stability are associated to the controllers, the LCL filters and the DC-link. Then, a sensibility analysis was made to these, which determined that the **inverse mode** is more flexible, because the ranges in which the system is stable of the critical parameters are bigger. It

also was noted that the inclusion of a damping resistor in series with the filter capacitor of the LCL filters, drastically affects the stability of the system to a point that it is mandatory its inclusion. From the analysis it was seen that its minimum value is small and can be compared to the value of the equivalent series resistor (ESR) of a capacitor, although it is recommended to add a separate resistor, as the ESR varies with the operation frequency and this can lead to problems if this is not taken into account. It is also recommended a small value for the resistor, as this “increases the power losses in the system and it greatly decreases the efficiency of the filter” (Kahlane, Hassaine, & Kherchi, 2014).

As future work, it is the study of more complex systems; as stated previously, various simplifications were made in this thesis, such as the exclusion of PLLs and the modelling of the DG as a constant power supply. The analysis of a system without these simplifications allows for a more complete representation. Besides, as states previously, the modularized modelling makes this work a starting point for a more complex analysis.

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APPENDIX

A. APPENDIX: EXPERIMENTAL RESULTS

The model of the network represented in figure 3.1 was tested experimentally at a reduced scale, in Imperial College London by Nathaniel Bottrell. The experimental setup is illustrated in figure A.1, and was designed according to the available laboratory facilities. The substation was experimentally implemented using a 90 kVA inverter labeled as "Grid Emulator 90 kVA" in figure A.1. A voltage and frequency controller was used to regulate the substation voltage to 230 V phase to neutral at 50 Hz. The SOP was experimentally tested using two 10 kVA inverters in a back-to-back configuration and is labelled as "B2B-SOP" in figure A.1. The SOP was programmed with the controllers described in Chapter 3 in reverse mode, meaning that the side not connected to the PV Inverter Emulator regulates the DC-bus. The parameters utilized in the experiment are the same as in the simulations and can be found in Chapter 3.

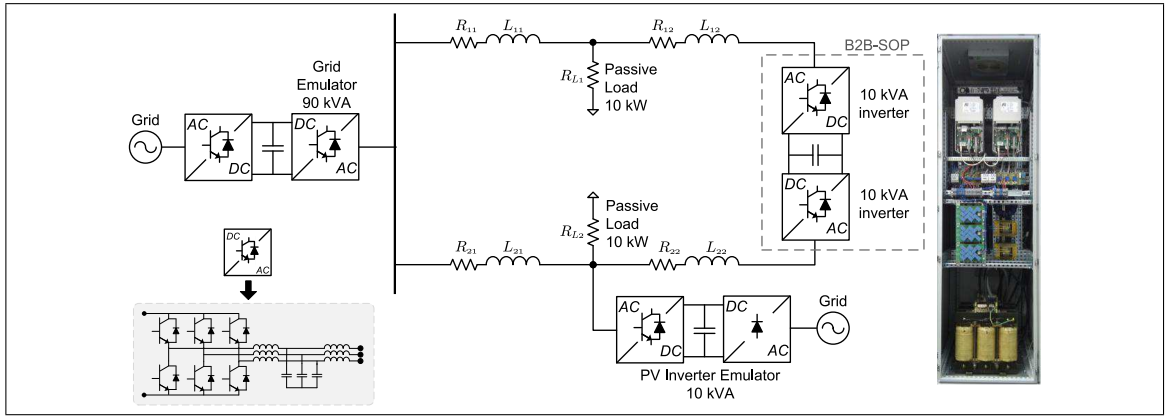


Figure A.1. Experimental setup.

The PV inverter was experimentally tested using a 10 kVA inverter which was programmed with an inner current controller and an output PQ controller. The converter was programmed to step through different P set-points which represented step-changing of load and generation on the feeder. The PV inverter was able to act as both as a load and as a generator by importing or exporting power. Load connected to the mid-point of the feeder caused the voltage to drop and generation caused the voltage to rise. The droop parameters of the SOP could be set to determine if the end of feeder voltage increases or decreases. In a feeder that normally contains a lot of generation, the droop voltage set-point could be reduced to enable more generation to connect. When there is a lot of load present, the droop set-point could be increased to allow more load to connect.

The experiment tested different droop set-points at different generation and load conditions, which was emulated by changing the power output of the inverter connected at the mid-point of the feeder.

Figure A.2 shows the voltage at the terminal of the SOP when the inverter connected to the mid-point of the feeder changed from 7.4 kW to -7.4 kW. Five experiments were performed, one with no SOP connected, others with the SOP droop set-point set changed from 220 V to 235 V at 5 V intervals. The voltage set-point of the 90 kVA inverter and power set-points of the inverter connected to the mid-point of the feeder were the same for each of the five experiments. Without the SOP there is a large voltage deviation of approximately 12 V at the end of the feeder (blue line). When the SOP was operating, the droop controller reduced the voltage deviation to approximately 4 V between maximum generation and maximum load for all set-points tested. The variation of SOP droop set-point was also able to change the end-of-feeder voltage. With the set point at 235 V, the voltage varied in a narrow range just below the set point. With set point at 220 V the variation was just above. The AC voltage and current output for the set-point of 230 V is shown in figure A.3.

The response to a load change from 3 kW to 4.3 kW was also tested. figure A.4 shows the terminal voltage when the SOP is not connected and when it is connected. figure A.5 shows the change in real power export and DQ current of the SOP for this load change. The time stamps of the SOP, substation and PQ inverter is not synchronised and subsequently there is a difference. It can be seen that the droop method is stable during this transient, the oscillations observed in the SOP export power, and DQ current in figure A.5 are damped and the SOP is stable.

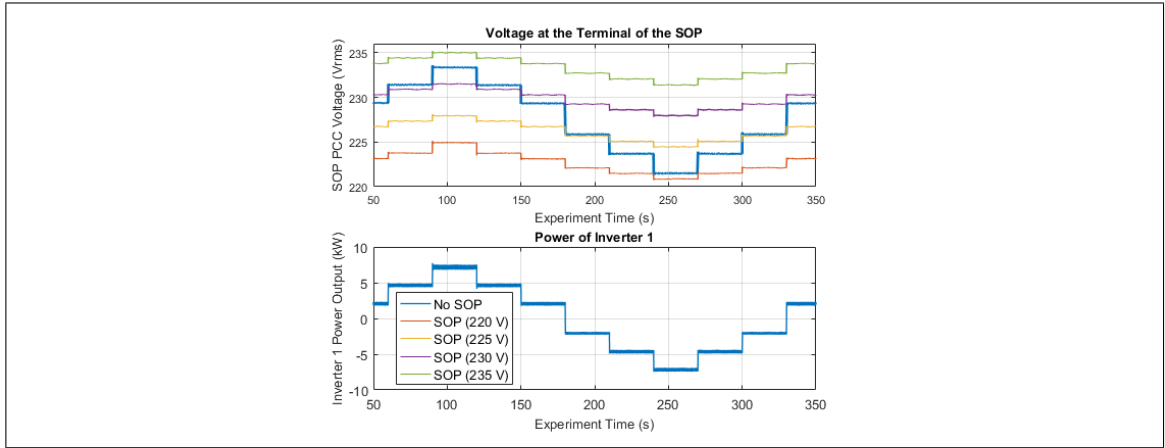


Figure A.2. The top graph shows the average phase to neutral RMS voltage of the three phases at the end of the feeder for when the SOP is disconnected ("No SOP") and when the SOP has a droop set-point of 220 V to 235 V at 5V intervals. The bottom graph shows the power of the inverter connected to the mid-point of the feeder. Positive power represents generation and negative power represents load.

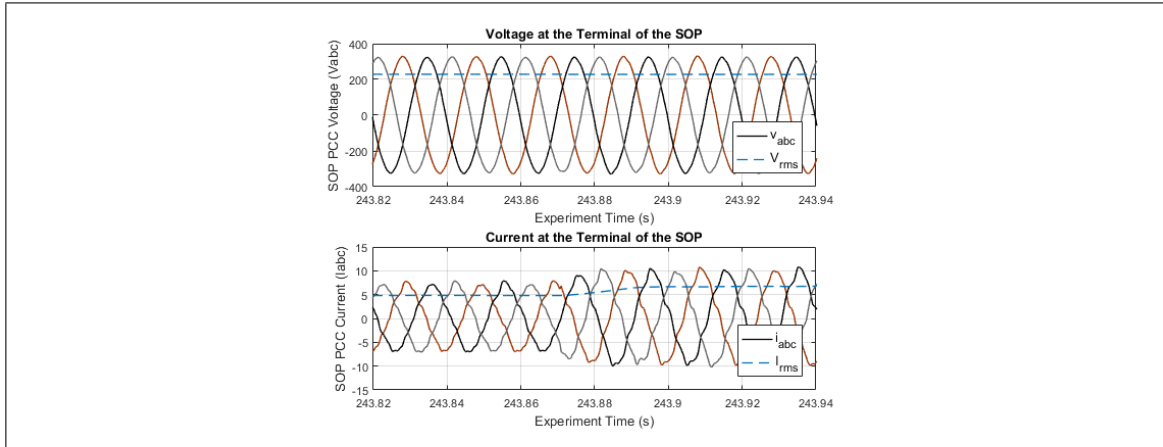


Figure A.3. The AC voltage and current at the terminals of the SOP showing a change in the current output from 5 Arms to 7 Arms and a voltage change of 228.6 Vrms to 227.9 Vrms.

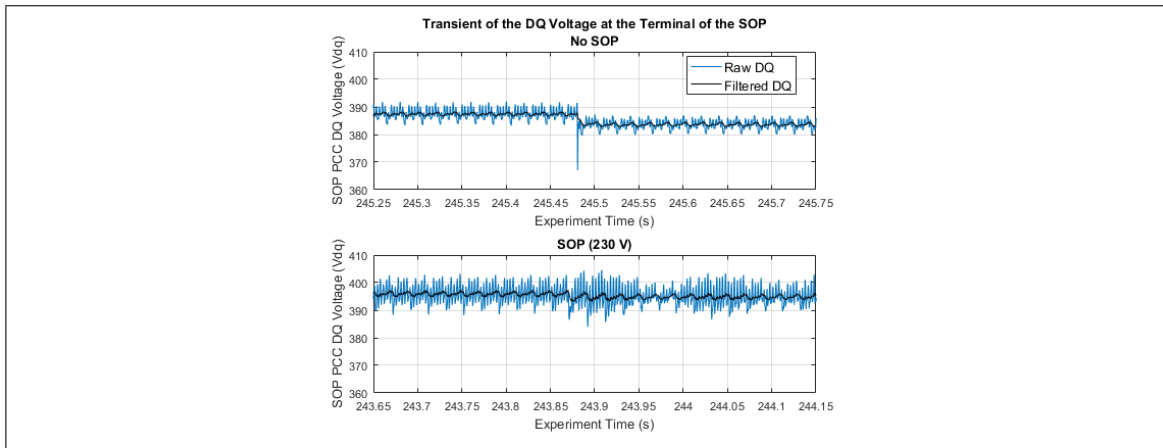


Figure A.4. The DQ voltage at the terminal of the SOP for a step load change from 4750 kW to 7250 kW. The top graph shows the voltage when no SOP is connected at the end of the feeder and the bottom graph shows the voltage when the SOP is connected and the droop function is set to 230 Vrms.

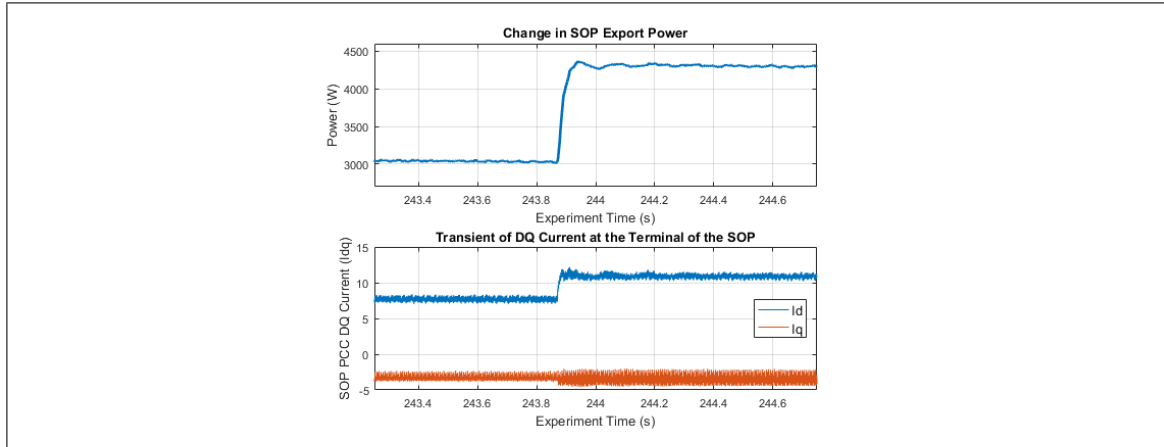


Figure A.5. SOP export power (top) and DQ current from the SOP (bottom) for a step load change from 3 kW to 4.3 kW.

B. APPENDIX: SUBMITTED PAPER

A paper with the results of this investigation was submitted for publication in "IEEE Transactions on Sustainable Energy" on December 5th. This manuscript is presented on the next page.

Stability Analysis of Soft-Open-Points in Distribution Networks with Distributed Generation

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Timothy C. Green, *Fellow Member, IEEE* and Felipe Nuñez, *Member, IEEE*,

Abstract—Distributed generation and charging of electric vehicles are growing rapidly and causing existing distribution networks to operate under stress with the potential to violate the power flow and voltage limits of feeders and transformers. An attractive solution to this problem is the additional control options provided by soft-open-points that allow controlled power flow between previously unconnected feeders and additional reactive power injection. This paper presents an analysis of the small-signal stability of an LV distribution network with a back-to-back inverter as soft-open-point and distributed generation operating in constant-power mode. The analysis reveals that the system is stable for any practical feeder length and load impedance as long as the controller gains are well chosen. Special attention is required on the design of the LC filters and coupling inductors, where a damping resistor is recommended to provide stability. The analysis has been verified by simulation and implementation in an experimental distribution network.

Index Terms—Soft-open-point (SOP), Active Distribution Network (ADN), power-flow control, grid-connected inverters, distributed generation, PV inverters, Constant power generation (CPG).

I. INTRODUCTION

THE reliability of the distribution network in modern cities can suffer a significant degradation due to the massively connection of distributed generation (DG) and charging of electric vehicles (EVs) [1]. The integration of several DG technologies, such as photovoltaic (PV), fuel cells, vehicle-to-grid, micro turbines, battery storage systems and small combined heat (CHP) plants, have turn the passive distribution grid into an active distribution network (ADN).

PV generation in low voltage residential areas has been integrated very fast due to the decreasing costs of the technology. However, high penetration levels of PV generation can produce overvoltage in low voltage feeders during high irradiance and low load periods, decreasing the steady state and transient stability of the network, specially the bus voltage magnitudes [2], [3], caused by the maximum power point (MPP) operation and the high variability of the solar irradiation. In countries with high PV integration, such as Germany, the PV systems with a peak power below 30 kW must limit the maximum generation power if they are not remotely controlled by the utility [4]. Additionally, PV inverters shall not actively regulate the voltage at the point of common coupling (PCC) according to standards such as IEEE 1547. Therefore, PV inverters work as constant power sources from a short-term perspective, decreasing damping and increasing the instabilities of the grid in comparison to standard grids with passive loads, even when they operate under ideal conditions [5]. Additionally,

the impact of LV residential PV penetration is very different according to the zone and customer profile [6]. On the other side, the large-scale integration of EVs in high populated urban areas poses a huge challenge for the distribution system operators (DSOs) because similarly to DG, they have no control over the locations and charging times. Some studies conclude the required network reinforcement can reach values up to 19% of total actual network costs in a situation without EVs [7].

The network expansion planning is becoming more complex and expensive due to the high degree of uncertainty introduced by DG and EVs, so the modernization of the LV distribution network is critical to integrate these technologies. A meshed distribution network can be an option but the upgrade cost of infrastructure could be prohibitive, it causes disruption and brings other technical problems such as circulating currents and fault current to increase, which can already be high in urban areas. A feasible solution is to migrate to a soft-meshed distribution network using power converters to control the power flow in the existing normal-open-points of the network, operating these as soft-open-points (SOP) [8], [9]. The benefits and feasibility of SOP technology had been proved [10], including its ability to address the problems caused by PV penetration [11] and to control the voltage in distribution networks [12]. However, the stability of a distribution network with soft-open-point and DG has not been studied.

The stability of grid-connected inverters have been widely covered for DG [13], but have not been addressed for SOPs. The interaction between distributed PV generation and SOPs could contribute to destabilize the system. As an example, the PV generator can produce an overvoltage in the feeder, whereupon the soft-open-point will attempt to reduce the voltage by extracting active power from the feeder, acting like an active load. Therefore, it is important to ensure small-signal stability of the system under small changes in operating conditions, load perturbations, DG operation and parameter variations.

This paper provides a small-signal model for a simplified LV distribution network with DG and a soft-open-point between feeders. The network consists in two feeders, one connected with a constant impedance load and the other connected to a DG working as a constant power source. The stability of the systems is analysed evaluating the eigenvalues from the state-space matrix of the linearized full model, allowing study interactions between states, participation analysis and eigenvalue trace analysis. This analysis is supported by simulation and implementation in an experimental distribution network.

II. CASE OF STUDY

The distribution network considered in this paper is illustrated in Fig. 1. The network has been simplified to focus on the analysis of the interaction between the soft-open-point and the DG in the LV network. The substation and transformers are not taken into account and are replaced by a three-phase voltage source with rating values of 400 V and 50 Hz. Each feeder is modeled as a constant impedance, one feeder has a passive load connected and the other feeder has a constant power source (distributed generator) connected. The soft-open-point is connected across the normally-open-point of the network, which is at the end of each feeder.

The DG is operated as a grid-supporting or feeding unit [14] that injects constant active power without voltage or frequency regulation support. Fig.1 shows a PV system as example, but can be any DG technology connected through a power inverter such as fuel cells, vehicle-to-grid, micro turbines, BESS, variable speed wind turbines or a CHP plant.

The back-to-back converter that operates as soft-open-point can offer support or correction to a number of features of the network such as voltage magnitude, unbalance currents, transformer thermal constraints and harmonics. However, for simplification in this paper the soft-open-point exclusively regulates the local voltage magnitude through active and reactive power; and the average model of the inverters is used. Usually the voltage is regulated by reactive power, but LV distribution networks have high R/X ratios, so active power can be equally or more effective than reactive power. Therefore, if active power is passing through the soft-open-point, one feeder would see the SOP as a constant power load and the other feeder would see it as a grid-feeding unit [14] with active and reactive power support.

III. MODELING

The model has a series of simplifications to reduce or eliminate specific characteristics introduced by high level controls, MPPT and PLL. The idea is to obtain a general and comprehensive benchmark model of the system reducing collateral factors to facilitate and clarify the analysis.

A. Soft-open-point (Back-to-back inverter)

The high-level or tertiary control of a soft-open-point will be a multi-objective controller that uses real and reactive power plus harmonic compensation to improve a variety of network conditions such as voltage magnitude, feeder currents, transformer over-load, unbalanced current, harmonic voltages and power losses in the network. This high-level control is not covered in this paper but it can be implemented using a set of algorithms tailored to each problem overseen by a priority list or an overall cost function.

The secondary and primary controls are illustrated in Fig. 2 for one side of the SOP; the other side control is identical, and they just differ in the calculation of the active power component. An EPPL and sequence calculator are present for the necessary decomposition of voltage and current in each loop. The stability of a soft-open-point in the presence of a power injection from a DG will be illustrated using real and

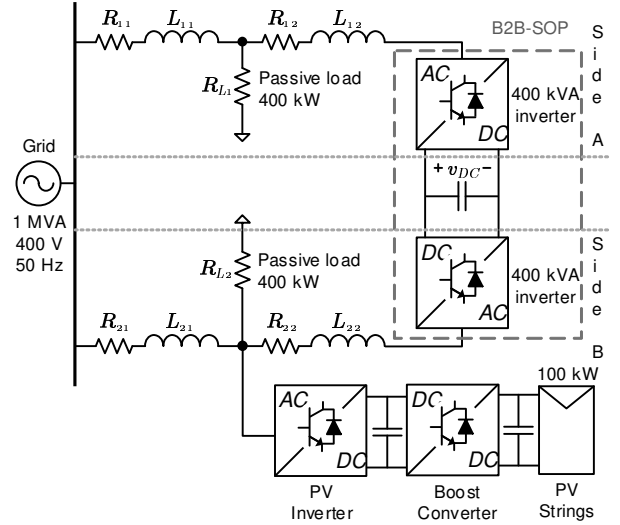


Fig. 1. Model of the network.

reactive power droop controllers responsible for supporting network voltage regulation. It is assumed that the network is balanced so that the impedance controllers and the negative and zero sequence are inactive and only the positive sequence controller is modelled.

The soft-open-point uses droop controllers in order to decide how much active power transfers between the feeders and how much reactive power injects or absorbs at each feeder. Both inverters maintains the quadrature voltage of their respective side through (1) and differs in how they calculate the active power transference in each feeder. The inverter in side A follows (2) in order to maintain the direct voltage of its side; and the inverter in side B maintains the DC-link voltage through (3).

$$Q^{*j} = k_Q^j \cdot (v_Q^* - v_{rms}^j), \quad (1)$$

$$P^{*A} = k_P^A \cdot (v_P^* - v_{rms}^A), \quad (2)$$

$$P^{*B} = k_P^B \cdot (v_{DC}^* - v_{DC}), \quad (3)$$

where $k_i^j, v_i^*, v_{rms}^j, v_{DC}^*, v_{DC} \in \mathbb{R}, i \in \{P, Q\}$, and $j \in \{A, B\}$.

The active and reactive power references calculated previously are introduced in the power controller to calculate the direct (4) and quadrature (5) reference currents of the inverter. Finally, the reference currents are introduced in the PR current controller shown in Fig. 3.

$$q\{i_{i_dq}\}^{*j} = \frac{P^{*j} \cdot d\{v_{o_dq}^j\} + Q^{*j} \cdot d\{v_{o_dq}^j\}}{d\{v_{o_dq}^j\}^2 + q\{v_{o_dq}^j\}^2} \quad (4)$$

$$d\{i_{i_dq}\}^{*j} = \frac{P^{*j} \cdot q\{v_{o_dq}^j\} - Q^{*j} \cdot d\{v_{o_dq}^j\}}{d\{v_{o_dq}^j\}^2 + q\{v_{o_dq}^j\}^2}, \quad (5)$$

B. Distributed Generator (DG)

The DG control loop utilizes a Power controller and a PI current controller, operating in the dq reference frame (Fig. 4). The power controller follows (4) and (5), the same as the SOP.

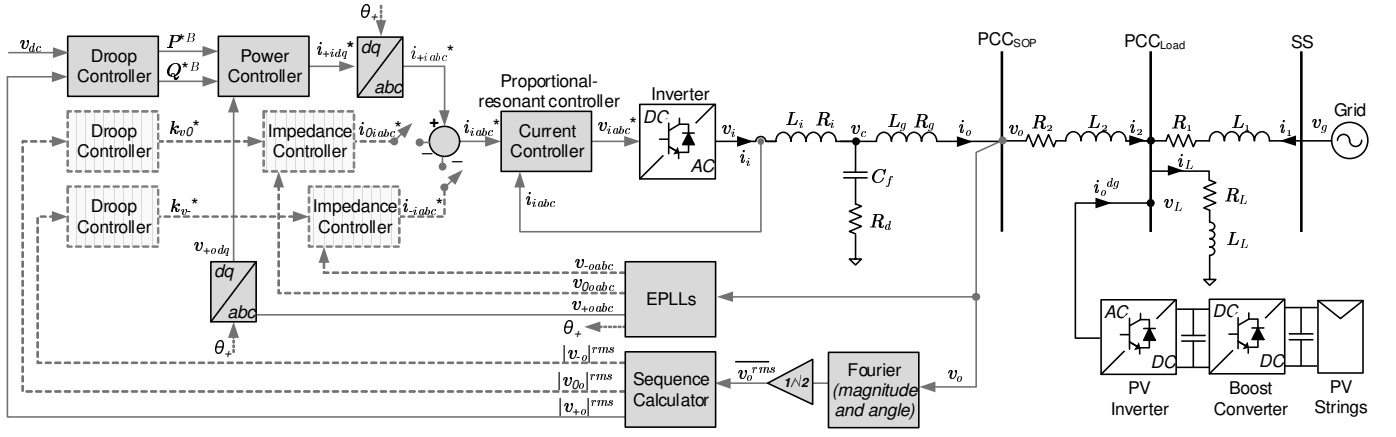


Fig. 2. Model of side B of the Soft-open-point.

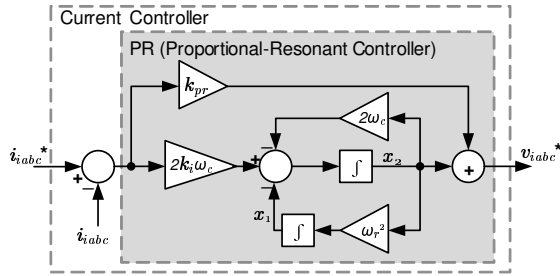


Fig. 3. Proportional-Resonant current controller of Fig. 2.

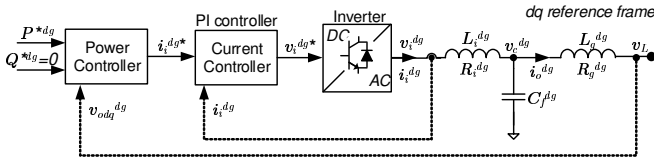


Fig. 4. DG Inverter working as Constant Power Generator (CPG).

The controller implemented is used to operate the DG with constant active power and represents a generic DG technology. As a PV string is being used as an example, the maximum power point tracker (MPPT) of the PV specifies the active power reference used in the Power controller. The MPPT can be implemented with several methods and each one of these adds different characteristics to the model, which makes the analysis more complex. Therefore and for simplicity, the MPPT and the DC-DC converter are not considered and they are replaced by a DC voltage source. This simplification eliminates all specific behaviours introduced by the PV system to the model, allowing an omnibus analysis that can represent any DG technology connected to the grid through a power inverter.

IV. STATE-SPACE MODEL REPRESENTATION

As mentioned before, the system is divided into two subsystems, in which side B controls the voltage of the DC-Link. Both sides are divided into three parts: the controllers and inverter; the LCL filter; and the feeder sections and passive

TABLE I
STATE VARIABLES OF THE SYSTEM ΔX

	Description	State variable
SOP	Output of integral 1 (PR controller)	$\Delta x_{1_dq}^j$
	Output of integral 2 (PR controller)	$\Delta x_{2_dq}^j$
	Output voltage of inverter	$\Delta v_{i_dq}^j$
	LCL filter input current	$\Delta i_{i_dq}^j$
	LCL filter output current	$\Delta i_{o_dq}^j$
	LCL filter capacitor voltage	$\Delta v_{c_dq}^j$
Lines and Load	Current from substation to PCC_{Load}	$\Delta i_{1_DQ}^j$
	Current from SOP to PCC_{Load}	$\Delta i_{2_DQ}^j$
	Current of the passive load	$\Delta i_{L_DQ}^j$
	Output voltage of SOP	$\Delta v_{o_DQ}^j$
	Voltage of the passive load	$\Delta v_{L_DQ}^j$
DG	Output of integral (PI controller)	$\Delta x_{3_dq}^j$
	Output voltage of inverter	$\Delta v_{i_dq}^j$
	LCL filter input current	$\Delta i_{i_dq}^j$
	LCL filter output current	$\Delta i_{o_dq}^j$
	LCL filter capacitor voltage	$\Delta v_{c_dq}^j$
DC Link	DC-Link capacitor voltage	Δv_{DC}

load. In addition to this, side B has an extra part, corresponding to the DG.

The inverters and LCL filter variables are defined in their own local dq reference frames, defined by the angle in the PCC_{SOP} of their respective side. The subsystems are integrated in a common DQ reference frame defined by the angle in the SS bus-bar. Therefore the matrices T are used to transform the variables from the local reference frames dq to the common reference frame DQ .

$$x_{a_DQ}^j = T^j x_{a_dq}^j \quad (6)$$

where $T^j \in \mathbb{R}^{2 \times 2}$ and $j \in \{A, B\}$.

A. Controllers and Inverters

As the droop and power controllers are non-linear, the system is linearized around the operation point (x_0, u_0) , given in

the appendices. Then, the non-linear controllers are linearized and combined in the Jacobian matrices J^A and J^B as

$$\begin{bmatrix} \Delta i_{i-dq}^{*A} \\ \Delta i_{i-dq}^{*B} \end{bmatrix} = \begin{bmatrix} J^A & 0_{2 \times 3} \\ 0_{2 \times 2} & J^B \end{bmatrix} \begin{bmatrix} \Delta v_{o-dq}^A \\ \Delta v_{o-dq}^B \\ \Delta v_{DC} \end{bmatrix}, \quad (7)$$

where $v_{DC} \in \mathbb{R}$, $j \in \{A, B\}$, $J^A \in \mathbb{R}^{2 \times 2}$, $J^B \in \mathbb{R}^{2 \times 3}$.

The model of the SOP current controllers are calculated from Fig. 3, where x_1 and x_2 are the integral outputs.

$$\begin{bmatrix} \Delta \dot{x}_{1-dq}^j \\ \Delta \dot{x}_{2-dq}^j \end{bmatrix} = A_{CC}^j \begin{bmatrix} \Delta x_{1-dq}^j \\ \Delta x_{2-dq}^j \end{bmatrix} + B_{CC}^j \begin{bmatrix} \Delta i_{i-dq}^{*j} \\ \Delta i_{i-dq}^{*j} \end{bmatrix} \quad (8)$$

$$\begin{bmatrix} \Delta v_{i-dq}^{*j} \end{bmatrix} = C_{CC}^j \begin{bmatrix} \Delta x_{1-dq}^j \\ \Delta x_{2-dq}^j \end{bmatrix} + D_{CC}^j \begin{bmatrix} \Delta i_{i-dq}^{*j} \\ \Delta i_{i-dq}^{*j} \end{bmatrix}, \quad (9)$$

where $A_{CC}^j, B_{CC}^j \in \mathbb{R}^{4 \times 4}$, $C_{CC}^j, D_{CC}^j \in \mathbb{R}^{2 \times 4}$, and $j \in \{A, B\}$.

The inverter is represented using the average model, as the first order transfer function $H(s) = v_{idq}/v_{idq}^* = 1/(1 + 1.5T_s s)$, where s is the Laplace variable. The functions includes the PWM and inverter delay [15].

$$\begin{bmatrix} \Delta v_{i-dq}^j \end{bmatrix} = A_{delay}^j \begin{bmatrix} \Delta v_{i-dq}^j \end{bmatrix} + B_{delay}^j \begin{bmatrix} \Delta v_{i-dq}^{*j} \end{bmatrix} \quad (10)$$

where $A_{delay}^j, B_{delay}^j \in \mathbb{R}^{2 \times 2}$, and $j \in \{A, B\}$.

B. LCL Filter model

The LCL filter model seen in Fig. 2 is the same for both sides of the SOP and its differential equations (11) are obtained by the dynamics of the capacitor, the inductors and the damping resistor.

$$\begin{bmatrix} \Delta \dot{i}_{i-dq}^j \\ \Delta \dot{i}_{o-dq}^j \\ \Delta \dot{v}_{c-dq}^j \end{bmatrix} = A_{LCL}^j \begin{bmatrix} \Delta i_{i-dq}^j \\ \Delta i_{o-dq}^j \\ \Delta v_{c-dq}^j \end{bmatrix} + B_{LCL}^j \begin{bmatrix} \Delta v_{i-dq}^j \\ \Delta v_{o-dq}^j \end{bmatrix} \quad (11)$$

where $A_{LCL}^j \in \mathbb{R}^{6 \times 6}$, $B_{LCL}^j \in \mathbb{R}^{6 \times 4}$, $j \in \{A, B\}$.

C. Distributed Generator (DG) model

The DG inverter uses the same nonlinear power controller of the SOP inverter (Fig. 4). A Jacobian matrix is also calculated to obtain the small-signal model.

$$\begin{bmatrix} \Delta i_{i-dq}^{*dg} \end{bmatrix} = J^{dg} \begin{bmatrix} \Delta v_{L-dq} \end{bmatrix} \quad (12)$$

where $J^{dg} \in \mathbb{R}^{2 \times 2}$.

The model of the DG current controller is a standard decoupled PI controller and is calculated in (13) and (14) where x_3^{dg} is the integral output.

$$\begin{bmatrix} \Delta \dot{x}_{3-dq}^{dg} \end{bmatrix} = A_{CC}^{dg} \begin{bmatrix} \Delta x_{3-dq}^{dg} \end{bmatrix} + B_{CC}^{dg} \begin{bmatrix} \Delta i_{i-dq}^{*dg} \\ \Delta i_{i-dq}^{*dg} \end{bmatrix} \quad (13)$$

$$\begin{bmatrix} \Delta v_{i-dq}^{*dg} \end{bmatrix} = C_{CC}^{dg} \begin{bmatrix} \Delta x_{3-dq}^{dg} \end{bmatrix} + D_{CC}^{dg} \begin{bmatrix} \Delta i_{i-dq}^{*dg} \\ \Delta i_{i-dq}^{*dg} \end{bmatrix} \quad (14)$$

where $A_{CC}^{dg}, C_{CC}^{dg} \in \mathbb{R}^{2 \times 2}$, $B_{CC}^{dg}, D_{CC}^{dg} \in \mathbb{R}^{2 \times 4}$.

The model of the LCL filter is calculated as in (11), and it does not include a damping resistor.

$$\begin{bmatrix} \Delta \dot{i}_{i-dq}^{dg} \\ \Delta \dot{i}_{o-dq}^{dg} \\ \Delta \dot{v}_{c-dq}^{dg} \end{bmatrix} = A_{LCL}^{dg} \begin{bmatrix} \Delta i_{i-dq}^{dg} \\ \Delta i_{o-dq}^{dg} \\ \Delta v_{c-dq}^{dg} \end{bmatrix} + B_{LCL}^{dg} \begin{bmatrix} \Delta v_{i-dq}^{dg} \\ \Delta v_{L-dq}^{dg} \end{bmatrix} \quad (15)$$

where $A_{LCL}^{dg} \in \mathbb{R}^{6 \times 6}$, $B_{LCL}^{dg} \in \mathbb{R}^{6 \times 4}$.

A transformation matrix T^{dg} is also used to transform the variables in the dq reference frame of the DG.

D. Passive components (line and R-L load)

The first section of the feeder connects the grid with PCC_{Load} and the second section connects PCC_{Load} with PCC_{SOP}.

$$\begin{bmatrix} \Delta \dot{i}_{1-DQ}^j \\ \Delta \dot{i}_{2-DQ}^j \\ \Delta \dot{i}_{L-DQ}^j \end{bmatrix} = A_{RL}^j \begin{bmatrix} \Delta i_{1-DQ}^j \\ \Delta i_{2-DQ}^j \\ \Delta i_{L-DQ}^j \end{bmatrix} + B_{RL}^j \begin{bmatrix} \Delta v_{g-DQ} \\ \Delta v_{o-DQ} \\ \Delta v_{L-DQ}^j \end{bmatrix} \quad (16)$$

where $A_{RL}^j, B_{RL}^j \in \mathbb{R}^{6 \times 6}$, $j \in \{A, B\}$.

E. State-space matrix of the system

The DC-Link capacitor voltage dynamics depends on the direct current that circulates between the inverters and the feeders [15]. Therefore, the dynamic of the DC-Link voltage is established in (17).

$$\begin{bmatrix} \Delta \dot{v}_{DC} \end{bmatrix} = A_{DC} \begin{bmatrix} \Delta v_{DC} \end{bmatrix} + B_{DC} \begin{bmatrix} d\{\Delta i_{i-dq}^A\} \\ d\{\Delta i_{i-dq}^B\} \end{bmatrix}, \quad (17)$$

where $A_{DC} \in \mathbb{R}$ and $B_{DC} \in \mathbb{R}^{1 \times 2}$.

The node voltages v_o and v_L are used as input in the previous subsystems. In order to define these variables, a small virtual capacitor (VC) is connected between these nodes and ground.

$$\begin{bmatrix} \Delta \dot{v}_{o-DQ}^j \\ \Delta \dot{v}_{L-DQ}^j \end{bmatrix} = A_{VC}^j \begin{bmatrix} v_{o-DQ}^j \\ v_{L-DQ}^j \end{bmatrix} + B_{VC}^j \begin{bmatrix} i_{1-DQ}^j \\ i_{2-DQ}^j \\ i_{o-DQ}^j \\ i_{L-DQ}^j \\ i_{o-DQ}^{DG} \\ i_{L-DQ}^{DG} \end{bmatrix}, \quad (18)$$

where $A_{VC}^j \in \mathbb{R}^{4 \times 4}$, $B_{VC}^j \in \mathbb{R}^{4 \times 10}$, and $j \in \{A, B\}$.

Finally, the state-space matrices of the system are obtained from all the previous equations and the state variables are listed in Table I, where $j \in \{A, B\}$.

$$\begin{bmatrix} \dot{X} \end{bmatrix} = \begin{bmatrix} \Delta \dot{X}^A \\ \Delta \dot{X}^B \\ \Delta \dot{X}^{dg} \\ \Delta \dot{v}_{DC} \end{bmatrix} = A \begin{bmatrix} \Delta X^A \\ \Delta X^B \\ \Delta X^{dg} \\ \Delta v_{DC} \end{bmatrix} + B \begin{bmatrix} \Delta v_{g-DQ} \end{bmatrix} \quad (19)$$

where $A \in \mathbb{R}^{55 \times 55}$ and $B \in \mathbb{R}^{55 \times 2}$.

V. STABILITY ANALYSIS

Stability of the operation point (x_0, u_0) can be established by analyzing the linearized system (19). To conduct the analysis, the values of the components were chosen according to the experimental setup, meanwhile the parameters of the controllers were obtained through analysis; these values are presented in the appendices. The low frequency eigenvalues of the matrix A are shown in Fig. 5. It can be seen that all the eigenvalues have negative real part, then by *Proposition 5.5.6* in [16], the linearized system is globally asymptotically controllable to (x_0, u_0) , which implies by *Theorem 19* in [16] that the original nonlinear system is locally asymptotically controllable to (x_0, u_0) , i.e., there exists a (linear) feedback law that renders the operation point (x_0, u_0) a locally asymptotically stable equilibrium for the closed-loop system.

An analysis was made using participation factors [17] to see how each state contribute to the different eigenvalues (Fig. 5). The states corresponding to the DC-link, the LCL filter of the B side and the controllers of both inverters of the SOP established the most critical eigenvalues; so it is important to carefully choose the parameters related with these states.

Fig. 6 shows eigenvalue traces when the parameters of the SOP current controller varies. It can be seen that the system is more sensitive to the proportional parameters. It is also noted that by increasing both values, the higher frequency poles tends to the right-half side and the lower frequency poles goes to the left side, increasing their damping.

In Fig. 7 the traces show the change in the Droop controllers parameter. As is seen on the figure, by increasing the value of either parameter, the system is closer to become unstable, and active power parameter has more influence on the stability. It is also important to note that k_p^A has a bigger effect on the stability than k_p^B . As these parameters indicate how much active and reactive power is transferred between the SOP and the feeders, these must be chosen carefully in order to achieve a faster response without the system becoming unstable. It is also noted that the system allows a higher value of active power than reactive power to be transferred through the SOP.

An inverse mode is established, in which the side A inverter regulates the DC-Link voltage and the side B inverter transfers power from the DC-Link into its feeder. The analysis of the eigenvalues for this mode showed that the system is stable for the same parameters. In Table II the stability conditions for the parameters of the controllers are shown; each of these are established when the other parameters remained constant. It is observed that in inverse mode the parameters has bigger ranges, allowing for a more flexible operation.

Fig. 8 shows the eigenvalue traces when the damping resistor of the SOP LCL Filters varies for both modes. It can be seen that an increment of the resistor increases the damping of the poles and improves the stability of the system. Although, a bigger resistor leads to a higher power dissipation on the filter. The trace also shows that the inverse mode requires a smaller damping resistor to stabilize the system. The minimum values of R_d in order to achieve stability are 3.6Ω and 1.5Ω for normal and inverse mode, respectively.

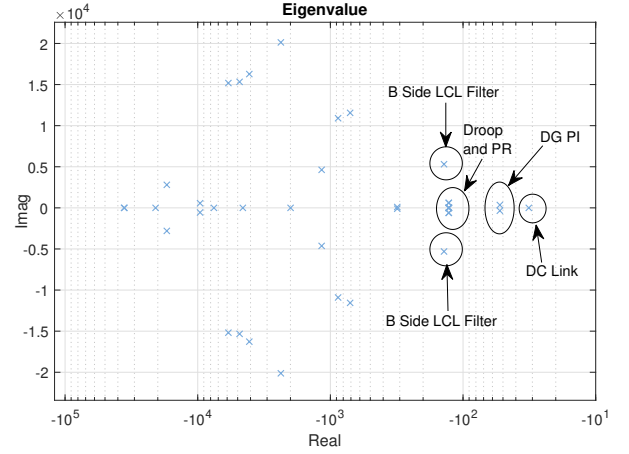


Fig. 5. Eigenvalues of the system.

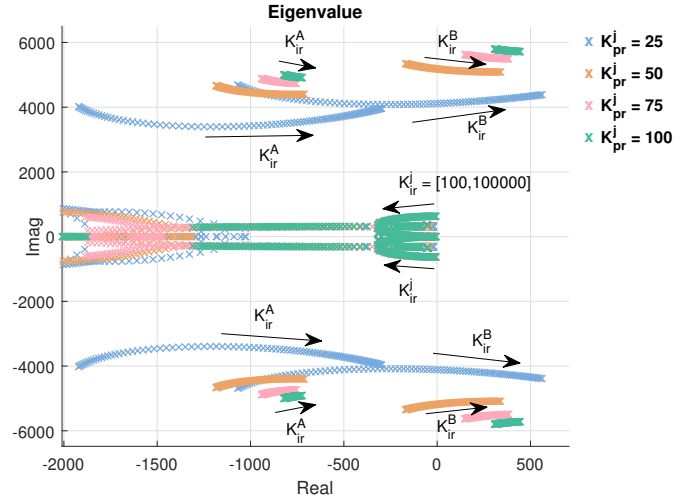


Fig. 6. Eigenvalue traces of PR Current controller. $j \in \{A, B\}$

TABLE II
STABILITY CONDITIONS FOR THE DROOP AND PR CONTROLLERS

Parameter	Normal mode (Side B regulates DC link)	Inverse mode (Side A regulates DC link)
k_{pr}^A	< 95000	< 7000
k_{pr}^B	< 60	< 106000
k_{ir}^A	< 270000	< 140000
k_{ir}^B	< 40000	< 230000
k_p^A	< 310	> -6300
k_p^B	> -4800	< 330
k_q^A	< 310	< 120
k_q^B	< 220	< 1000

Fig. 9 shows the eigenvalue traces when the DC-Link capacitor value varies. From this analysis, it can be seen that the system in normal mode is unstable if $C_{DC} = [50, 100]\mu F$; in reverse mode, the value of the capacitor does not make the system unstable. Therefore, the value of the capacitor must be bigger than $0.1mF$; although a bigger value, will decrease the damping of some eigenvalues.

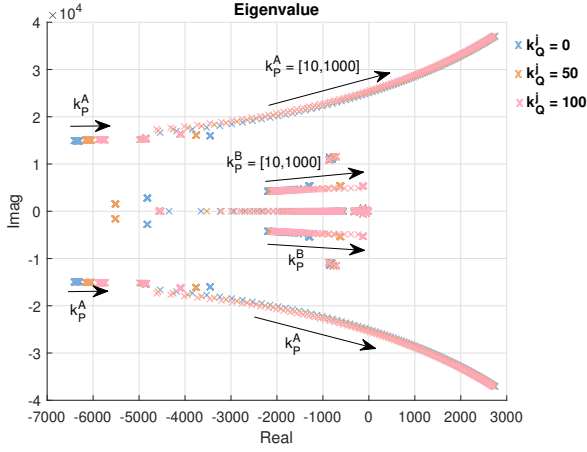
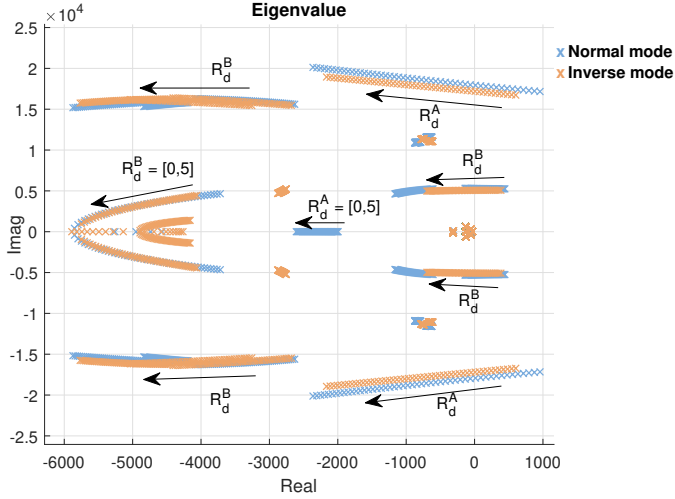
Fig. 7. Eigenvalue traces of Droop controller. $j \in \{A, B\}$ 

Fig. 8. Eigenvalue trace of Damping resistor.

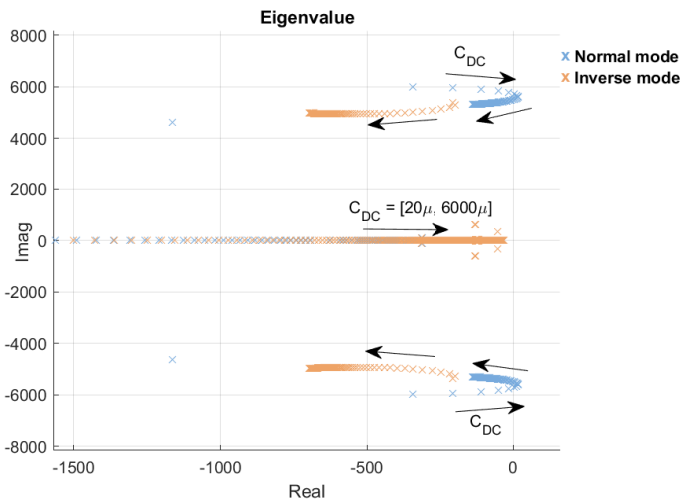


Fig. 9. Eigenvalue trace of DC Link Capacitor.

VI. EXPERIMENTAL RESULTS

The model of the network represented in Fig. 1 was tested experimentally at a reduced scale, as illustrated in Fig. 10,

according to the available laboratory facilities. The substation was experimentally implemented using a 90 kVA inverter labeled as "Grid Emulator 90 kVA" in Fig. 10. A voltage and frequency controller was used to regulate the substation voltage to 230 V phase to neutral at 50 Hz. The SOP was experimentally tested using two 10 kVA inverters in a back-to-back configuration and is labelled as "B2B-SOP" in Fig. 10. The SOP was programmed with the controllers described above in reverse mode, meaning that the side not connected to the PV Inverter Emulator regulates the DC-bus. The parameters utilized in the experiment can be found in the appendices.

The PV inverter was experimentally tested using a 10 kVA inverter which was programmed with an inner current controller and an output PQ controller. The converter was programmed to step through different P set-points which represented step-changing of load and generation on the feeder. The PV inverter was able to act as both as a load and as a generator by importing or exporting power. Load connected to the mid-point of the feeder caused the voltage to drop and generation caused the voltage to rise. The droop parameters of the SOP could be set to determine if the end of feeder voltage increases or decreases. In a feeder that normally contains a lot of generation, the droop voltage set-point could be reduced to enable more generation to connect. When there is a lot of load present, the droop set-point could be increased to allow more load to connect.

The experiment tested different droop set-points at different generation and load conditions, which was emulated by changing the power output of the inverter connected at the mid-point of the feeder.

Fig. 11 shows the voltage at the terminal of the SOP when the inverter connected to the mid-point of the feeder changed from 7.4 kW to -7.4 kW. Five experiments were performed, one with no SOP connected, others with the SOP droop set-point set changed from 220 V to 235 V at 5 V intervals. The voltage set-point of the 90 kVA inverter and power set-points of the inverter connected to the mid-point of the feeder were the same for each of the five experiments. Without the SOP there is a large voltage deviation of approximately 12 V at the end of the feeder (blue line). When the SOP was operating, the droop controller reduced the voltage deviation to approximately 4 V between maximum generation and maximum load for all set-points tested. The variation of SOP droop set-point was also able to change the end-of-feeder voltage. With the set point at 235 V, the voltage varied in a narrow range just below the set point. With set point at 220 V the variation was just above. The AC voltage and current output for the set-point of 230 V is shown in Fig. 12.

The response to a load change from 3 kW to 4.3 kW was also tested. Fig. 13 shows the terminal voltage when the SOP is not connected and when it is connected. Fig. 14 shows the change in real power export and DQ current of the SOP for this load change. The time stamps of the SOP, substation and PQ inverter is not synchronised and subsequently there is a difference. It can be seen that the droop method is stable during this transient, the oscillations observed in the SOP export power, and DQ current in Fig. 14 are damped and the SOP is stable.

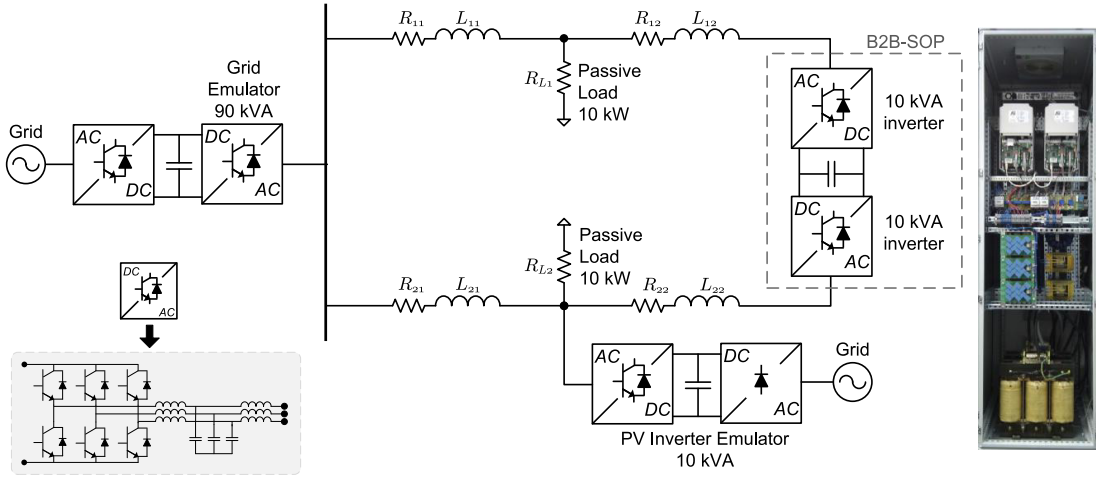


Fig. 10. Experimental setup.

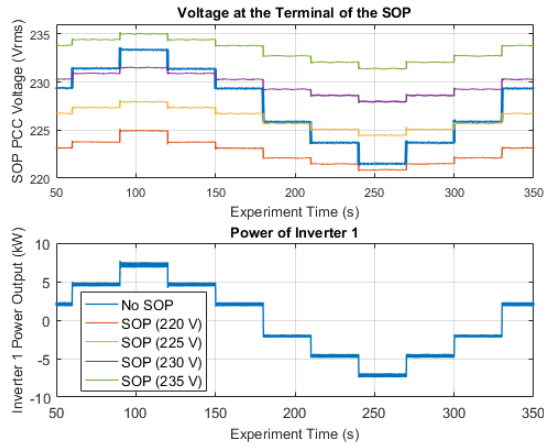


Fig. 11. The top graph shows the average phase to neutral RMS voltage of the three phases at the end of the feeder for when the SOP is disconnected ("No SOP") and when the SOP has a droop set-point of 220 V to 235 V at 5V intervals. The bottom graph shows the power of the inverter connected to the mid-point of the feeder. Positive power represents generation and negative power represents load.

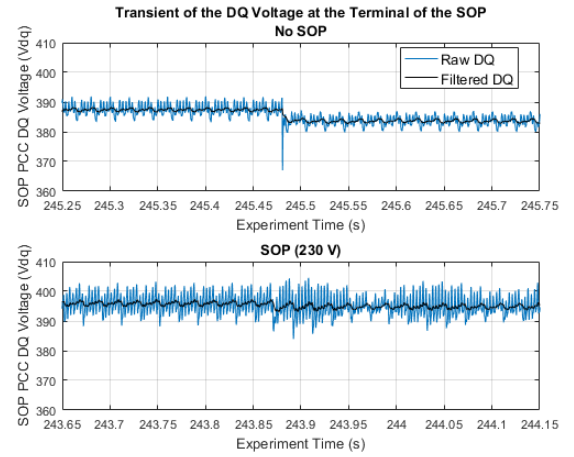


Fig. 13. The DQ voltage at the terminal of the SOP for a step load change from 4750 kW to 7250 kW. The top graph shows the voltage when no SOP is connected at the end of the feeder and the bottom graph shows the voltage when the SOP is connected and the droop function is set to 230 Vrms.

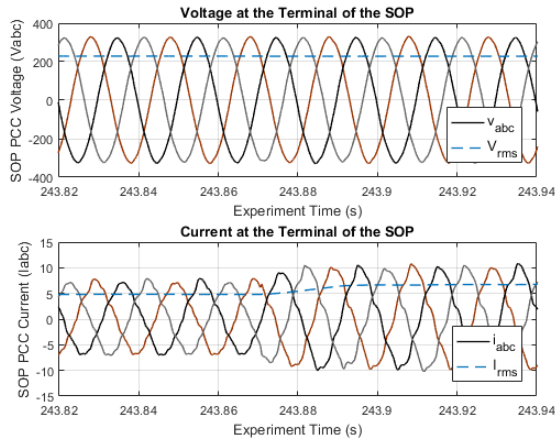


Fig. 12. The AC voltage and current at the terminals of the SOP showing a change in the current output from 5 Arms to 7 Arms and a voltage change of 228.6 Vrms to 227.9 Vrms.

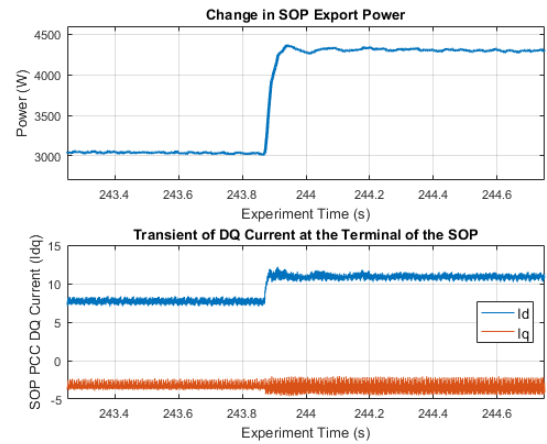


Fig. 14. SOP export power (top) and DQ current from the SOP (bottom) for a step load change from 3 kW to 4.3 kW.

VII. CONCLUSION

In this paper a small-signal stability analysis was performed for a distribution network containing passive loads and grid-tied inverters in a PV distributed generator and a soft-open point. The SOP was tested in two modes, in each mode one of the inverters acts as an active load to its feeder, while the other acts as a controlled voltage power source; both inverters also provide regulation of reactive power.

The analysis shows that choosing properly the parameters for the controllers and filters, the linearized system is globally asymptotically controllable to an operation point for standard length feeders and load impedance, then the non-linear model is locally asymptotically controllable to the operation point. It could be seen that an increase in the parameters of the controllers leads to instability, so these must be chosen carefully in order to avoid it. It is important also to note that the inclusion of a damping resistor connected in series to the capacitor of the SOP LCL filters greatly increment the stability of the system in exchange of an increment of power losses, so it is encourage to add it, but its value must be analyzed in order to reduce these inconveniences.

The droop algorithm was successful in reducing the variation of voltage between the operation under peak load and peak generation of the feeder, enabling the feeder to operate in a wider range of conditions. Further, the set-point of the droop regulator could be used to move the average voltage at the end of feeder up or down. The experimental results verified that the SOP droop-controller is stable and well damped when operation in a low voltage distribution network.

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APPENDIX A
PARAMETERSTABLE III
PARAMETERS OF THE SYSTEM

	Description	Parameter	Value
SOP	Filter capacitor	C_f^j	24.6 [μF]
	Damping resistor	R_d^j	5 [Ω]
	Filter inductor	L_f^j	2.3 [mH]
		R_i^j	0.1 [Ω]
	Coupling inductor	L_g^j	0.93 [mF]
		R_g^j	0.03 [Ω]
DG (dg)	Switching frequency	f_s^j	10 [kHz]
	Filter capacitor	C^{dg}	8.8 [μF]
	Filter Inductor	L_f^{dg}	2.3 [mH]
		R_f^{dg}	0.1 [Ω]
	Coupling Inductor	L_c^{dg}	0.93 [mF]
Feeder		R_c^{dg}	0.03 [Ω]
	SS-load line	R_1^j	0.47 [Ω]
		L_1^j	1.2 [mF]
	SOP-load line	R_2^j	0.1 [Ω]
		L_2^j	1.2 [mF]
	Passive Load	R_L^j	16 [Ω]
		L_L^j	0.01 [mF]
	Grid frequency	ω	$2\pi \cdot 50$ [rad/s]
	Virtual Capacitor	C_{VC}	1 [pF]

TABLE IV
PARAMETERS OF THE CONTROLLERS

	Description	Parameter	Value
SOP	Current Controller	k_{ir}^j	10000
		k_{pr}^j	50
		ω_c^j	$2\pi \cdot 0.1$
		ω_r^j	$2\pi \cdot 50$
		k_P^A	+100 [W/V]
	Droop Controller	k_P^B	-100 [W/V]
		$k_Q^{\{A,B\}}$	100 [W/V]
		v_P^*	230 [V]
		v_Q^*	230 [V]
		v_{DC}^*	600 [V]
DG (dg)	Current Controller	$k_{i_{dg}}^{dg}$	100
		k_p^{dg}	1
	Power Controller	P^{*dg}	10 [kW]
		Q^{*dg}	0 [kW]

TABLE V
OPERATION POINT VALUES

Parameter	Side A value	Side B value
$x_{1_dq}^j$	4.7×10^7 6×10^6	4.6×10^7 -3.8×10^5
$x_{2_dq}^j$	480 60	472 -3.8
$v_{i_dq}^j$	392 -11.2	400 -5.9
$i_{i_dq}^j$	1.37 -1.1	-1.3 -0.32
$i_{o_dq}^j$	1.4 -2.2	-1.3 -0.8
$v_{c_dq}^j$	392 -8.6	400 -2.5
$i_{1_DQ}^j$	22.9 2.9	1.3 3.1
$i_{2_DQ}^j$	1.4 -1.3	-1.3 -0.8
$i_{L_DQ}^j$	24.3 0	25 1.2
$v_{o_DQ}^j$	390 -9	400 -2.1
$v_{L_DQ}^j$	398 -9.3	400 -1.7
Parameter	Value	
$x_{3_dq}^{dg}$	397.3	46.7
$v_{i_dq}^{dg}$	396.7	64.4
$i_{i_dq}^{dg}$	25.1	1.5
$i_{o_dq}^{dg}$	25.2	0.4
$v_{c_dq}^{dg}$	398	4
v_{DC}	593.3	
v_{g_DQ}	400	0

APPENDIX B STATE-SPACE EQUATIONS

The output voltage v_{rms}^j is calculated in (20), where $j \in \{A, B\}$, and used in the droop controllers (3) and (1) to calculate P^* and Q^* .

$$v_{rms}^j = \sqrt{\frac{d\{v_{o_dq}^j\}^2 + q\{v_{o_dq}^j\}^2}{3}} \quad (20)$$

The Droop and power controllers (2) - (5) are linearized and integrated into a Jacobian matrix. Which will be different if for the side controlling the DC-link voltage.

$$J^A = J_{vodq^A}^{idq^A} + J_{PQ^A}^{idq^A} \cdot J_{vodq^A}^{PQ^A} \quad (21)$$

$$J^B = J_{vodq^B}^{idq^B} + J_{PQ^B}^{idq^B} \cdot J_{vodq^B}^{PQ^B} \quad (22)$$

$$J_{dq} = \begin{bmatrix} J_{(1)}^B & J_{(2)}^B \end{bmatrix} \quad J_{DC} = \begin{bmatrix} J_{(3)}^B \end{bmatrix} \quad (23)$$

$$J_{vodq^A}^{idq^A} = \begin{bmatrix} J_{vodq_{11}^A}^{idq^A} & J_{vodq_{12}^A}^{idq^A} \\ J_{vodq_{21}^A}^{idq^A} & J_{vodq_{22}^A}^{idq^A} \end{bmatrix} \quad (24)$$

$$J_{vodq^B}^{idq^B} = \begin{bmatrix} J_{vodq_{11}^B}^{idq^B} & J_{vodq_{12}^B}^{idq^B} & 0 \\ J_{vodq_{21}^B}^{idq^B} & J_{vodq_{22}^B}^{idq^B} & 0 \end{bmatrix} \quad (25)$$

$$\begin{aligned} J_{vodq_{11}^j}^{idq^j} &= -J_{vodq_{22}^j}^{idq^j} \\ &= \frac{P^{*j}(q\{v_{o_dq}^j\}^2 - d\{v_{o_dq}^j\}^2) - 2Q^{*j}d\{v_{o_dq}^j\}q\{v_{o_dq}^j\}}{(d\{v_{o_dq}^j\}^2 + q\{v_{o_dq}^j\}^2)^2} \end{aligned} \quad (26)$$

$$\begin{aligned} J_{vodq_{12}^j}^{idq^j} &= -J_{vodq_{21}^j}^{idq^j} \\ &= \frac{Q^{*j}(d\{v_{o_dq}^j\}^2 - q\{v_{o_dq}^j\}^2) - 2P^{*j}d\{v_{o_dq}^j\}q\{v_{o_dq}^j\}}{(d\{v_{o_dq}^j\}^2 + q\{v_{o_dq}^j\}^2)^2} \end{aligned} \quad (27)$$

$$J_{vodq^A}^{PQ^A} = \frac{-1}{\sqrt{3(d\{v_{o_dq}^A\}^2 + q\{v_{o_dq}^A\}^2)}} \begin{bmatrix} k_p^A \cdot d\{v_{o_dq}^A\} & k_p^A \cdot q\{v_{o_dq}^A\} \\ k_q^A \cdot d\{v_{o_dq}^A\} & k_q^A \cdot q\{v_{o_dq}^A\} \end{bmatrix} \quad (28)$$

$$J_{vodq^B}^{PQ^B} = \begin{bmatrix} 0 & 0 & -B_0^{-1} \\ -\frac{k_q^B \cdot d\{v_{o_dq}^B\}}{B_0} & -\frac{k_q^B \cdot q\{v_{o_dq}^B\}}{B_0} & 0 \end{bmatrix} \quad (29)$$

$$B_0 = \sqrt{3(d\{v_{o_dq}^B\}^2 + q\{v_{o_dq}^B\}^2)}$$

$$J_{PQ^j}^{idq^j} = \frac{1}{d\{v_{o_dq}^j\}^2 + q\{v_{o_dq}^j\}^2} \begin{bmatrix} d\{v_{o_dq}^j\} & q\{v_{o_dq}^j\} \\ q\{v_{o_dq}^j\} & -d\{v_{o_dq}^j\} \end{bmatrix} \quad (30)$$

where $j \in \{A, B\}$.

The transformation matrices are:

$$T^j = \begin{bmatrix} \cos(\delta^j) & \sin(\delta^j) \\ -\sin(\delta^j) & \cos(\delta^j) \end{bmatrix} \quad (31)$$

where $j \in \{A, B, dg\}$.

Where the values of $\delta^{\{A,B\}}$ are the difference between the angle in the PCC_{SOP} , of sides A and B , and the angle in SS . Meanwhile, δ^{dg} is the difference between the angle in PCC_{load} and SS .

The matrices for the current controllers are:

$$A_{CC}^j = \begin{bmatrix} 0 & \omega & (\omega_r^j)^2 & 0 \\ -\omega & 0 & 0 & (\omega_r^j)^2 \\ -1 & 0 & -2\omega_c^j & \omega \\ 0 & -1 & -\omega & -2\omega_c^j \end{bmatrix} \quad C_{CC}^j = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (32)$$

$$B_{CC}^j = 2k_{ir}^j \omega_c^j \cdot \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \quad D_{CC}^j = k_{pr}^j \cdot \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \quad (33)$$

where $j \in \{A, B\}$.

The inverter dynamic is described by the transfer function (34), where s is the Laplace variable.

$$H(s) = \frac{v_{i,dq}^j}{v_{i,dq}^{*j}} = \frac{1}{1 + 1.5T_s s} \quad (34)$$

where T_s is the sampling time and $j \in \{A, B\}$. It will be set as $T_s = \frac{1}{f_s}$, where f_s is the switching frequency of the inverters. The matrices of the system are shown in (35), where $j \in \{A, B\}$

$$A_{delay}^j = \begin{bmatrix} -\frac{1}{1.5T_s} & \omega \\ -\omega & -\frac{1}{1.5T_s} \end{bmatrix} \quad B_{delay}^j = \begin{bmatrix} \frac{1}{1.5T_s} & 0 \\ 0 & \frac{1}{1.5T_s} \end{bmatrix} \quad (35)$$

The matrices of the LCL SOP filter are:

$$A_{LCL}^j = \begin{bmatrix} -\frac{R_i^j}{L_i^j} & \omega & 0 & 0 & -\frac{1}{L_i^j} & 0 \\ -\omega & -\frac{R_i^j}{L_i^j} & 0 & 0 & 0 & -\frac{1}{L_i^j} \\ 0 & 0 & -\frac{R_g^j}{L_g^j} & \omega & \frac{1}{L_g^j} & 0 \\ 0 & 0 & -\omega & -\frac{R_g^j}{L_g^j} & 0 & \frac{1}{L_g^j} \\ Z_0 & R_d^j \omega & Z_1 & -R_d^j \omega & Z_2 & \omega \\ -R_d^j \omega & Z_0 & R_d^j \omega & Z_1 & -\omega & Z_2 \end{bmatrix} \quad (36)$$

$$Z_0 = \frac{1}{C_f^j} - \frac{R_d^j R_i^j}{L_i^j} \quad Z_1 = -\frac{1}{C_f^j} + \frac{R_d^j R_g^j}{L_g^j} \quad Z_2 = -\frac{R_d^j}{L_i^j} - \frac{R_d^j}{L_g^j} \quad (37)$$

$$B_{LCL}^j = \begin{bmatrix} \frac{1}{L_i^j} & 0 & 0 & 0 \\ 0 & \frac{1}{L_i^j} & 0 & 0 \\ 0 & 0 & -\frac{1}{L_g^j} & 0 \\ 0 & 0 & 0 & -\frac{1}{L_g^j} \\ \frac{R_d^j}{L_i^j} & 0 & \frac{R_d^j}{L_g^j} & 0 \\ 0 & \frac{R_d^j}{L_i^j} & 0 & \frac{R_d^j}{L_g^j} \end{bmatrix} \quad j \in \{A, B\} \quad (38)$$

The matrices of the Lines and Loads are:

$$A_{RL}^j = \begin{bmatrix} -\frac{R_1^j}{L_1^j} & \omega & 0 & 0 & 0 & 0 \\ -\omega & -\frac{R_1^j}{L_1^j} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{R_2^j}{L_2^j} & \omega & 0 & 0 \\ 0 & 0 & -\omega & -\frac{R_2^j}{L_2^j} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{R_L^j}{L_L^j} & \omega \\ 0 & 0 & 0 & 0 & -\omega & -\frac{R_L^j}{L_L^j} \end{bmatrix} \quad (39)$$

$$B_{RL}^j = \begin{bmatrix} \frac{1}{L_1^j} & 0 & 0 & 0 & -\frac{1}{L_1^j} & 0 \\ 0 & \frac{1}{L_1^j} & 0 & 0 & 0 & -\frac{1}{L_1^j} \\ 0 & 0 & \frac{1}{L_2^j} & 0 & -\frac{1}{L_2^j} & 0 \\ 0 & 0 & 0 & \frac{1}{L_2^j} & 0 & -\frac{1}{L_2^j} \\ 0 & 0 & 0 & 0 & \frac{1}{L_L^j} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{L_L^j} \end{bmatrix} \quad (40)$$

where $j \in \{A, B\}$.

The virtual capacitor matrices are:

$$A_{VC}^j = \begin{bmatrix} 0 & \omega & 0 & 0 \\ -\omega & 0 & 0 & 0 \\ 0 & 0 & -\omega & 0 \\ 0 & 0 & 0 & -\omega \end{bmatrix} \quad (41)$$

$$B_{VC}^A = \frac{1}{C_{VC}} \begin{bmatrix} 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & -1 \end{bmatrix} \quad (42)$$

$$B_{VC}^B = \frac{1}{C_{VC}} \begin{bmatrix} 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & -1 \end{bmatrix} \quad (43)$$

where $j \in \{A, B\}$.

The DG matrices are the following:

$$J_{11}^{dg} = -J_{22}^{dg} = \frac{(q\{v_{L,dq}\}^2 - d\{v_{L,dq}\}^2)P^{*dg} - 2d\{v_{L,dq}\}q\{v_{L,dq}\}Q^{*dg}}{(d\{v_{L,dq}\}^2 + q\{v_{L,dq}\}^2)^2} \quad (44)$$

$$J_{12}^{dg} = J_{21}^{dg} = \frac{(d\{v_{L,dq}\}^2 - q\{v_{L,dq}\}^2)Q^{*dg} - 2d\{v_{L,dq}\}q\{v_{L,dq}\}P^{*dg}}{(d\{v_{L,dq}\}^2 + q\{v_{L,dq}\}^2)^2} \quad (45)$$

$$A_{CC}^{dg} = \begin{bmatrix} 0 & \omega \\ -\omega & 0 \end{bmatrix} \quad B_{CC}^{dg} = k_i^{dg} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \quad (46)$$

$$C_{CC}^{dg} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad D_{CC}^{dg} = \begin{bmatrix} k_p^{dg} & 0 & -k_p^{dg} & -\omega L_i \\ 0 & k_p^{dg} & \omega L_i & -k_p^{dg} \end{bmatrix} \quad (47)$$

$$A_{LCL}^{dg} = \begin{bmatrix} -\frac{R_i^{dg}}{L_i^{dg}} & \omega & 0 & 0 & -\frac{1}{L_i^{dg}} & 0 \\ -\omega & -\frac{R_i^{dg}}{L_i^{dg}} & 0 & 0 & 0 & -\frac{1}{L_i^{dg}} \\ 0 & 0 & -\frac{R_g^{dg}}{L_g^{dg}} & \omega & \frac{1}{L_g^{dg}} & 0 \\ 0 & 0 & -\omega & -\frac{R_g^{dg}}{L_g^{dg}} & 0 & \frac{1}{L_g^{dg}} \\ \frac{1}{C_f^{dg}} & 0 & -\frac{1}{C_f^{dg}} & 0 & 0 & \omega \\ 0 & \frac{1}{C_f^{dg}} & 0 & -\frac{1}{C_f^{dg}} & -\omega & 0 \end{bmatrix} \quad (48)$$

$$B_{LCL}^{dg} = \begin{bmatrix} \frac{1}{L_i^{dg}} & 0 & 0 & 0 \\ 0 & \frac{1}{L_i^{dg}} & 0 & 0 \\ 0 & 0 & -\frac{1}{L_g^{dg}} & 0 \\ 0 & 0 & 0 & -\frac{1}{L_g^{dg}} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (49)$$

The DC-Link matrices are:

$$A_{DC} = [0] \quad B_{DC} = \begin{bmatrix} -\frac{3}{4C_{DC}} & -\frac{3}{4C_{DC}} \end{bmatrix} \quad (50)$$

If the numbers in the subscripts indicates the columns of its matrix, the state space matrices are the following:

$$A^A = \begin{bmatrix} A_{CC}^A & 0_{4 \times 2} & B_{CC(2)}^A & 0_{4 \times 10} & A_{auxA1} \\ B_{delay}^A C_{CC}^A & A_{delay}^A & B_{delay}^A D_{CC(2)}^A & 0_{2 \times 10} & A_{auxA2} \\ 0_{6 \times 2} & B_{LCL(1)}^A & A_{LCL}^A & 0_{6 \times 6} & A_{auxA3} \\ 0_{6 \times 4} & 0_{6 \times 4} & 0_{6 \times 4} & A_{RL}^A & B_{RL(2,3)}^A \\ 0_{4 \times 8} & B_{VC(1)}^A T^A & 0_{4 \times 2} & B_{VC(2,3,4)}^A & A_{VC}^A \end{bmatrix} \quad (51)$$

$$A_{auxA1} = [B_{CC(1)}^A J^A (T^A)^{-1} \quad 0_{4 \times 2}] \quad (52)$$

$$A_{auxA2} = [B_{delay}^A D_{CC(1)}^A J^A (T^A)^{-1} \quad 0_{2 \times 2}] \quad (53)$$

$$A_{auxA3} = [B_{LCL(2)}^A (T^A)^{-1} \quad 0_{6 \times 2}] \quad (54)$$

$$B^A = \begin{bmatrix} 0_{12 \times 2} \\ B_{RL(1)}^A \\ 0_{4 \times 2} \end{bmatrix} \quad (55)$$

$$A^B = \begin{bmatrix} A_{CC}^B & 0_{4 \times 2} & B_{CC(2)}^B & 0_{4 \times 10} \\ B_{delay}^B C_{CC}^B & A_{delay}^B & B_{delay}^B D_{CC(2)}^B & 0_{2 \times 10} \\ 0_{6 \times 4} & B_{LCL(1)}^B & A_{LCL}^B & 0_{6 \times 6} \\ 0_{6 \times 4} & 0_{6 \times 4} & 0_{6 \times 4} & A_{RL}^B \\ 0_{4 \times 8} & B_{VC(1)}^B T^B & 0_{4 \times 2} & B_{VC(2,3,4)}^B \\ 0_{2 \times 5} & 0_{2 \times 5} & 0_{2 \times 5} & 0_{2 \times 5} \\ 0_{2 \times 5} & 0_{2 \times 5} & 0_{2 \times 5} & 0_{2 \times 5} \\ 0_{6 \times 5} & 0_{6 \times 5} & 0_{6 \times 5} & 0_{6 \times 5} \end{bmatrix} \quad (56)$$

$$\begin{bmatrix} B_{CC(1)}^B J_{dq}(T^B)^{-1} & 0_{4 \times 4} & 0_{4 \times 4} & 0_{2 \times 4} \\ B_{delay}^B D_{CC(1)}^B J_{dq}(T^B)^{-1} & 0_{2 \times 4} & 0_{2 \times 4} & 0_{2 \times 4} \\ B_{LCL(2)}^B (T^B)^{-1} & 0_{6 \times 4} & 0_{6 \times 4} & 0_{6 \times 4} \\ B_{RL(2)}^B & B_{RL(3)}^B & 0_{6 \times 5} & 0_{6 \times 5} \\ A_{VC}^B & 0_{4 \times 6} & B_{VC(5)}^B T^{dg} & 0_{4 \times 2} \\ B_{CC(1)}^{dg} J^{dg}(T^{dg})^{-1} & A_{CC}^{dg} & 0_{2 \times 2} & A_{auxB1} \\ B_{delay}^{dg} D_{CC(1)}^{dg} J^{dg}(T^{dg})^{-1} & B_{delay}^{dg} C_{CC}^{dg} & A_{delay}^{dg} & A_{auxB2} \\ B_{LCL(2)}^{dg} (T^{dg})^{-1} & 0_{6 \times 2} & B_{LCL(1)}^{dg} & A_{LCL}^{dg} \end{bmatrix}$$

$$A_{auxB1} = \begin{bmatrix} B_{CC(2)}^{dg} & 0_{2 \times 4} \end{bmatrix} \quad (57)$$

$$A_{auxB2} = \begin{bmatrix} B_{delay}^{dg} D_{CC(2)}^{dg} & 0_{2 \times 4} \end{bmatrix} \quad (58)$$

$$B^B = \begin{bmatrix} 0_{4 \times 2} & B_{CC(1)}^B J^{DC} \\ 0_{2 \times 2} & B_{delay}^B D_{CC(1)}^B J^{DC} \\ 0_{6 \times 2} & 0_{6 \times 2} \\ B_{RL(1)}^B & 0_{6 \times 2} \\ 0_{14 \times 2} & 0_{14 \times 2} \end{bmatrix} \quad (59)$$

Finally, the state space vectors and matrices of the system are:

$$\Delta x^j = [\Delta x_{1,dq}^j \Delta x_{2,dq}^j \Delta v_{i,dq}^j \Delta i_{i,dq}^j \Delta i_{o,dq}^j \Delta v_{c,dq}^j \Delta i_{1-DQ}^j \Delta i_{2-DQ}^j \Delta i_{L-DQ}^j \Delta v_{o-DQ}^j \Delta v_{L-DQ}^j] \quad (60)$$

$$\Delta x^{dg} = [\Delta x_{3,dq}^{dg} \Delta v_{i,dq}^{dg} \Delta i_{i,dq}^{dg} \Delta i_{o,dq}^{dg} v_{c,dq}^{dg}] \quad (61)$$

$$A = \begin{bmatrix} A^A & 0_{22 \times 32} & 0_{22 \times 1} \\ 0_{32 \times 22} & A^B & X^0 \\ X^1 & X^2 & X^3 \end{bmatrix} \quad (62)$$

where $A \in \mathbb{R}^{55 \times 55}$, $A^A \in \mathbb{R}^{22 \times 22}$, $A^B \in \mathbb{R}^{32 \times 32}$, $j \in \{A, B\}$, $X^0 \in \mathbb{R}^{32 \times 1}$, $X^1 \in \mathbb{R}^{1 \times 22}$, $X^2 \in \mathbb{R}^{1 \times 32}$, $X^3 \in \mathbb{R}$.

$$X^0 = \begin{bmatrix} B_{CC(1,2)}^B J_{DC} \\ B_{delay}^B D_{CC(1)}^B J_{DC} \\ 0_{26 \times 2} \end{bmatrix} \quad (63)$$

$$X^1 = [0_{1 \times 6} \quad B_{DC(1)} \quad 0_{1 \times 14}] \quad (64)$$

$$X^2 = [0_{1 \times 6} \quad B_{DC(2)} \quad 0_{1 \times 24}] \quad (65)$$

$$X^3 = 0 \quad (66)$$

$$B = \begin{bmatrix} B^A \\ B^{B2} \\ 0_{1 \times 2} \end{bmatrix} \quad B^{B2} = \begin{bmatrix} 0_{2 \times 12} \\ B_{RL(1,2)}^B \\ 0_{2 \times 14} \end{bmatrix} \quad (67)$$

where $B \in \mathbb{R}^{55 \times 2}$, $B^A \in \mathbb{R}^{22 \times 2}$, $B^{B2} \in \mathbb{R}^{32 \times 2}$.